Cooperative dynamics of active heterogeneous systems

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Abstract:

Collective motion is fundamental in flocking or schooling dynamics and represent one of the most fascinating sights in nature. In this paper, to describe collective motion, we use one of the most representative models called the Vicsek Model. The main features of this model is a noise-driven phase transition between ordered and disordered phase and its mathematical and computational simplicity. We reproduce its basic results and continue our work adding a leader to study how leadership affects its dynamics, showing that this new ingredient breaks the phase transition.

I. INTRODUCTION

In nature, collective motion plays an important role in life being behavior. Examples of that fact are seen worldwide and are well-known since it can be seen in our everyday life. Representative examples, which highlight that fact, are schools of fish providing protection or outmaneuvering, in a matter of seconds, a predator, a flock of birds moving around or migrating in formation or a sheep herd.

Although those are the most intuitive examples, there is a wide range of other systems where collective motion takes place. From an overcrowded street to cells or bacteria, even non-living systems show that kind of collective phenomena [1], like micrometer-sized silver chloride (AgCl) particles acting as an autonomous micro-motor in an special environmental conditions [2].

This wide range of scales and systems got the attention of researchers from several fields. Biologists were the first who started studying that phenomena [3], followed by computer graphics researchers, like C.W. Reynolds, who created realistic animation of birds flocking by setting basic rules like collision avoidance and velocity matching, for instance [4]. However, it was not until T.Vicsek proposed his model that the physics community took interest in that field [5].

T.Vicsek presented a model where each individual is constantly updating its velocity direction by averaging with its closest neighbors, being the system fully determined by the position and velocity direction of each particle [5]. From a mathematical and computational point of view, Vicsek model is quite simple and doesn't require big computational efforts and from a physical view, this model shows a phase transition between ordered and disordered phase, which makes this model really interesting for a physicist. For those reasons, we decided to use Vicsek Model to study collective motion.

The structure of our work is the following: in Section II, we explain how Vicsek Model works, the way we implemented it and finally we present its basic results. After that, in Sec. III we introduce a new role (leader) in our system and explore how our system evolves and if the phase transition still takes place in function of the fraction of followers.

II. STANDARD VICSEK MODEL

The Vicsek Model is based in N self-propelled particles which are able to move in a d-dimensional box of sides L with constant speed v_0. Each particle interacts with the other ones through averaging its velocity direction with other particles inside a range of interaction defined in terms of an euclidean distance of radius R_0. Since perfect interaction is not realistic, a noise term η is introduced in order to simulate difficulty in gathering or processing information, etc.

In a 2-dimensional box, every particle is characterized by two variables, its position $\mathbf{r_i(t)}$ and its velocity direction, which can be represented with only one angle $\boldsymbol{\theta}\mathbf{(t)}$ arbitrarily chosen between $[-\pi , \pi]$. By knowing how both parameters evolve over time, we can fully determine our system's evolution. Then, given that each particle dynamics depend just on its velocity, its future position can be determined following the Eq.(1):

$$\mathbf{r_i}(t+\Delta t) = \mathbf{r_i}(t) + \mathbf{v_i}(t)\Delta t, \qquad (1)$$

where Δt represents our integration time step and $\mathbf{v_i}$ the velocity at time t, which can be decomposed in terms of $\boldsymbol{\theta}\mathbf{(t)}$:

$$\mathbf{v_i}(t) = v_0(cos(\theta_i(t)), sin(\theta_i(t))) \qquad (2)$$

It must be remarked that the only time dependence is found in $\boldsymbol{\theta}_i(t)$ so it is the parameter which rules the motion. The conditions that defines $\boldsymbol{\theta}_i(t)$'s time dependence are two, the way interaction between particles is summarized and the impossibility of perfect alignment. As said before, each particle averages its direction with all individuals inside its radios of interaction with a noise term added to fulfill the second condition. Those conditions lead to:

$$\theta_i(t+\Delta t) = arg(\sum_{j\in R_0} \mathbf{v}_j(t)) + \eta\xi_i \qquad (3)$$

where $\boldsymbol{\eta}$ is a parameter which allows us to control the noise intensity and ranges from 0 to 1, $\boldsymbol{\xi}$ is an uniformly distributed random number set between $[-\pi ,\pi]$ and arg(...)=arctan(v_y / v_x) is the velocity direction.

