Computational Mechanics Tools
Course simulation project: Dynamics Analysis of a train wheel with Abaqus

Magdalena Pérez Lanfranco
Samuel Parada Bustelo

Polytechnic University of Catalonia

January 13, 2017

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. References

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. References

Wheel squeal

- Acoustic pollution is one of the most common issues with rail transit in inhabited areas.
- One of the relevant sources of acoustic pollution is the so-called wheel squeal.

Wheel squeal is a high frequency, high pressure level sound caused by the vibration of the wheels that results in unacceptable noisy environments around the rail systems.

- Usually produced between 2 and 8 kHz.
- Characteristics of wheel squeal vary from vehicle to vehicle. It mainly depends on,
  - *Geometric factors*: wheel size and shape, curvature of rail, etc.
  - *Environmental factors*: temperature, humidity of air, etc.
  - *Dynamic factors*: linear velocity, angular velocity,...
- According to [3] , there are up to 60 identified different parameters that can affect wheel squeal.

Outline

1. Introduction
2. **Problem statement**
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. References

- Analyze the dynamic response of a train wheel
- Squeal is generated between 2 and 8 kHz
- Study cases
  - Analyze if some modes of the wheel generate squeal.
  - Coupling between the wheel eigenfrequencies and wheel frequency due to rotation.
  - Coupling between position of sleepers and squeal phenomena.

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. References

The equation of motion is,

Free undamped system

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0} \tag{1}$$

where, in general (multidegree-of-freedom system)

- **M** is the so-called mass matrix
- **K** is the stiffness matrix of the system
- **u** is the displacement vector

- Solution of this equation can be found by assuming a solution of the form

$$\boldsymbol{u}(t) = \boldsymbol{a}\phi(t) \tag{2}$$

where $\boldsymbol{a}$ is a vector of constant parameters and $\phi(t)$ is just a function of time.

- The configuration of the system, given by the vector

$$\boldsymbol{a} = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{Bmatrix}$$

is known as the *mode shapes*.

- Solution procedure $\boldsymbol{u}(t) = \boldsymbol{a}\phi(t) \;\Rightarrow\; \ddot{\boldsymbol{u}} = \boldsymbol{a}\ddot{\phi}(t)$
- Plug this into the former equation, rearrange and after some algebra, it yields

$$-\omega^2 \boldsymbol{M}\boldsymbol{a} + \boldsymbol{K}\boldsymbol{a} = 0 \;\Rightarrow\; \boldsymbol{K}\boldsymbol{a} = \omega^2 \boldsymbol{M}\boldsymbol{a} \;\Rightarrow\; \boldsymbol{M}^{-1}\boldsymbol{K}\boldsymbol{a} = \omega^2 \boldsymbol{a}$$

and this is an eigenvalue problem

Eigenvalue problem

$$\boldsymbol{D}\boldsymbol{a} = \omega^2 \boldsymbol{a} \;\Rightarrow\; det(\boldsymbol{D} - \omega^2 \boldsymbol{I}) = 0 \tag{3}$$

where $\boldsymbol{D} = \boldsymbol{M}^{-1}\boldsymbol{K}$ is the so-called *dynamical matrix* and $\boldsymbol{I}$ is the identity matrix of the required order [5].

- By finding the solution for this problem, we obtain as many couples $\omega, \boldsymbol{a}$ as degrees of freedom of the system.
- ω_i are the eigenvalues or *natural frequencies*.
- $\boldsymbol{a}_i$ are the eigenvectors or *mode shapes*.
- This information could be used to determine if an excitation is close to one of the natural frequencies what could cause the so-called resonance phenomena.

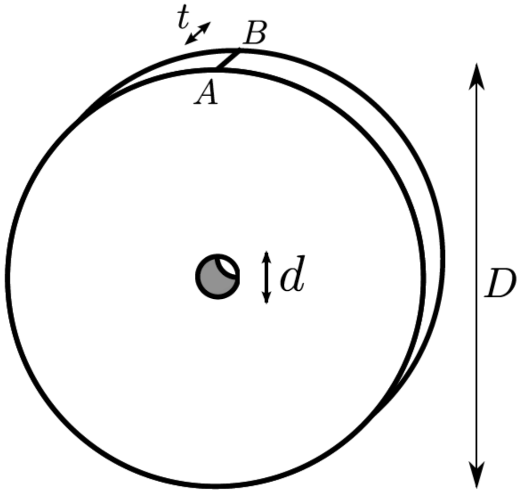
Outline

1 Introduction
2 Problem statement
3 Theoretical Framework
4 Methodology
- Wheel geometry
- Simulation setup
- Boundary conditions
- Mesh

5 Results
- Eigenfrequencies
- Eigenmodes
- Frequency of rotation coupling
- Sleeper contact frequency coupling
- Stick-slip transition wheel/rail

6 Conclusions
7 References

- Wheel data (geometry and materials)



Width	t	[m]	0.05
Internal diameter	d	[m]	0.10
External diameter	D	[m]	1.00
Density	ρ	[Kg/m^3]	7800
Yong Modulus	E	[Pa]	$210E9$
Poisson Ratio	ν	[-]	0.25

Figure 1: Geometry and material data for the train wheel.

- Wheel is sketched using the Abaqus CAD mode.Sketch the 2D model (two concentric circles) and then extrude.
- Create the section and assign the material. In this case, linear elastic behavior is assumed.

