

## Nonlinear Elastic Block

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### Part one Identify in the code (file,lines) the following items:

(a) The definition of the example (loading,geometry).

The file named `preprocessing.m` is the definition of the example, or to be more concise, from **line 8 (switch example)** to **line 59 (end)**.

(b) The choice of solution method (newton's method with or without line-search).

The file named `main_buckling.m` (**line18,19,20**) and the file named `main_incremental_iterative.m` (**line20,21**) are the choice of solution. According to the comment of the code, we can choose different method.

(c) The implementation of the solution method

The file named `equilibrate.m` is the implementation of the solution method, or to be more precise, from **line9(switch options.method)** to **line94(end)**.

(d) The implementation of the incremental-iterative strategy, with smart initial guesses for imposed displacements.

The file named `main_buckling.m` (**line65,loop on the load increments**) and the file named `main_incremental_iterative.m` (**line39,loop on the load increments**) are the implementation of the incremental-iterative strategy.

(e) The introduction of random perturbations in the initial guesses of the solution method.

The file named `main_buckling.m` (**line76,  $x=x+\text{rand}(\text{size}(x))*0.001$** ) and the file named `main_incremental_iterative.m` (**line50,  $x=x+\text{rand}(\text{size}(x))*0.001$** ).

### Part two Report and discuss a selection of results that illustrate the feature of the

#### nonlinear model vis a vis the following features of linear models:

- Proportionality of the response with respect to the loads.
- Symmetry of the response with respect to the sign of the loads.
- Uniqueness of solutions.
- Stability of the solutions.

The comparison is facilitated by the fact that the code also performs a linear-elasticity analysis, which is compared graphically with the result of the nonlinear analysis. Comment on the physical validity of the solutions given by both models. Finally, sum up your conclusions.

- Proportionality of the response with respect to the loads.

It is clear that for linear models, the response has a direct proportion to the loads, while for nonlinear models, there is a nonlinear relationship between the response and the loads.

In the beginning of response, the nonlinear models can be approximated by linear models, or it follows the Hook's law. However, in a little larger displacement, there is not such a relationship.

- Symmetry of the response with respect to the sign of the loads.

According to the experiment (b), we can get the following figure 1. Symmetry of the two models can be concluded.

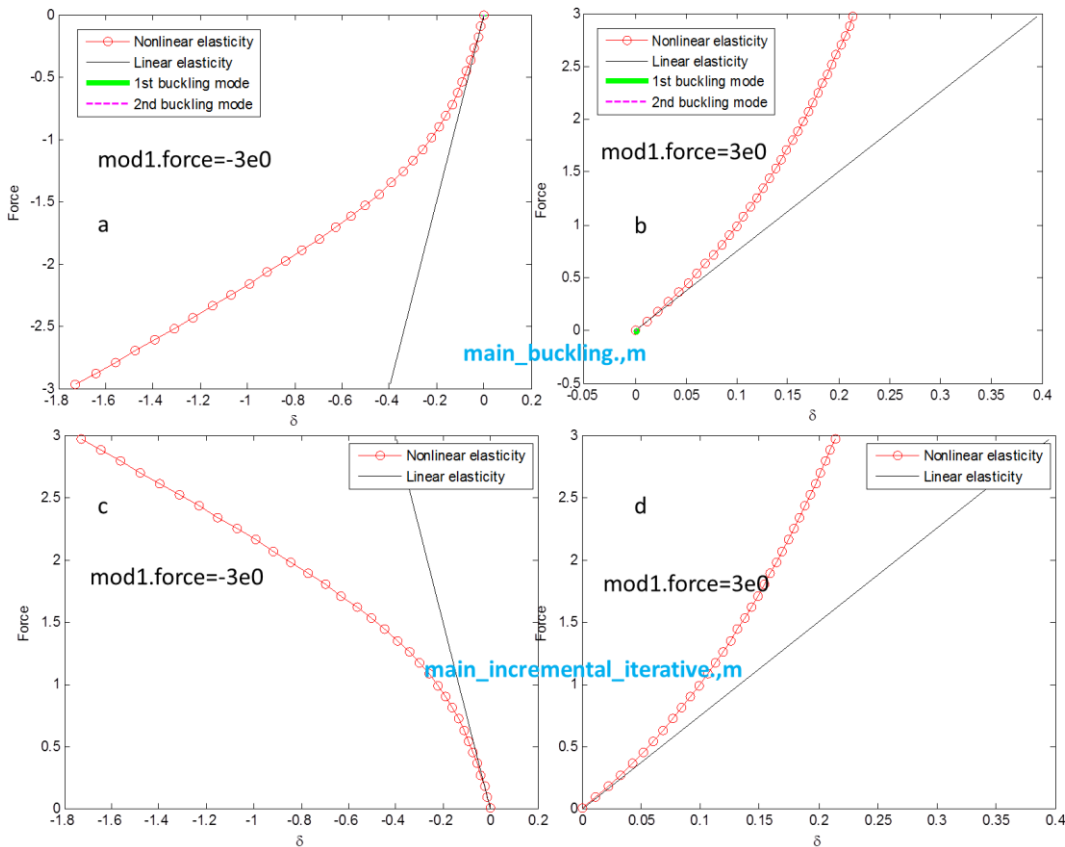


Figure 1: symmetry of the two models

Comparing figure1a with figure 1b, we find that the only difference is sign of the loads (-3e0 and 3e0 respectively). In figure1a and figure 1b, the slope of linear elasticity is the same. However, as nonlinear elasticity, the sign of the load would affect the absolute displacement, which means that it is not symmetric. It is easy to draw the conclusion that the linear elasticity is symmetric while the nonlinear elasticity is asymmetric. The same results can be obtained from `main_incremental_iterative.m`.

