Communication Skills 1

Assignment # 2: Extended Abstract

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Communication strategy: Writing an extended abstract implies the existence of question, hypothesis and results subject to discussion. Therefore, I wrote the requested text from a personal, not published document about a problem of mechanics in modelling the arterial segment, which allows the following structure: title, short abstract, introduction, method, results and discussion, conclusion, and references.

Stiffness and elastic collapse analysis of a model for the mechanical deformation of the arterial segment

Abstract

Elastic instability of the arteries is associated to a wide variety of symptoms, some of which require medical intervention. To study this situation, the arterial segment was modelled as a thick walled, anisotropic cylindrical pressure vessel subjected to large strains, where assumption of axisymmetric deformation pattern allowed for a straightforward solution of the static equilibrium equation. Analysis of the stiffness by means of intraluminal pressure-radial deflection suggested that axial stretching has the most influence in this sense. However, in the tendency to elastic collapse, its role depends on the orientation of reinforcing fibres. The study also indicates that, whilst wall thickness is not as relevant in the stiffness behaviour, it certainly is in the collapse scenario.

Key words

Arterial Mechanics, Hyperelasticity, Soft Biological Tissues

List of symbols

R_i, R_o, r_i, r_o	Internal and external radii, not-deformed and deformed configurations.
α, p_i	Opening angle upon longitudinal cut, intraluminal pressure.
λ_z, L	Axial stretch, segment length.
Θ_0, H	Material coordinate in circumferential direction, wall thickness.
c, k_1	Matrix stiffness, fibre stiffness factor.

Introduction

Arterial pseudo-coarctation is a condition associated to its excess length and tortuosities thereof [1], which may lead to serious complications such as aneurysms [2, 3]. This situation has been modelled as the elastic collapse or buckling of a thick walled cylindrical pressure vessel [4, 5]. However, the treatment has separated the collapse itself from the pressure-radial deflection scenario naturally preceding it. One of the challenges in bringing these two aspects together is dealing with the large amount of material and geometric parameters that emerge.

In this work, a dimensional analysis of the pressure-radial deflection study from Barrera [6, 7] allowed identification of the most influential factors. It was then extended to the elastic collapse analysis, where the intersection of the pressure-radial deflection and the critical buckling pressure curves make clear the relative incidence of various factors in this sense.

Method

The arterial segment is deemed to undergo three deflection instances, according to Figure 1. The corresponding axisymmetric deformation gradient tensor is given by:

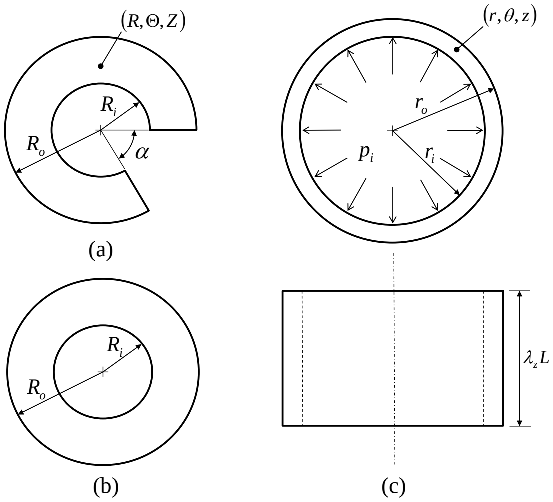


Figure 1. a) Open configuration; b) closed, unloaded configuration; c) configuration loaded due to intraluminal pressure. (R,Θ,Z) and (r,θ,z) are material and spatial coordinates of a material point before and after any deformation.

$$\underline{F} = \begin{bmatrix} \frac{\partial r}{\partial R} & 0 & 0 \\ 0 & \frac{\pi r}{\Theta_0 R} & 0 \\ 0 & 0 & \lambda_z^0 \end{bmatrix}. \tag{1}$$

The deformation energy function was taken from Holzapfel et al. [8], whilst using the deformation gradient along with the stress-strain relations from hyperelasticity [9] yield Cauchy stresses. The deformation pattern makes equilibrium in the radial direction the relevant one, which yields an indirect relation between the intraluminal pressure and radial deflection. This expression is used to analyse the arterial segment stiffness in the light of the model parameters. These were organised in non-dimensional numbers: non-dimensional pressure and radial deflection are given by

$$\Pi_1 = \frac{p_i}{k_1} \quad \Pi_2 = \frac{r_i}{R_i}. \tag{2}$$

Non-dimensional material and parameter factors are the following:

$$\Pi_3 = \frac{c}{k_1} \quad \Pi_4 = \frac{H}{R_i} \quad \Pi_5 = \lambda_z. \tag{3}$$

Extension to the collapse scenario is based upon the typical, Euler column buckling analysis. The balance equation between moment of forces due to support reaction and those due to the wall stresses attained by simultaneous intraluminal pressure-radial deflection, provide the means to produce a relationship between radial deflection and buckling pressure. The intersection of the latter and the pressure-radial deflection indicates the point of actual collapse.

