

Computational Solid Mechanics

Assignment 1

Damage Models

By

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Master in numerical method in engineering

1. Part I: Rate-independent

1.1. Continuous damage models

The Figure 1 shows different continuous damage models. The Figure 1a is corresponded to symmetric damage models. It can be seen that the elastic traction region and the elastic compression region are the same. The Figure 1b shows only-tension damage model. As can be seen, it has an infinite elastic compression region. On the other hand, the tension elastic region is small compared with the compression region. Finally, the Figure 1 c shows a non-symmetric model. It shows that the compression elastic region is larger than the elastic traction region, this behaviour is related with the parameter "n". This parameter takes into account the rate between compression and traction region. All of these models were plot in the Stress-Stress space.

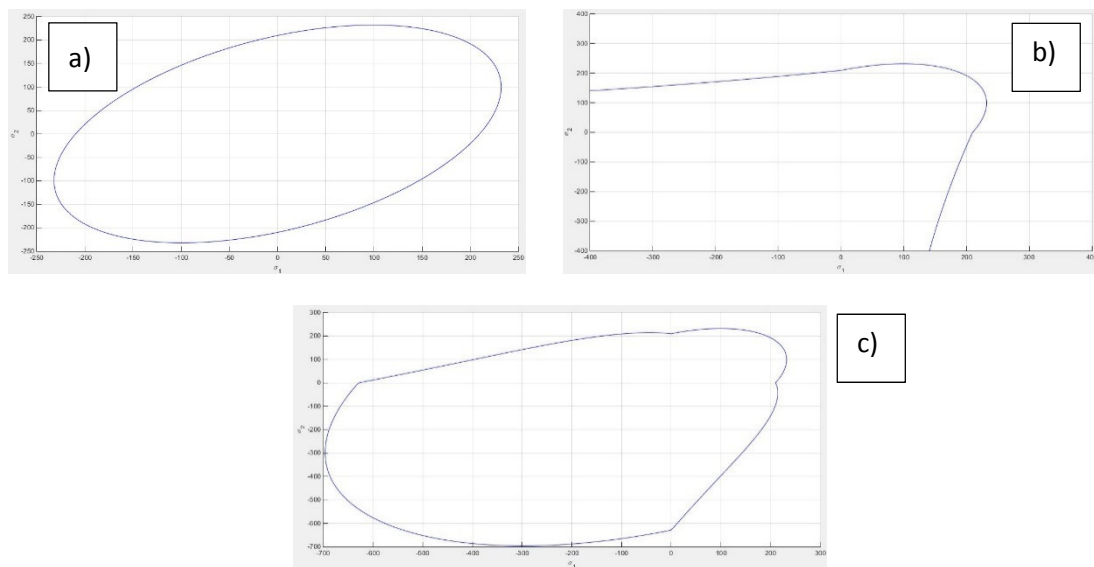


Figure 1: Three different models are shown a) Symmetric model, b) Only tension, c) Non-symmetric model.

1.2. Linear and exponential Hardening and softening

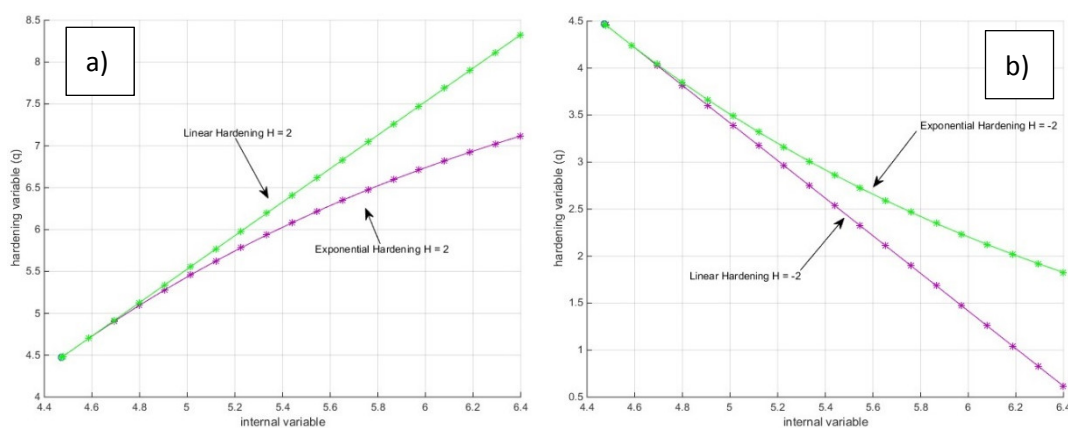


Figure 2: Linear and exponential hardening law a) Hardening. b) Softening.

In the Figure 2 can be shown two different hardening laws, linear and exponential. Is quite obvious the different between them. In this case the linear and exponential hardening were plot taking into account a hardening modulus $H = 2$ for hardening behaviour and $H = -2$ for softening behaviour.

1.3. Loading and unloading path

1.3.1. Uniaxial loading and unloading

The loading path and material parameters are shown in the table below.

Table 1: Total loading path and material parameters.

Load Path 1		Load Path 2		Load Path 3		Material parameters			
$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	E	σ_{yied}	ν	H
250	0	-600	0	350	0	2000	200	0.3	-0.1

The loading path and the material parameters, shown before, were used not only for symmetric model, but also for only tension and non-symmetric models. Exponential hardening law was used as well.

Symmetric model

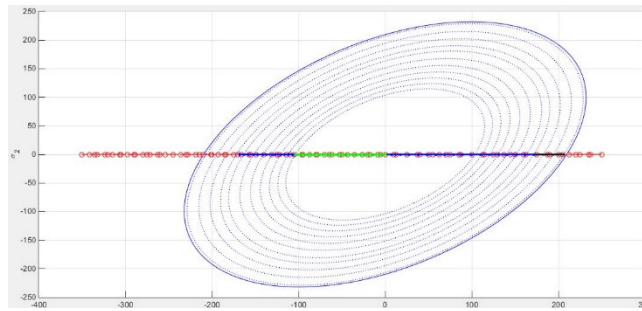


Figure 3: Symmetric model behaviour for uniaxial loading and unloading.

The Figure 3 shows the behaviour of the symmetric model when it is submitted by a uniaxial loading and unloading. It can be seen how the damage surface is shrink by the effect of the softening with each time step. The distance between different damage surface keep an exponential relation due to an exponential hardening law was used.

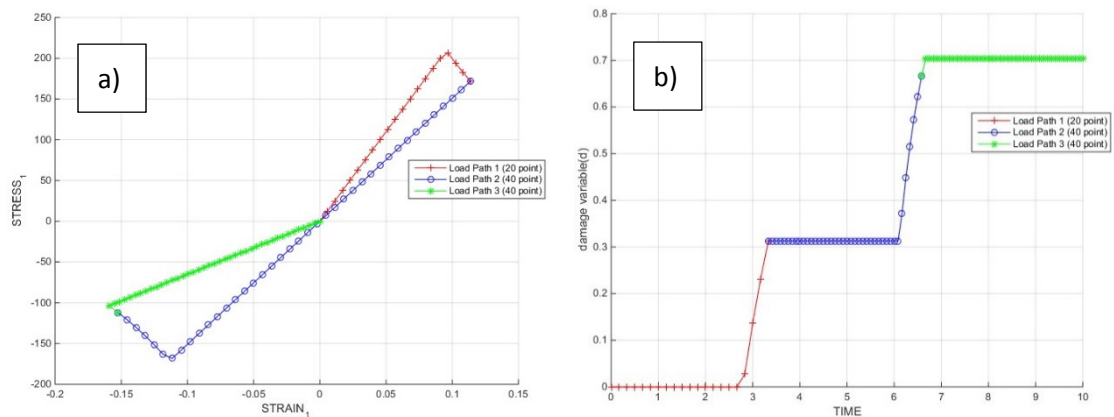


Figure 4: Results for symmetric model. a) Strain – Stress curve, b) Damage variable (d) – Time.

In the Figure 4 can be observed the strain – stress plot and the damage variable – time plot. Several remarks can be said. First of all, the behaviour of the model was as expected. The load path 1 produced an elastic loading followed by a damage loading (red line). In the load path 2 (blue line) can be seen an elastic unloading, followed by a compressive loading, and the load path 3 showed an unloading until the origin of the plot. Second, a reduction of the slope

correspond to the blue and the green straight lines was observed (the elastic unloading / compressive loading). That reduction of the two slopes was due to a material degradation. This important remark is in accordance with the Figure 4b. Finally, in the Figure 4b can be seen that the horizontal lines not only represent a constants value of the damage in the material but also, represent the different elastics loading and elastics unloading stages as well.

Only tension model

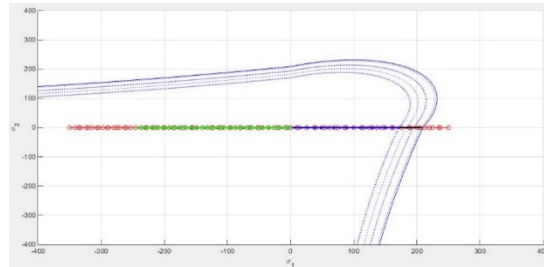


Figure 5: Only tension model behaviour for uniaxial loading and unloading.

The Figure 5 shows the behaviour of the only tension model when it is submitted by the loading path described above. It can be seen that the radial distance between different damage surface keep an exponential relation in accordance with the exponential hardening law used.

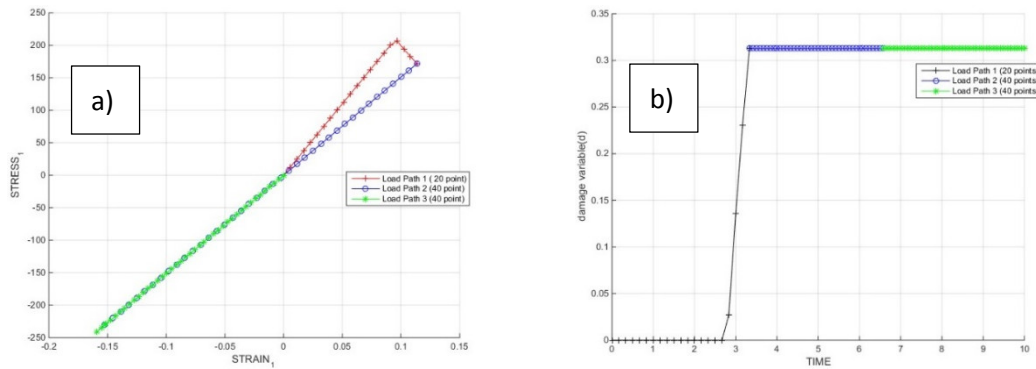


Figure 6: Results for only tension model. a) Strain – Stress curve, b) Damage variable (d) – Time.

In the Figure 6a can be observed three elastic paths. The first one corresponds to the elastic loading until reach the yield stress (straight red line). The second one (straight blue line) corresponds to the tensile unloading and compressive loading and the third one (straight green line) corresponds to the compressive unloading. It has been observed a degradation of the material. The difference between the slopes of the straight red and blue line means a degradation of the material. This degradation can be detected in the Figure 6b where the horizontal blue and green line point the damage level in the material. According to this graphic, a constant level of damage means an elastic loading or unloading path.

Non-symmetric model

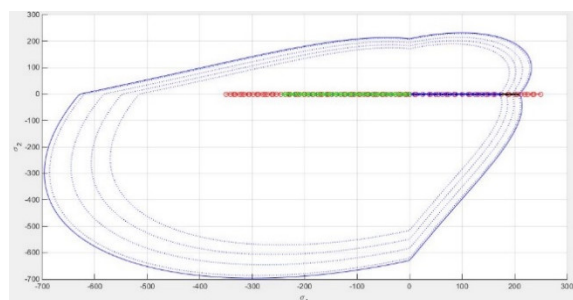


Figure 7: Non-symmetric model behaviour for uniaxial loading and unloading.

This model keeps a similar behaviour as it described before. The different between them is that this model has a finite elastic region.

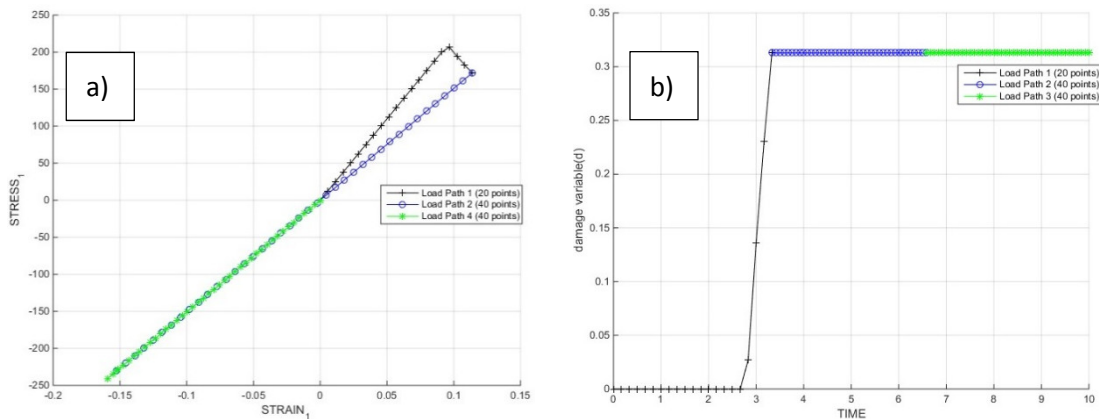


Figure 8: Non-symmetric model behaviour for uniaxial loading and unloading.