At this point, we need to define a suitable order parameter so as to capture the statistical information of the collective motion and to analyze the transition between ordered and disordered phase. A possibility is to define it as shown in Eq.(4):

$$|\phi(t)| = \frac{1}{Nv_0}|\sum_{i=1}^{N} \mathbf{v}_i(t)| \qquad (4)$$

This order parameter gives us information about the disorder of our system in function of η.

On the one hand, when η is equal to 0, we reach the maximum order possible because all particles are aligned exactly to the same direction. Therefore, there is no cancellation in the summation, so our order parameter reaches its maximum value $\phi = 1$. However, if we increase slightly the noise intensity η, that perfect alignment is broken, although all particles move approximately at the same direction, as seen in Fig.(1). Because of that, there are some cancellation in the sum and our order parameter decreases its values.

On the other hand, when we have a high noise intensity η, the term $\eta\ \xi_i(t)$ introduces a huge distortion to the alignment and produces a random distribution of each particle's direction. That produces the cancellation of the sum, which makes the order parameter vanish, reaching $\phi = 0$ when $\eta = 1$.

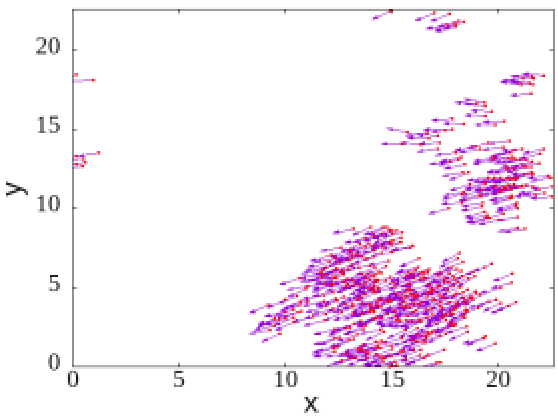

FIG. 1: Final ordered configuration when η=0.05.

Finally, one can estimate the error of the order parameter by computing its variance as follows:

$$Var = <\phi^2> - <\phi>^2 \qquad (5)$$

Once we have already talked about Vicsek Model from a dynamical and statistical point of view, we must concert about the computational strategy to follow:

First of all we must initialize our system by setting N particles moving with velocity $v_0 = 0.3$ inside a 2-D box of side L, with density $\rho = N/L^2$. Given that there is no privileged position nor direction, we must set them uniformly distributed ranging all possible values [0,L] for the positions and $[-\pi , \pi]$ for the velocity direction.

Secondly, because of the fact that we are working with a finite sized system, we need to reduce finite size effects. To do so, we work with periodic boundary conditions (PBC) where a particle that exits from one side, enter form the opposite one, fulfilling the conservation of the number of particles.

Finally, when considering the distance between particles, in order to know if a particle is inside the range of interaction, one must take into account PBC since particles that are at a distance greater than L/2 are, in fact, at a shorter distance when considering PBC. For instance, consider two particles in the opposite frontier of the box, its distance is greater than L/2 but they could interact because of the PBC.

At this point, we begin our simulations by studying the order parameter in function of the noise intensity η for different sizes but with constant density $\rho = 1$.

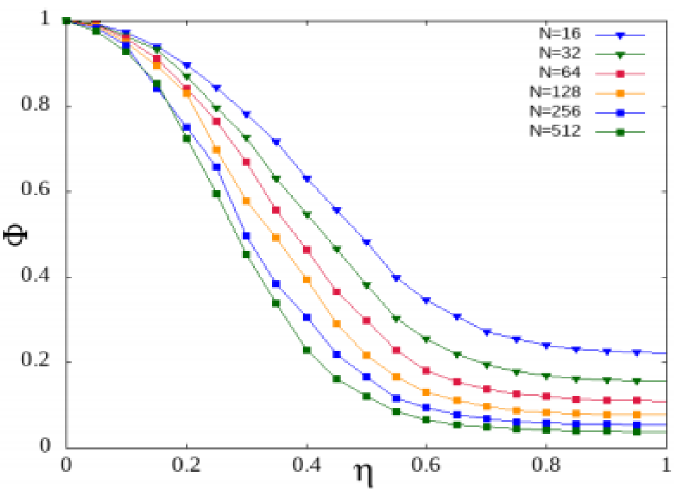

FIG. 2: Order parameter in function of the noise intensity for different sizes, with ρ=1.

