

Universitat Politecnica de Catalunya
Master on Numerical Methods in Engineering
Computational Mechanics Tools

ASSIGNMENT 2
PDE - TOOLBOX

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1. PROBLEM TO SOLVE:

$$u_t - \Delta u = f \quad \text{in } \Omega = [0, 1]^2.$$

Source term:

$$f(x, y, t) = -3e^{-3t}.$$

Initial condition at $t = 0$:

$$u(x, y, t = 0) = x^2 + xy - y^2 + 1.$$

Boundary conditions:

$$u_n(x = 0, y, t) = -y,$$

$$u_n(x = 1, y, t) = 2 + y,$$

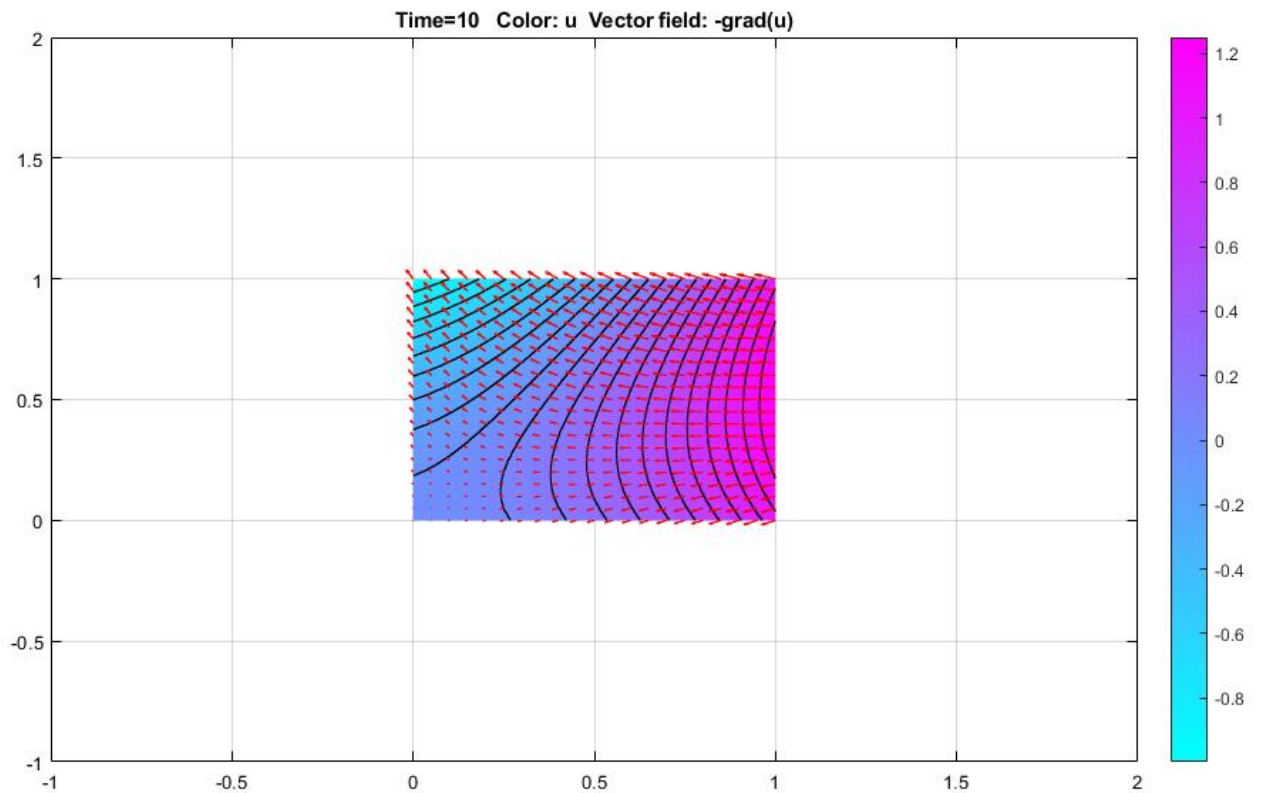
$$u(x, y = 0, t) = x^2 + e^{-3t},$$

$$u_n(x, y = 1, t) = x - 2,$$

Analytical solution of the problem:

