Assignment 3 - Seminar Critical Review: "Basic ideas on the coupling of virtual element and boundary element methods" - Comunication Skills I

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In the reviewed seminar, G.N. Gatica presents his work on coupling virtual element (XEM) and boundary element methods (BEM). His aim is to show how these numerical methods for solving partial differential equations in any domain can combine together, finding that doing so for two dimensional problems, the error of the XEM is reduced by using some terms well known in the BEM, instead of using approximations. The presented method is certainly a step forward into precise randomly meshed methods.

The author begins the seminar describing XEM basic ideas, which are using more general meshes and substituting classic bi-lineal forms for approximated ones. He then shows how to couple XEM and BEM with a simple problem, where a domain is ruled by a Poisson equation and it is surrounded by an other domain ruled by Laplace equation. This problem has two boundaries, one separating the two domains and the other one limiting the outer domain. He solves this problem for both two dimensional and three dimensional cases, using the proposed combined method.

For the two dimensional case, Gatica first uses BEM to find the weak form of the problem and all the equations needed. Then he discretizes the domain using XEM, and he explains that the elements are arbitrary polygons with only special convexity conditions applied. These elements are not known on a explicit way, but it is enough to know the traces and the Laplacian to be able to evaluate the integrals over the elements. Traces are usually approximated in XEM, but now they are given by BEM, and the error is reduced. This is the reason why coupling both methods is interesting. The speaker also shows the error analysis of the method, and points that the method is expensive computationally, but that it is worthed using it when a more general discretization in required.

The reason to explain the three dimensional case is to show how traces are not provided by BEM in this case, and then a different coupling is needed. The researcher

shows how it is possible to combine both methods by introducing the traces and the normal derivatives as unknowns, by analogy with the FEM-BEM coupling, although the performance is not increased as in the two dimensional case.

The XEM is starting to be a popular tool for simulations in some fields, and a method that improves XEM precision is surely useful. Also from the mathematical point of view the coupling on the two dimensional case is elegant and the error analysis shows that this is a well studied work. In conclusion, I consider this method as an important tool to consider when using the virtual elements method.

References

Gabriel N. Gatica, CIMNE Seminar, Basic ideas on the coupling of virtual element and boundary element methods. https://youtu.be/LChg3fZjnPM