From Fig.(2) it can be seen how the order parameter evolves in function of η for different sizes. When the noise intensity is low, for all sizes the order parameter is close to 1. While we increase the intensity, the order parameter gets lower in a different way depending on the size. Although for all sizes the order parameter decreases, for bigger systems it decreases faster. It is due to finite size effects. Furthermore, it must be remarked that $\phi(\eta)$ should tend to zero when η tends to 1, however this value is never reached, and not only that, for small sizes, ϕ is not even close to 0. The reason is because when the noise intensity tends to 1, our system's configuration is basically a set of N random vectors, which, from a mathematical point of view, its order parameter goes as $\phi \sim 1/\sqrt{N}$ [6].

To determine the existence of a true phase transition and when it takes place, we must take a close look to the parameter of order and its evolution over time. An evidence of phase transition can be found in the behavior of the order parameter. Fluctuations on it become more and more important while it approaches to the phase transition critical point, being infinite when the transition takes place.

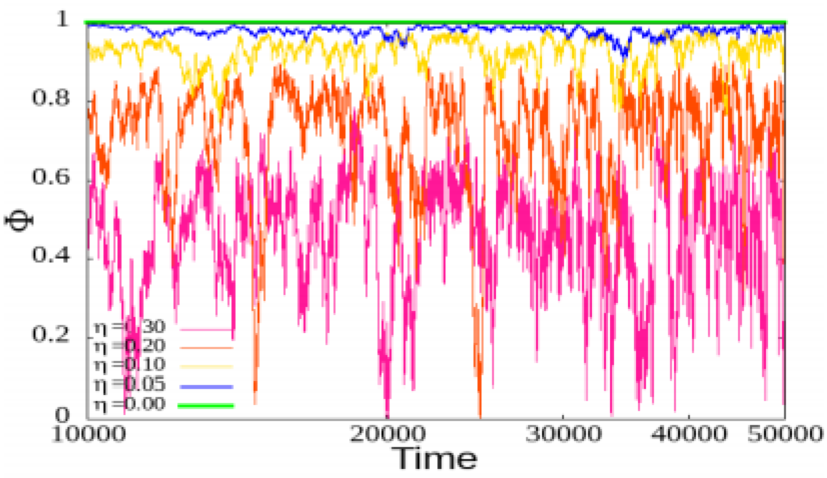

FIG. 3: Order parameter over time with log scale in the x-axis once the system reached an stationary state.

Analyzing Fig.(3), we clearly see how $\phi(t)$ fluctuates when we increase η. When the noise intensity is far from its critical value, it doesn't fluctuates as much as it does while we approach its critical value. When we get closer, its fluctuations start being considerable until it reaches its maximum value at $\eta = \eta_c$. In order to determine η_c with precision, we must quantify the magnitude of the fluctuations. To do so, with analogy with ferromagnetism, we define the susceptibility as in Eq.(6):

$$\chi = NVar \qquad (6)$$

Recalling Eq.(5) we get:

$$\chi = N(<\phi^2> - <\phi>^2) \qquad (7)$$

Plotting χ versus η allows us to get some information about the phase transition. Once again, making the analogy with ferromagnetism, the presence of a peak is a clear evidence of a second-order phase transition between ordered and disordered phase which takes place where the peak is found.

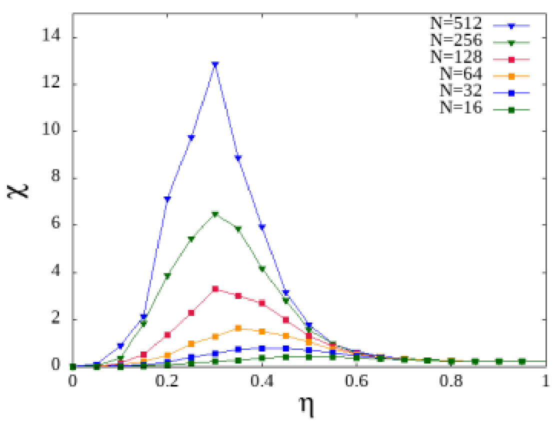

FIG. 4: Susceptibility in function of η for different sizes.