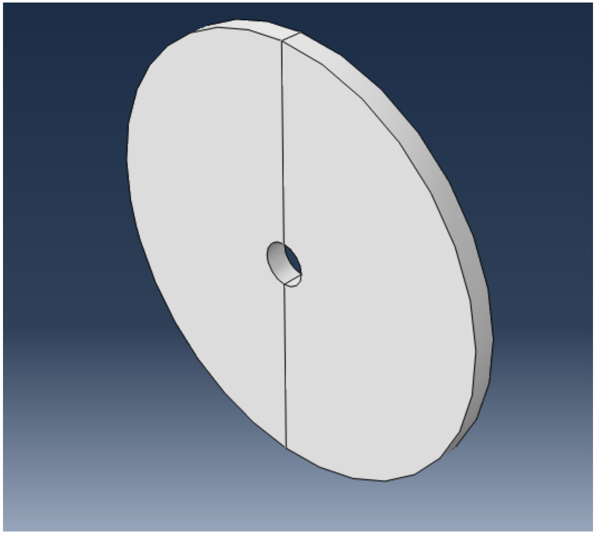


Figure 2: Train wheel CAD model.

- Create a step and set the procedure to "Linear perturbation" and select "Frequency".
- Select value and introduce the desired number of eigenvalues (frequencies), 10

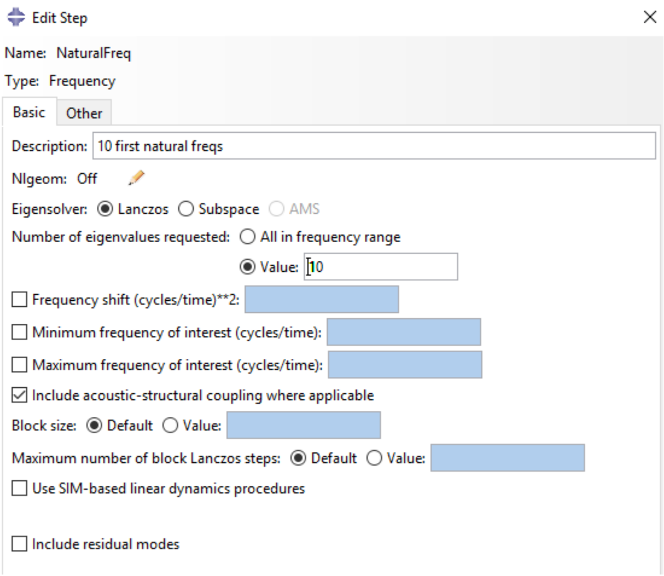


Figure 3: Setup for the eigenvalue problem.

- Assign boundary conditions for the dynamic analysis
  - Prescribe displacement of nodes in the hole ⇒ encastre
  - Prescribe displacement of nodes of line AB ⇒ X-symmetry

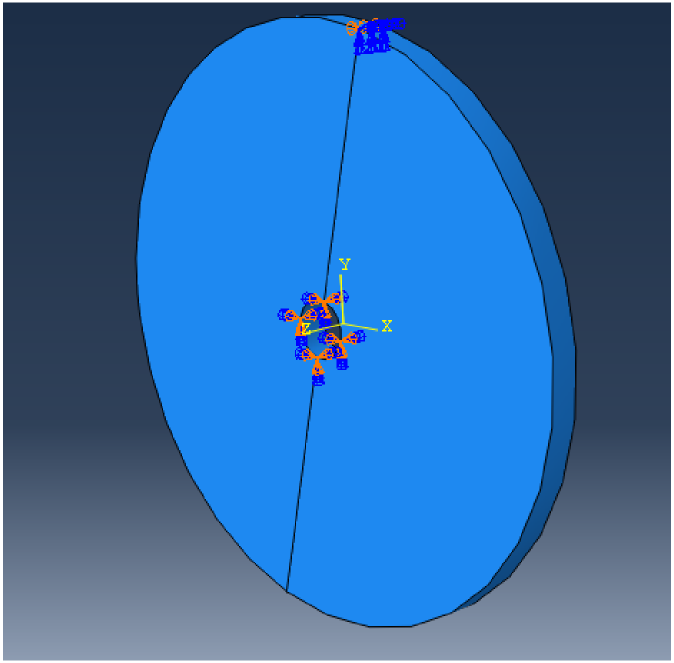


Figure 4: Representation of the boundary conditions

- Wheel is discretized with hexahedra finite elements.
- Use approximate global size equal to 0.05.