$$u(x, y, t) = x^2 + xy - y^2 + e^{-3t}$$

2. SOLUTION OF THE PROBLEM WITH $t_{\text{end}} = 10$:

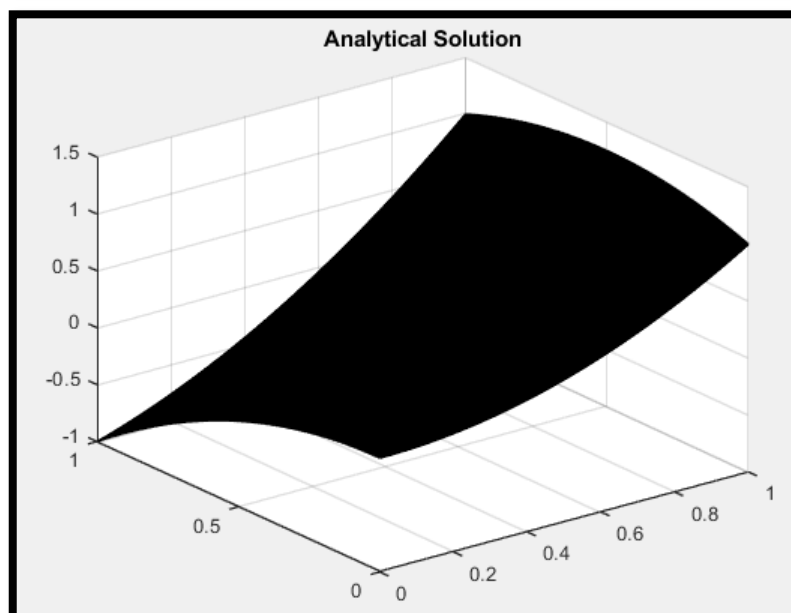
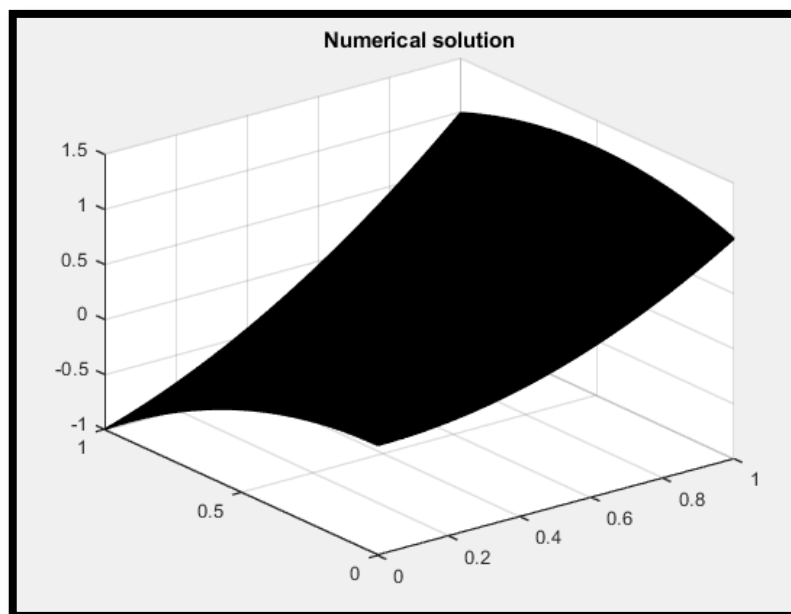