Results and discussion

Pressure-radial deflection curves for the non-dimensional factors, Figure 2, indicate that those belonging to the form Π_5, such as λ_z, have the most influence in the arterial segment stiffness.

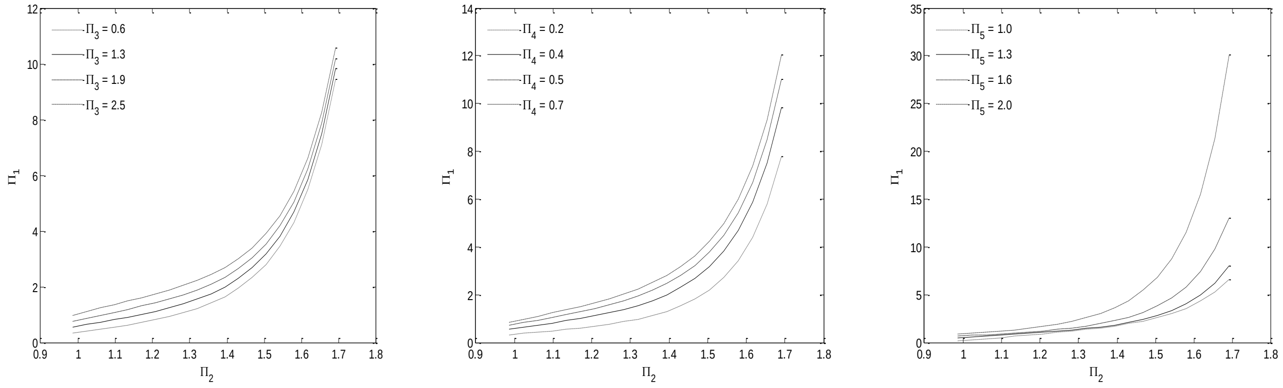
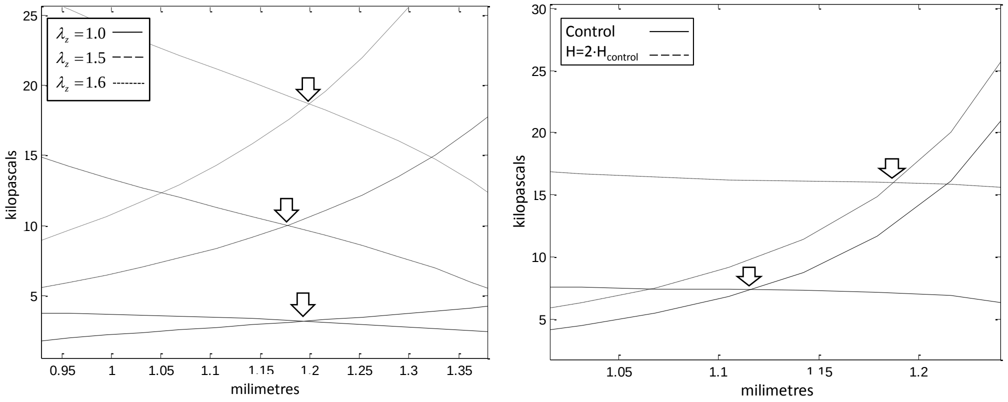


Figure 2. Radial deflection-intraluminal pressure non-dimensional curves for various non-dimensional factors.

In regards of the elastic collapse, the critical pressure and the pressure-radial deflection curves intersection indicates its onset. Although reinforce fibre angles of ~30° indicate that there is a value of λ_z below which the tendency for collapse increases and above which decreases, the reported effect of λ_z decreasing this tendency is recovered after changing the fibre angles to ~60° (Figure 3a). This suggests that the role of λ_z depends on the fibre orientation.



(a)

(b)

Figure 3. a) Collapse pressure-radial deflection for various axial stretches; b) effect of wall thickness in collapse pressure-radial deflection.

Finally, albeit wall thickness H belongs to the group Π_4, not very influential in the arterial stiffness, it has a significant effect in decreasing the tendency to collapse, according to Figure 3b.

Conclusion

The analysis applied produced a conjoint interpretation of the effect of material and geometric factors in both the intraluminal pressure-radial deflection stiffness and the tendency to elastic collapse of the arterial segment. Although λ_z was clearly the most influential factor in arterial radial stiffness, its role in the elastic collapse is mediated by the reinforcing fibre orientation. Then again, the arterial wall thickness, whilst not showing relevance in the pressure-radial deflection curve, can be as influential as λ_z in the tendency to elastic collapse.

The study also sets a frame onto which more sophisticated material models can be easily implemented and analysed, mainly thanks to judicious application of dimensional analysis.

References

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