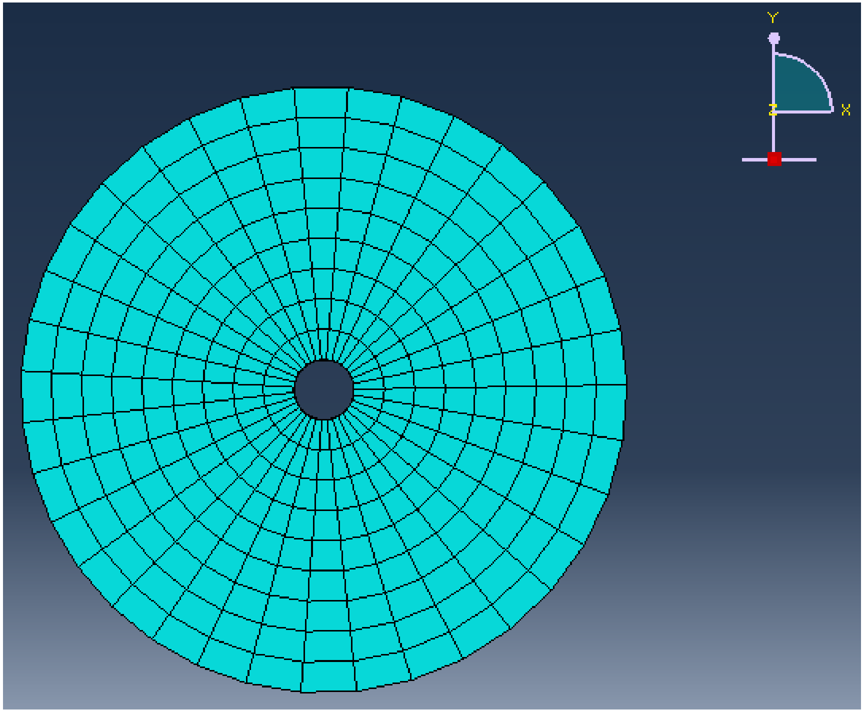


Figure 5: Mesh with hexahedra elements

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. References

- Eigenfrequencies of the wheel are collected in Table 20,

Mode	Frequency $[Hz]$
1	24.759
2	25.116
3	26.256
4	38.749
5	48.386
6	82.899
7	87.851
8	133.11
9	141.63
10	146.99

Table 1: Wheel's natural frequencies

- In the next figures, we show the eigenmodes. Original shape is also included for comparison.