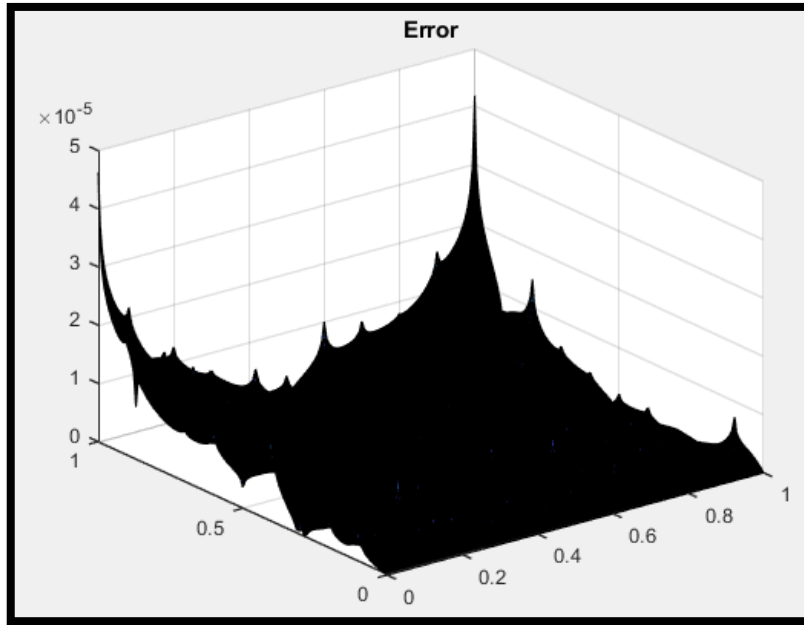
number of elements	324	1296	5184	20736	82944
h	0.0786	0.0393	0.0196	0.0098	0.0049

$$h = \sqrt{\frac{2}{N}}$$

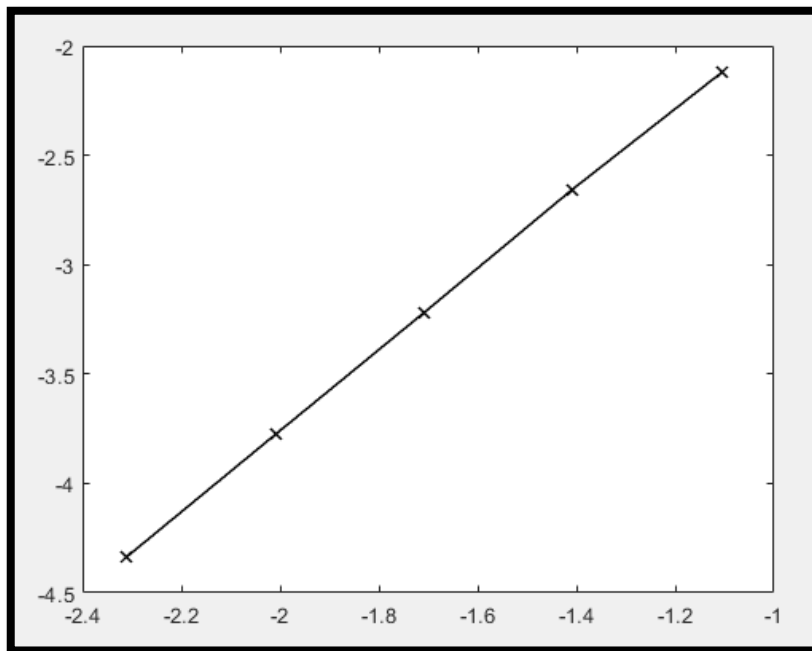
h	0.0786	0.0393	0.0196	0.0098	0.0049
error	7.6000E-03	2.2000E-03	6.0532E-04	1.6781E-04	4.6282E-05

Results for $h = 0,0049$:

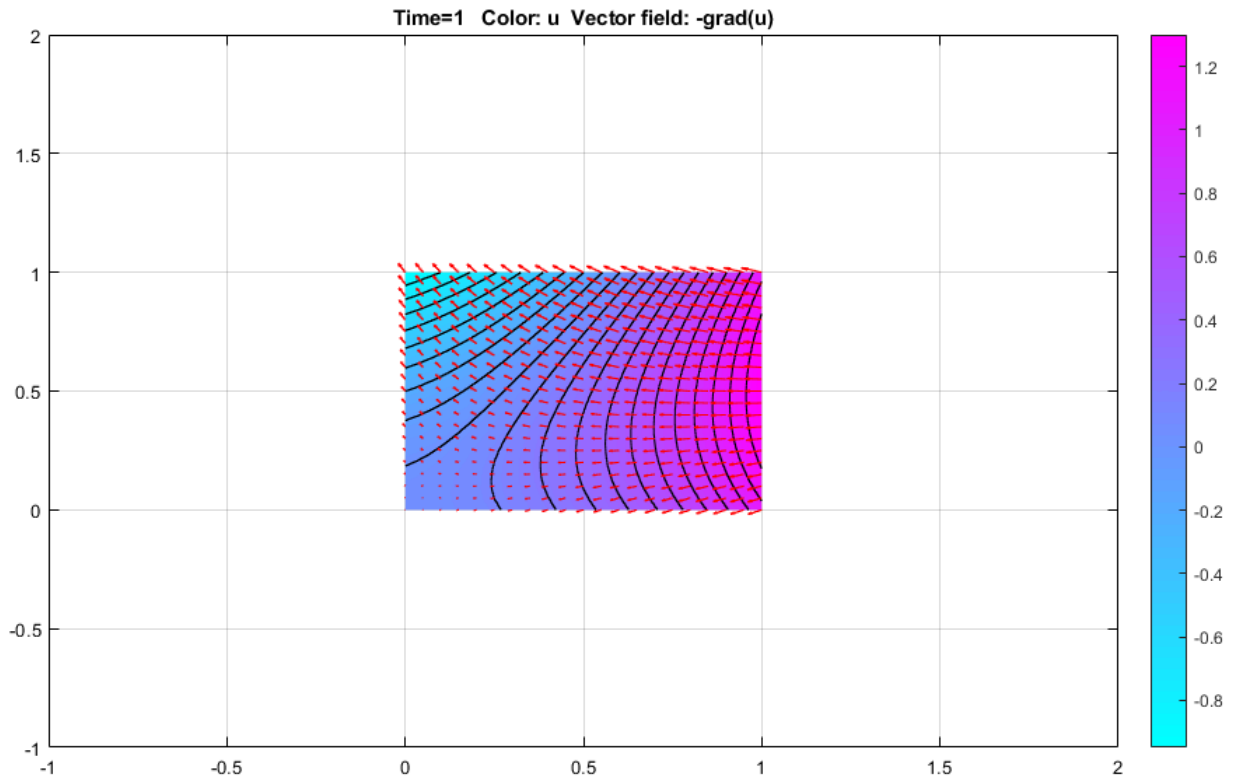




Convergence:



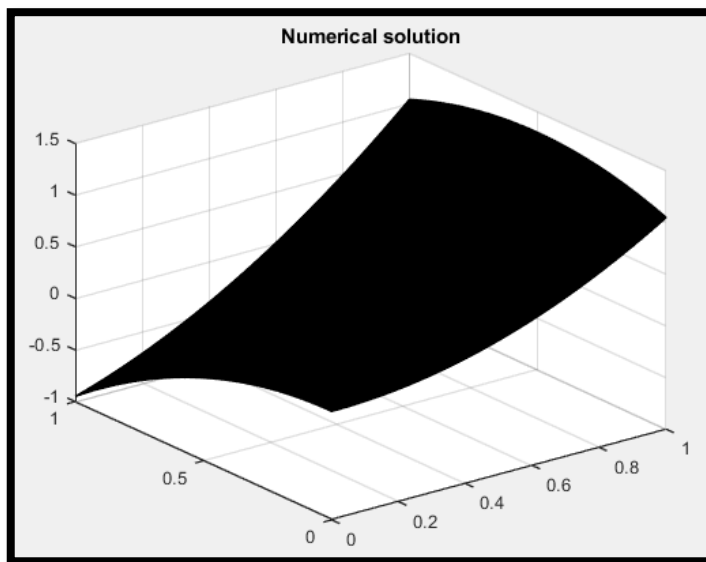
3. SOLUTION OF THE PROBLEM WITH $t_{\text{end}} = 1$:

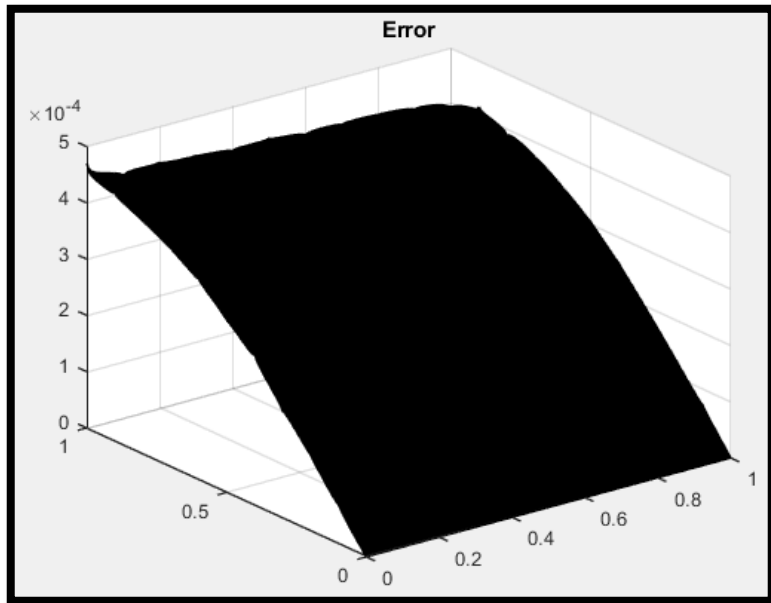
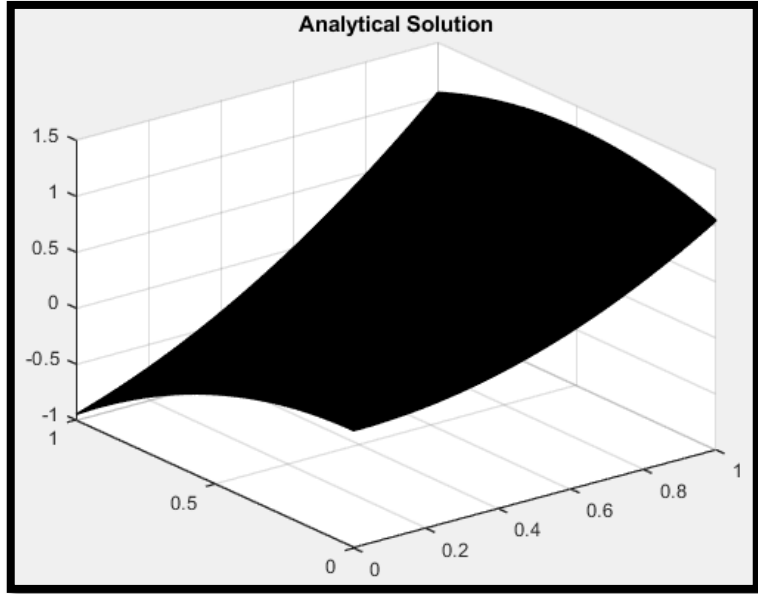


number of elements	324	1296	5184	20736	82944
h	0.0786	0.0393	0.0196	0.0098	0.0049

h	0.0786	0.0393	0.0196	0.0098	0.0049
error	8.2000E-03	2.1000E-03	1.1000E-03	5.8246E-04	4.7057E-04

Results for $h = 0,0049$:





Convergence:

