Advanced Fluid Mechanics

Homework 3, Section 2

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Note: Solution to the first problem is in handwriting which has already submitted during the class on 3, Dec.

The wave pattern which results from the breaking of the diaphragm is shown in the following x-t diagram,figure 1:

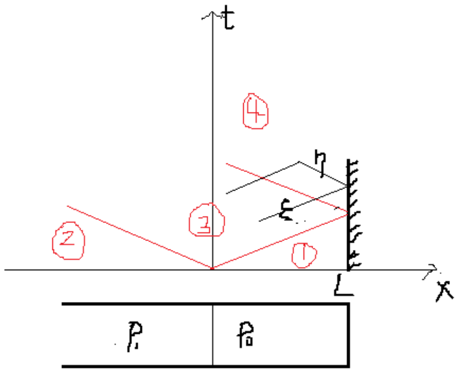

Figure 1 x-t digram

In the positive x direction, the wave will be reflected as a wave of some form upon striking the closed end of the shock tube.

Analysis of the four regions:

Region 1 has not yet been influenced by the outgoing wave from the origin so maintains its initial conditions of $u = 0, p = p_0$.

Region 2 still maintains its previous state with $u = 0, p = p_1$

Region 3 is influenced by an expansion wave to the negative x direction and an shock wave to the positive x direction. In this region, using characteristic line and Riemann invariant, we have the following equation governing this region.

$$\frac{u}{a_0} = \frac{1}{2\gamma}\frac{p_1}{p_0} - 1 \dots\dots\dots\dots\dots\dots\dots\dots\dots. ①$$

$$\frac{p}{p_0} = \frac{1}{2}\frac{p_1}{p_0} + 1 \dots\dots\dots\dots\dots\dots\dots\dots\dots. ②$$

Region 4 is the region we are going to obtain expression for the velocity and pressure. This region is always behind the wave which is reflected from the closed end of the tube.

Since the pressure in region 4 is unknown, we developed a characteristic line ε from the end tube to the region 3 where the pressure is already known.

As the reflected shockwave region 4, $\eta = \text{constant}$ can be used to describe this region.

Assuming the pressure in region 4 is p_4. On the end of the tube, p=p_4, u= 0. Hence,

$$\frac{u}{a_0} - \frac{1}{\gamma}\frac{p}{p_0} = -\frac{1}{\gamma}\frac{p_4}{p_0} \dots\dots\dots\dots\dots\dots\dots ③$$

In order to evaluate p_4, we can use $\varepsilon = \text{constant}$ characteristic,

$$\frac{u}{a_0} + \frac{1}{\gamma}\frac{p}{p_0} = \frac{1}{\gamma}\frac{p_4}{p_0} \dots\dots\dots\dots\dots\dots\dots ④$$

According to equation ① and ②, we can rearrange equation ④ as follows,

$$\frac{u}{a_0} + \frac{1}{\gamma}\frac{p}{p_0} = \frac{1}{\gamma}\frac{p_4}{p_0} = \frac{1}{2\gamma}\frac{p_1}{p_0} - 1 + \frac{1}{2\gamma}\frac{p_1}{p_0} + 1 = \frac{1}{\gamma}\frac{p_1}{p_0}$$

So,

$$p_4 = p_1$$

Along the characteristic line: $\eta = \text{constant}$, we thus have,

$$\frac{u}{a_0} - \frac{1}{\gamma}\frac{p}{p_0} = -\frac{1}{\gamma}\frac{p_1}{p_0} \dots\dots\dots\dots\dots\dots\dots ⑤$$

Along the characteristic line: $\varepsilon = \text{constant}$, we thus have,

$$\frac{u}{a_0} + \frac{1}{\gamma}\frac{p}{p_0} = \frac{1}{\gamma}\frac{p_1}{p_0} \dots\dots\dots\dots\dots\dots. \dots ⑥$$

So,

$$\mathrm{u} = 0$$

Finally, we can conclude that the pressure behind the wave is equal to p_1 and the velocity is zero.