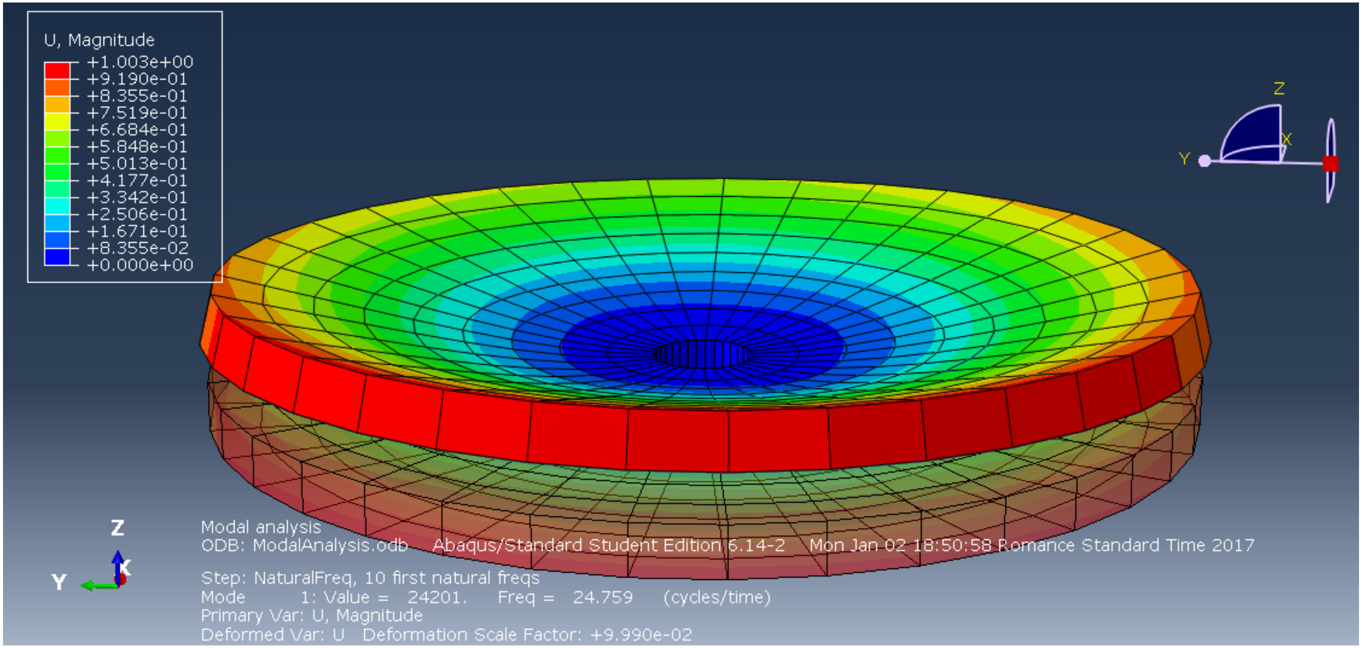


Figure 6: Mode 1

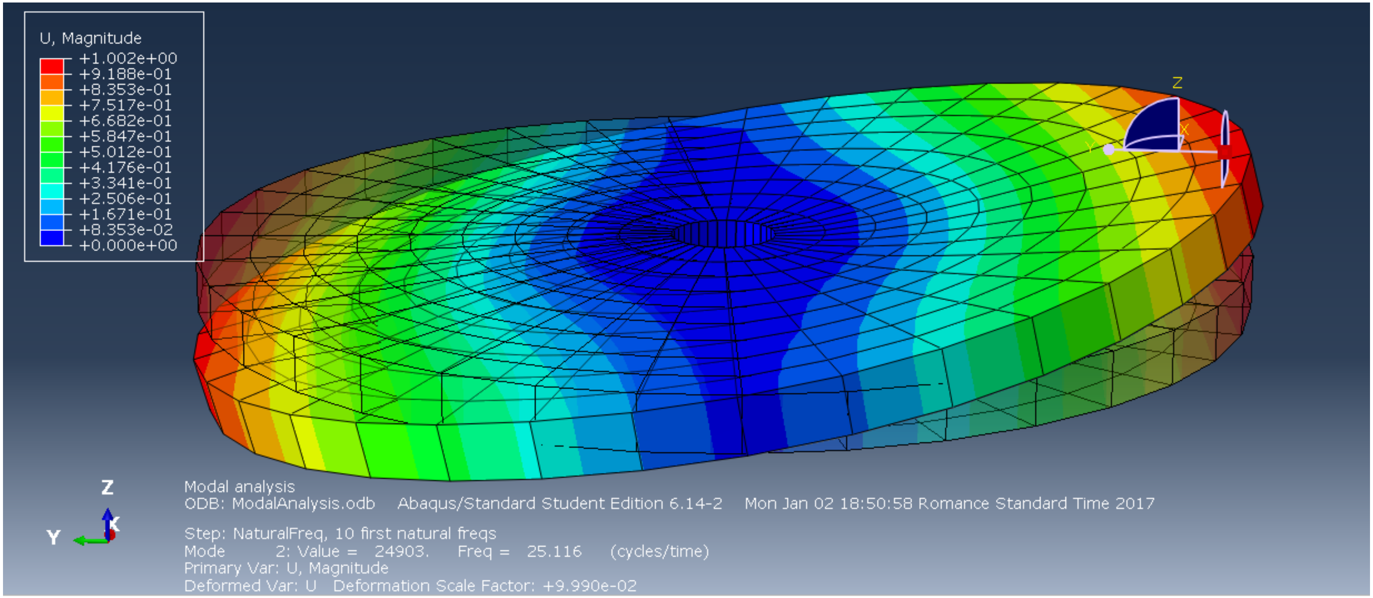


Figure 7: Mode 2

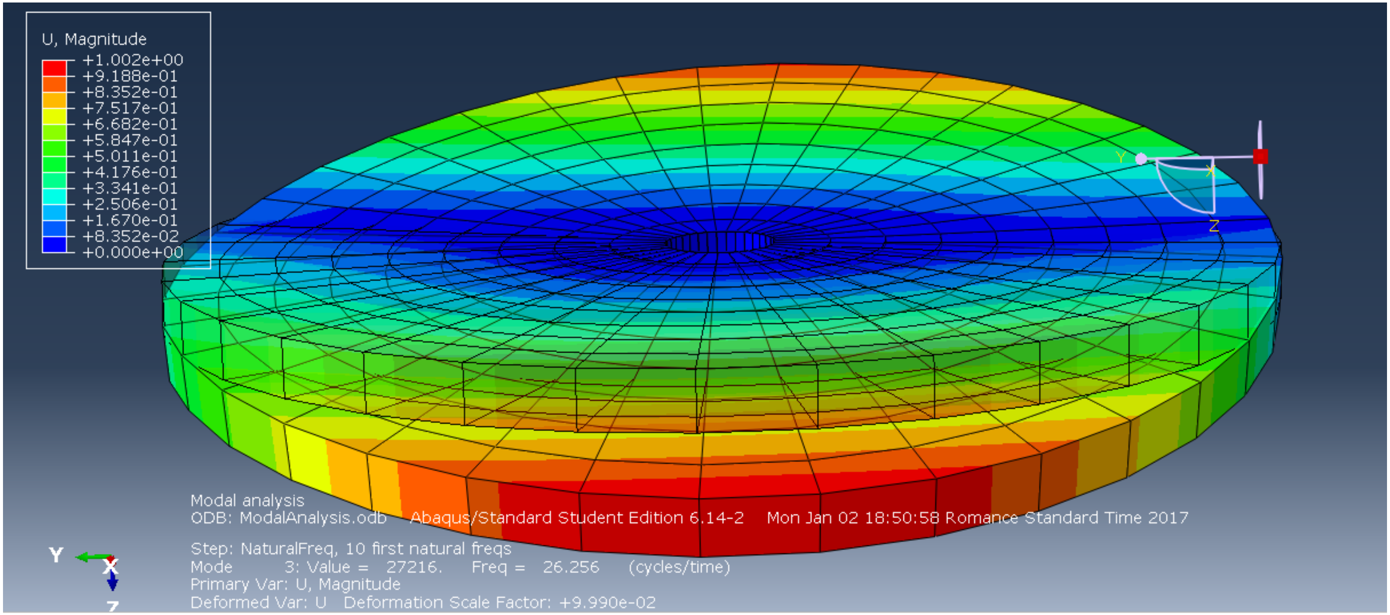


Figure 8: Mode 3

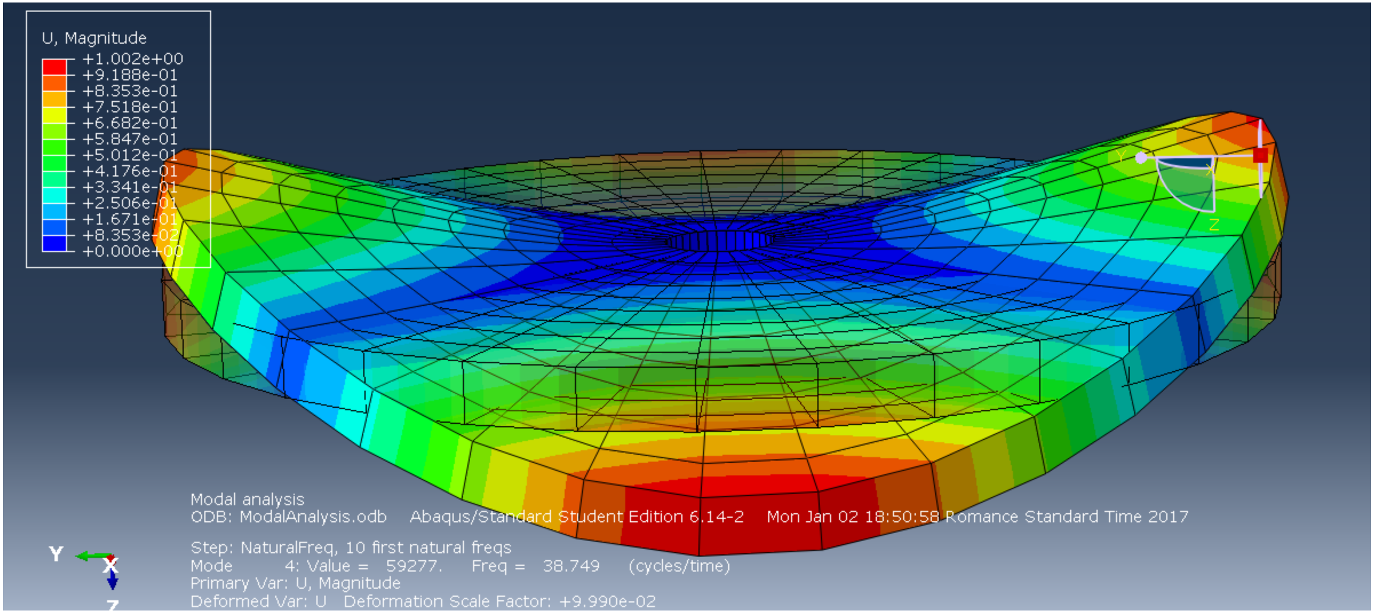


Figure 9: Mode 4

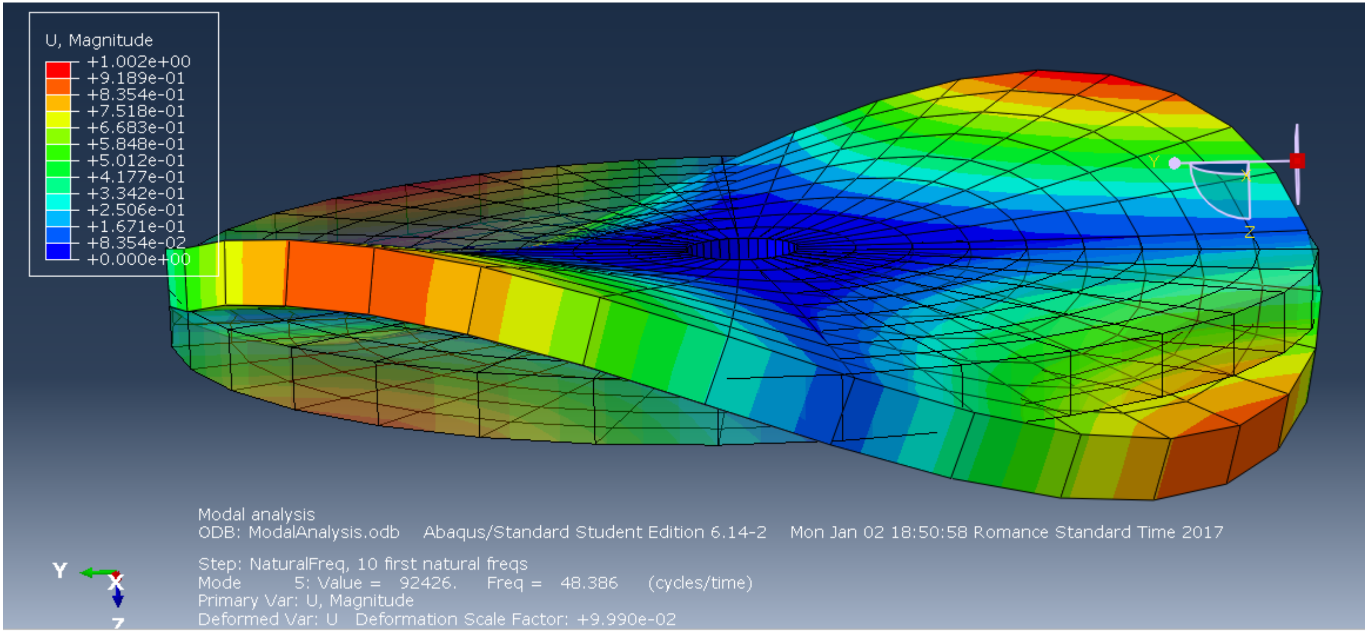


Figure 10: Mode 5

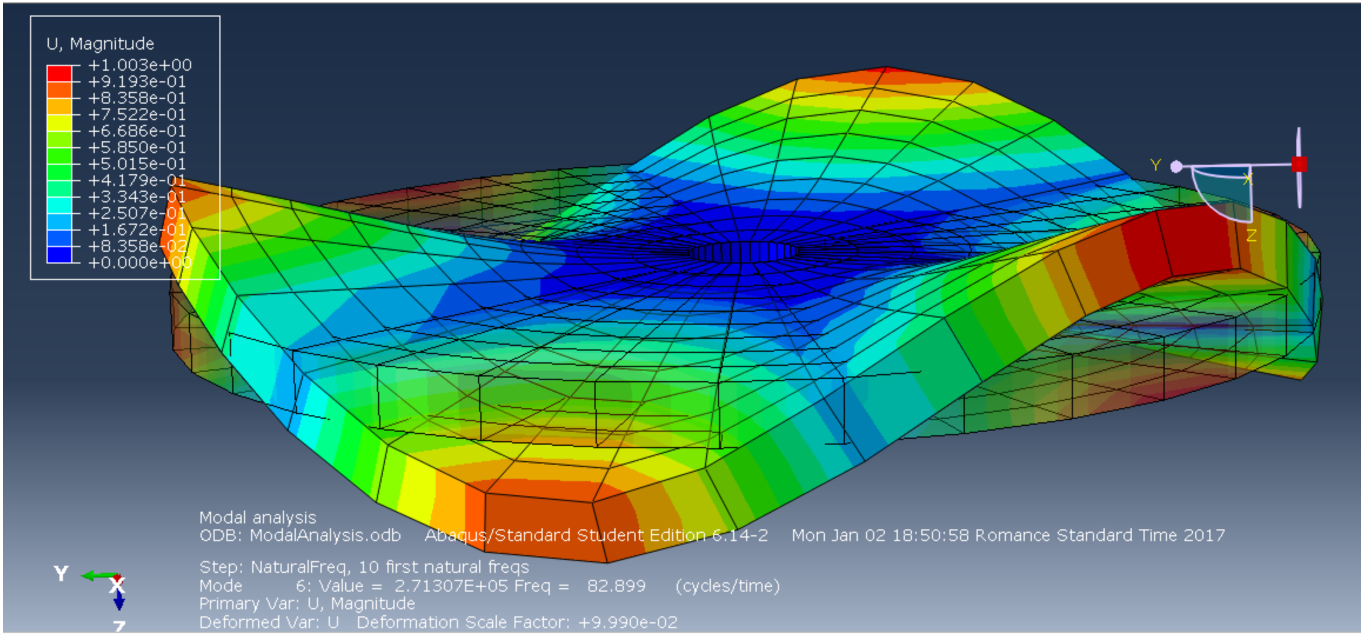


Figure 11: Mode 6

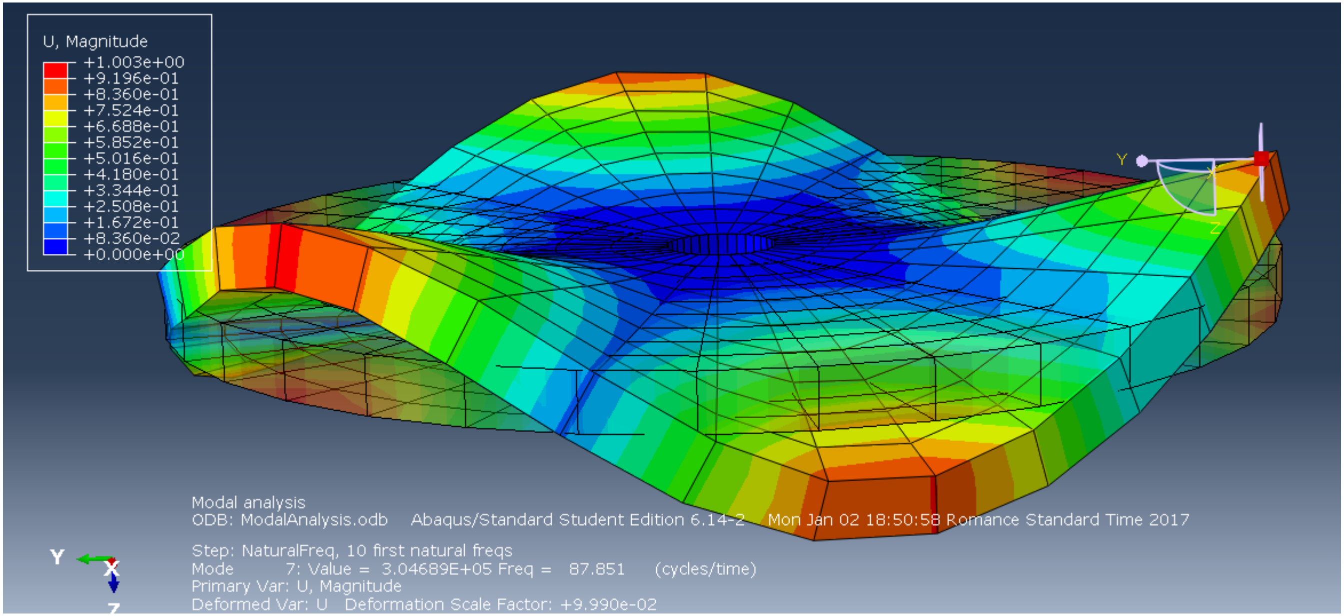


Figure 12: Mode 7

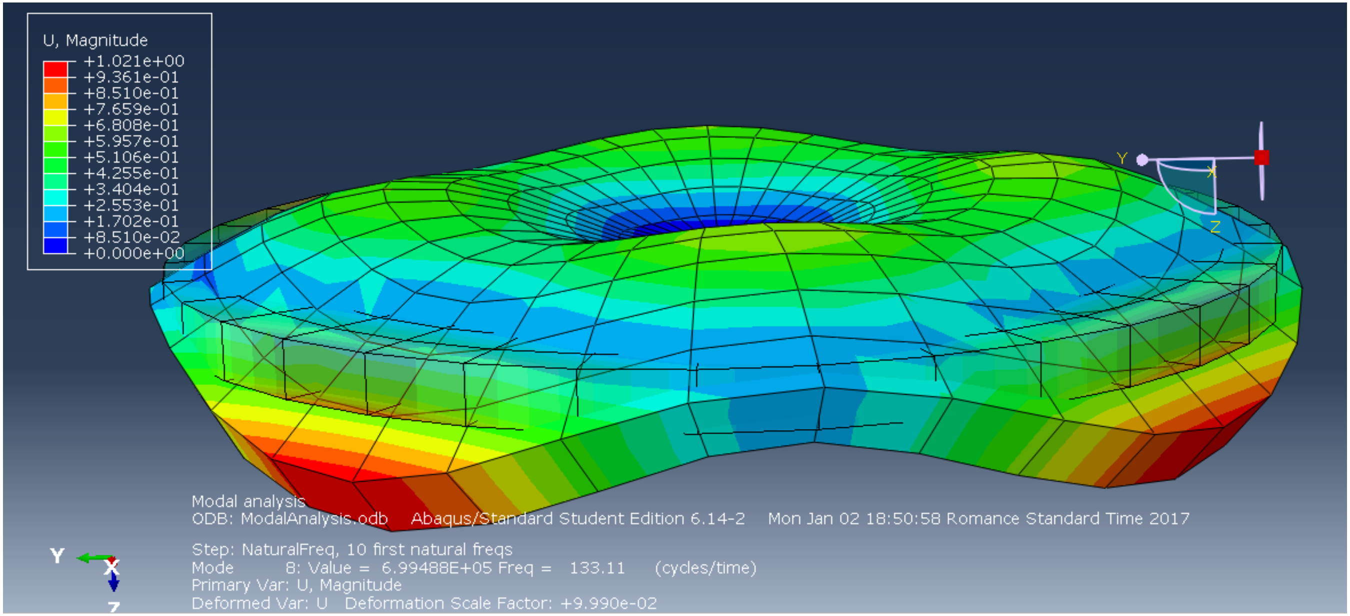


Figure 13: Mode 8

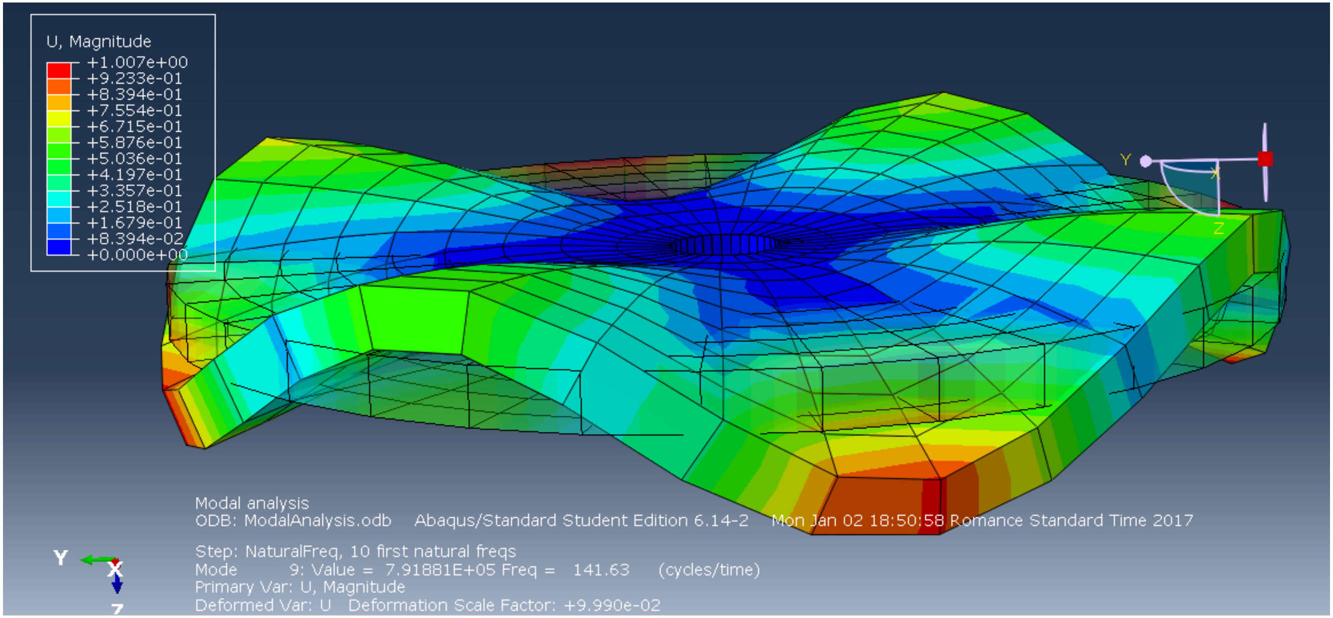


Figure 14: Mode 9

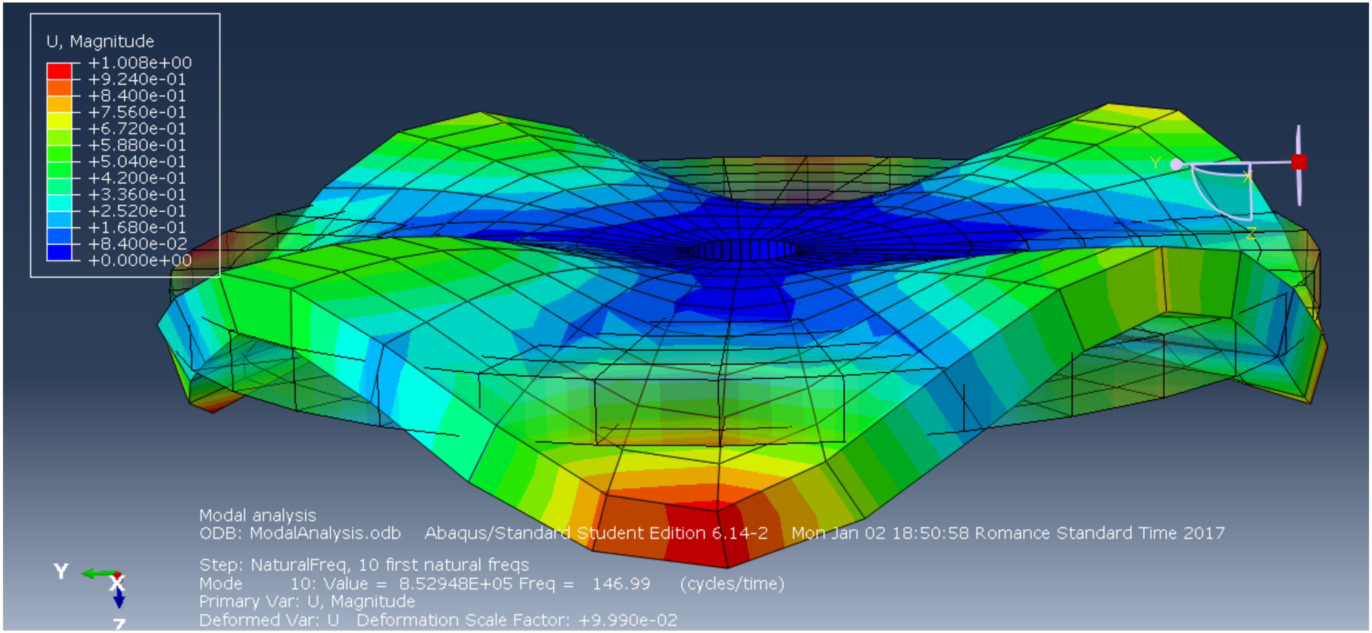


Figure 15: Mode 10

- Study the possible coupling between natural frequencies and frequency of rotation when the train travels at $v_{max} = 350\ km/h = 97.22\ m/s$
- Calculate angular velocity

$$\omega_{max} = \frac{v_{max}}{R} = \frac{97.22\ m/s}{0.5m} = 194.44\ rad/s$$

Angular velocity

$$\omega_{max} = 194.44\ \frac{rad}{s} \cdot \frac{1\ rev}{2\pi\ rad} = 30.946\ rev/s\ [Hz] \qquad (4)$$