- Uniqueness of solutions.

In experiment(c), we select the result of example 2 for analysis figure2.

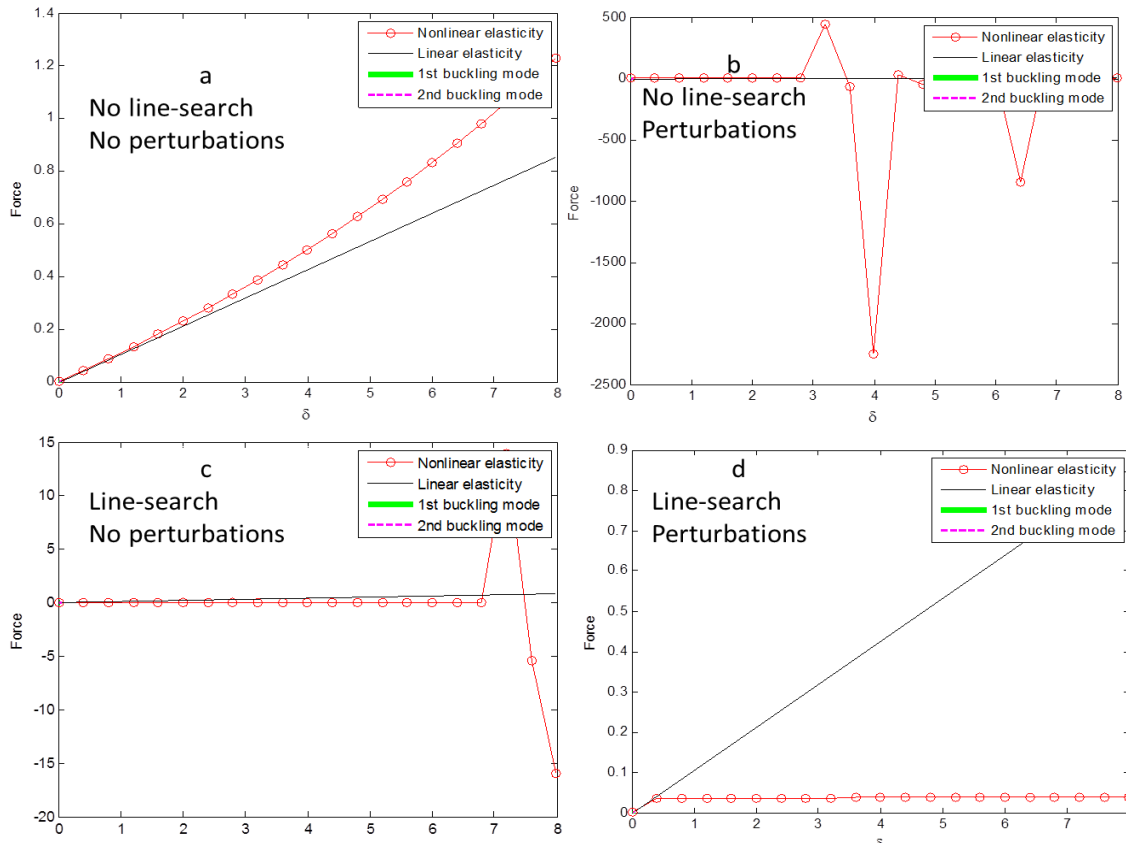


Figure2: Solutions for example 2

For linear models, we can draw the conclusion from figure2a, figure2b, figure2c and figure2d that the solution is unique. Although it seems that the slope is different for the four figures, actually they are same; all is from 0 to about 0.87 vertically.

For nonlinear models, solutions are not unique. In figure2a, the nonlinear model curve rising faster and faster along the displacement increasing. In figure2b, figure2c, there are fluctuations around the linear model. The fluctuation can be very large and can be very small, so this is unstable solution for the model. In this case, it is necessary to perturb the system and thus get figure2d, so this model is the fact in reality.

- Stability of the solutions.

According to experiment (d), we can compare the solution using newton's method with and without line-search. If the solutions are different, this means they are unstable, showing in figure2 (example4) and figure 3(example5).