A peak is found in the susceptibility versus noise intensity (Fig.4), being a clear evidence of the existence of a phase transition in the Vicsek Model.

Once we have already reproduced Vicsek's Model results, we can move an step forward by introducing new features in the model. Our first move is to introduce a new role in the system.

III. LEADERSHIP

When considering social interaction, leadership is an important feature to take into consideration. Several species of animals follow a hierarchical structure where some individuals are followed by the rest of the community, playing the role of a leader.

In this part, we are interested in the effect produced by a single leader in the collective motion and, particularly, how the introduction of that new feature affects the behavior of the Standard Vicsek Model from a statistical point of view.

A non-local leader is introduced, in such a way that its followers (informed individuals) interact with it no matter at what distance is found and considering it to have an immutable velocity $\mathbf{v}_{\mathbf{leader}}$. In spite of having this privilege, if an individual is a follower, it interacts with the leader by taking into consideration its direction with the same weight as a normal individual when averaging with its surrounding neighbors, as shown in Eq.(8):

$$\theta_i(t+\Delta t) = arg(\sum_{j\in R_0} \mathbf{v}_j(t) + \epsilon_i \mathbf{v}_{leader}) + \eta\xi_i \qquad (8)$$

where $\boldsymbol{\varepsilon}_{\mathbf{i}}$ is a coefficient equal to 1 if the i-th particle is a follower and 0 otherwise.

With this slight modification, we proceed the same way as in the no-leader case to see if there is a phase transition, it is, we take a close look to the parameter of order behavior and quantify its oscillations via the susceptibility $\boldsymbol{\chi}$.

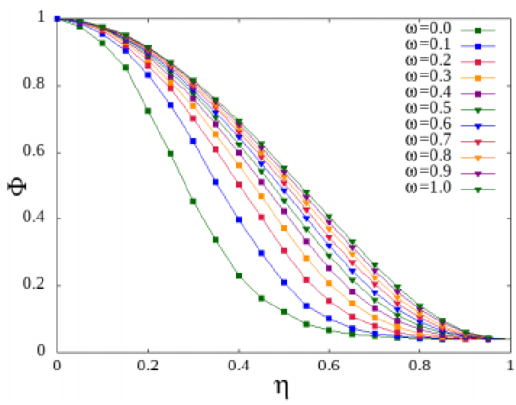

FIG. 5: Order parameter as a function of η for different fractions of informed individuals, with N=512 and ρ=1.

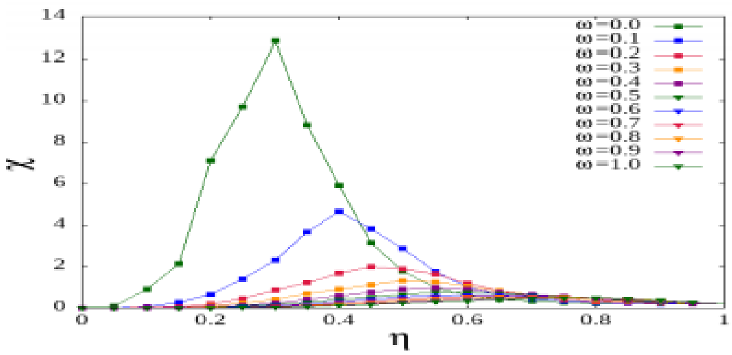

FIG. 6: Susceptibility versus η for different ω with N=512

From FIG.(5) it can be clearly seen that the system increases its order state when increasing the fraction of informed individuals $\boldsymbol{\omega}$. FIG.(6) shows how this peak vanishes when the fraction of informed individuals increase. This fact means that the phase transition is broken and no longer exist or in other words, the system is always in an ordered phase.

IV. CONCLUSION

This work reproduced the behavior of the so called Vicsek Model and proved that there exist a noise-driven phase transition between ordered and disordered phase. Then, a new role was introduced (leader) and it has been seen that the previous phase transition no longer exist, remaining our system in an ordered phase no matter the noise intensity applied.

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