Mode	Frequency $[Hz]$
1	24.759
2	25.116
3	26.256
4	38.749
5	48.386
6	82.899
7	87.851
8	133.11
9	141.63
10	146.99

- ω_{max} is close to the third mode frequency $\Rightarrow$ resonance may appear.
- Wheel will be suffering severe deformations over time.
- For high enough values of amplitude, wheel can enter into stick-slip transition $\Rightarrow$ squeal

- Analyze the possible coupling between the sleepers and the wheel.
- Sleepers are the transverse beams holding the train rails located every 60 cm in this case.
- Assume a linear velocity v for the train. Thus, the frequency at which the train will pass by a sleeper is,

Contact frequency with sleepers

$$v \, \frac{m}{s} \cdot \frac{1 \; cycle}{0.6 \; m} = f_c \, \frac{cycle}{s} [Hz] \qquad (5)$$

Mode	Frequency $[Hz]$	Velocity of coupling $[m/s]$
1	24.759	14.85
2	25.116	15.06
3	26.256	15.75
4	38.749	23.24
5	48.386	29.02
6	82.899	49.73
7	87.851	52.71
8	133.11	79.86
9	141.63	84.97
10	146.99	88.19

Table 2: Traveling speeds that cause coupling

- If the frequency of contact f_c is equal to any of the natural frequencies $\Rightarrow$ resonance $\Rightarrow$ vibration of the wheel will be amplified and the stick-slip transition phenomenon will appear.
- Coupling seems to be specially relevant for low speeds $\Rightarrow$ consider the case when the train is entering or leaving a station.

The stick-slip phenomenon is a type of spontaneous motion that can occur while two objects are sliding over each other

- Two surfaces alternating between two states: sticking to each other and sliding over each other.
- If a force large enough is applied to one of the surfaces, it will start sliding and friction coefficient decreases from μ_s to μ_d.
- When it happens between wheel/rail surfaces, the wheel can oscillate and radiate the squeal noise [1].

- Squeal can be also generated when the train passes through a curved section or a switch between rails.
- During curve passages wheels suffer lateral creepage as they are not perfectly aligned.