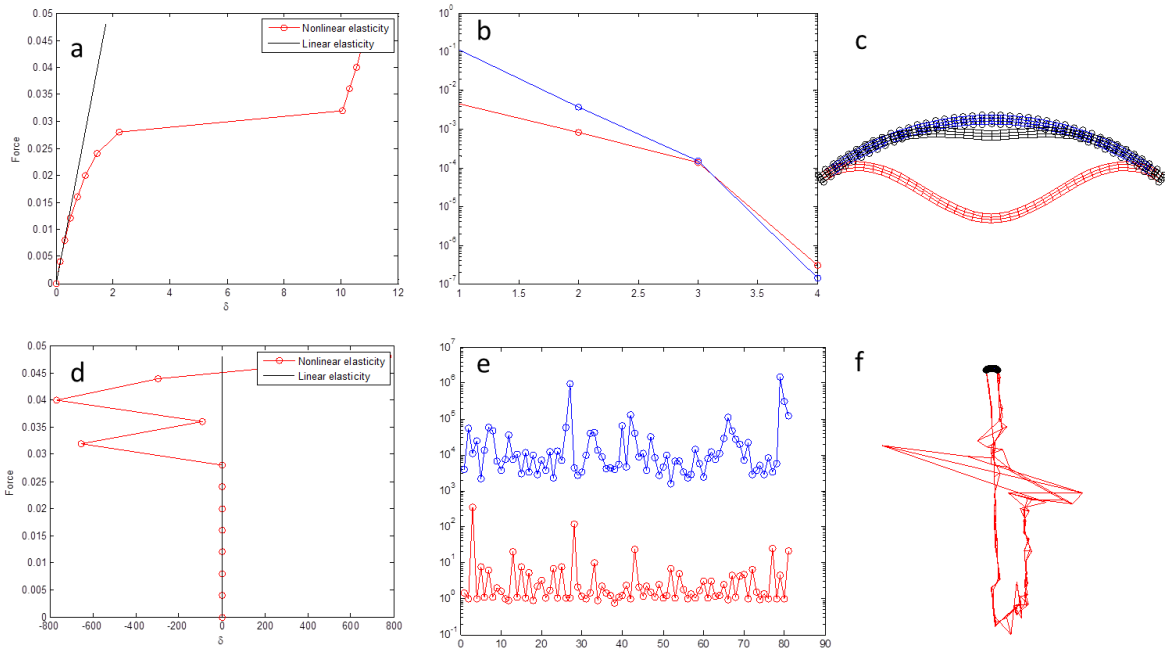


Figure3 : with line-search: a,solutions, b,error, c, mesh; without line-search:d, solutions,e,error, f,mesh

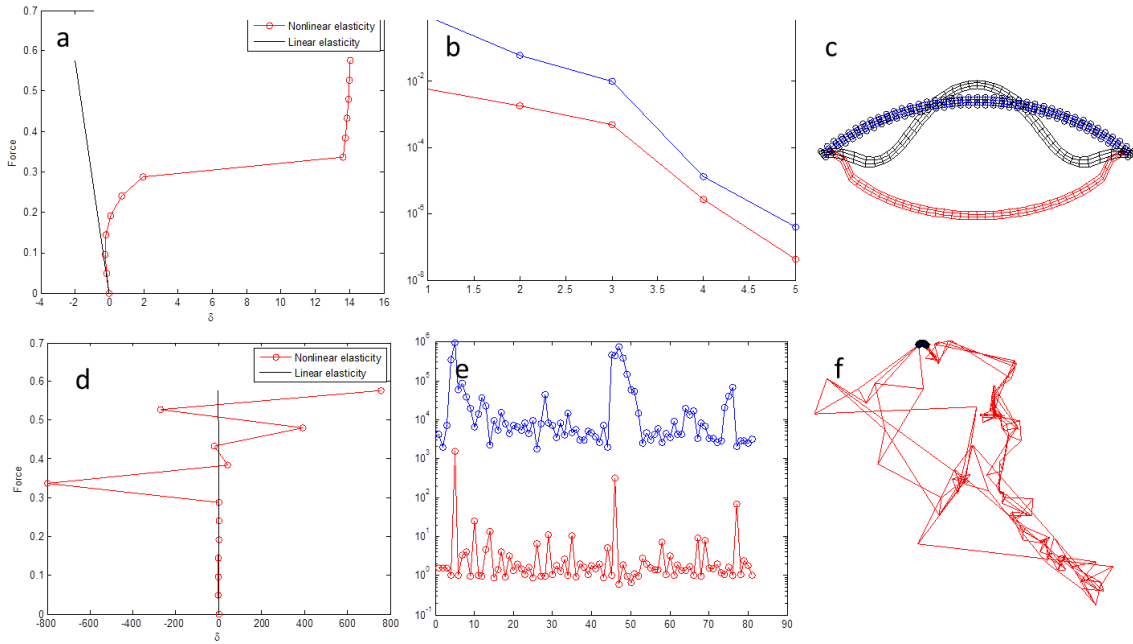


Figure4: with line-search: a,solutions, b,error, c, mesh; without line-search:d, solutions,e,error, f,mesh

From figure 2 and figure 3, we can see that the solutions with line-search method are stable and show a convergence. However, the solutions without line-search method are unstable and the error is very large, so in reality, they are hardly ever observed. This results are always true for both linear and nonlinear elasticity.