- The rolling angle, α, is the angle between the rolling direction and the direction of the movement.

Lateral creepage

The lateral creepage is defined as the tangent of the rolling angle

$$LC = \tan\alpha \tag{6}$$

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. **Conclusions**
7. References

- Wheel squeal is still a complicated issue with varied causes [2]: geometric, atmospheric, material-related, etc.
- More research is needed to fully understand this phenomena and find reliable solutions.
- Solutions? Change location of sleepers to an arbitrary one; lubrication stations at critical points...

To completely eradicate wheel squeal?

MAGLEV TRAIN $\Rightarrow$ no wheel vibration

Outline

1. Introduction
2. Problem statement
3. Theoretical Framework
4. Methodology
   - Wheel geometry
   - Simulation setup
   - Boundary conditions
   - Mesh
5. Results
   - Eigenfrequencies
   - Eigenmodes
   - Frequency of rotation coupling
   - Sleeper contact frequency coupling
   - Stick-slip transition wheel/rail
6. Conclusions
7. **References**

- Beer, F.,G., Janssens, M.,H.,A., Kooijman, P. *Squeal noise of rail-bound vehicles in influenced by lateral contact position.* Journal of Sound and Vibration, 267, p. 497-505, 2003.
- Steenbergen, M.,J. Esveld, C. *Relation between the geometry of rail welds and the dynamic wheel-rail response: numerical simulations for measured welds.* IMECH E, Part F: J. Rail and Rapid Transit, 220:409-423, 2006.
- Chin-Quee, D., *Wheel squeal in rail transit systems*, Richmond Hill, Ontario.
- Wayside Steering Committee, *Rail wheel noise fact sheet*
- Rao, S., S., *Mechanical vibrations*, Prentice Hall, 2005.
- Rao, S., S. *Vibrations of continuous systems*, Wiley, 2007.

- Hanson, D.; Jiang, J.; Dowdell, B; Dwight, R; *Curve squeal: causes, treatments and results*. Inter.Noise 2014, Melbourne, Australia
- Konowrocki, R; Bajer, C; *Friction rolling with lateral slip in rail vehicles*. Journal of Theoretical and Applied Mechanics