As it said before, these two models (only tension and non-symmetric model) have shown a similar behaviour for the loading path applied. As it can be noted that the Figure 6a and Figure 8a are similar, while Figure 6b and Figure 8b keep certain similitude.

1.3.2. Uniaxial - biaxial loading and unloading

The Table 2 shows the loading path and material parameters used.

Table 2: Total loading path and material parameters.

Load Path 1		Load Path 2		Load Path 3		Material parameters			
$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	E	σ_{yied}	ν	H
250	0	-250	-250	100	100	2000	200	0.3	-0.1

The loading path and the material parameters, shown above, were used not only for symmetric model, but also for only tension and non-symmetric models. Exponential hardening law was used as well.

Symmetric model

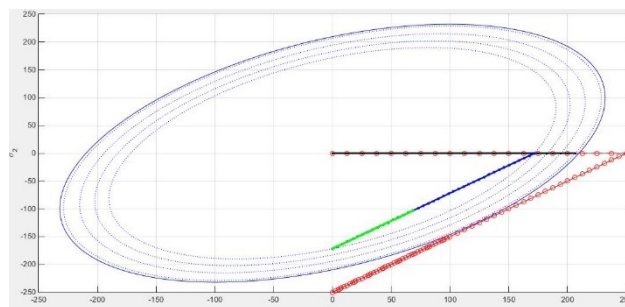


Figure 9: Symmetric model behaviour for uniaxial – biaxial loading and unloading.

In the Figure 9 can be noted the behaviour of the model when it was submitted by the loading path described in the table above. It can be observed that the first load path, correspond to the uniaxial loading, crosses the damage surface. Then a biaxial tensile/compressive loading and unloading was applied showing that those load paths never crossed through the damage surface keeping the material on the elastic region. These observations are in accordance with the Figure

10a and b, where it can be seen an elastic loading followed by a damage loading and finally an elastic tension/compression loading and unloading.

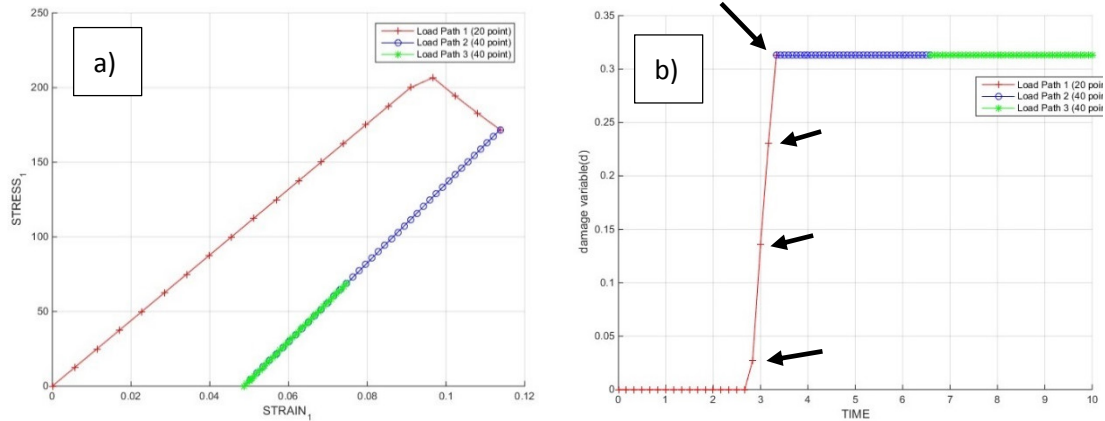


Figure 10: Results for symmetric model. a) Strain – Stress curve, b) Damage variable (d) – Time.

The arrows in the Figure 10b mark four points belong to the damage region. Those four points are outside of the elastic region and they can be seen in the Figure 9.

Only tension model

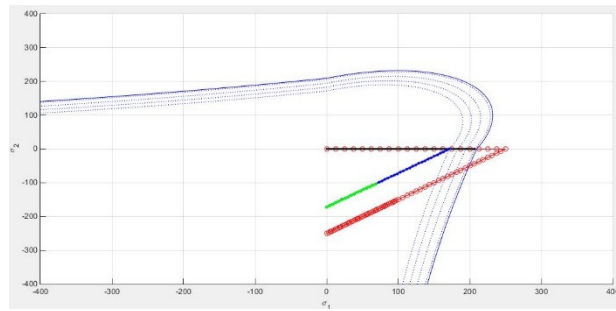


Figure 11: Only tension model behaviour for uniaxial – biaxial loading and unloading.

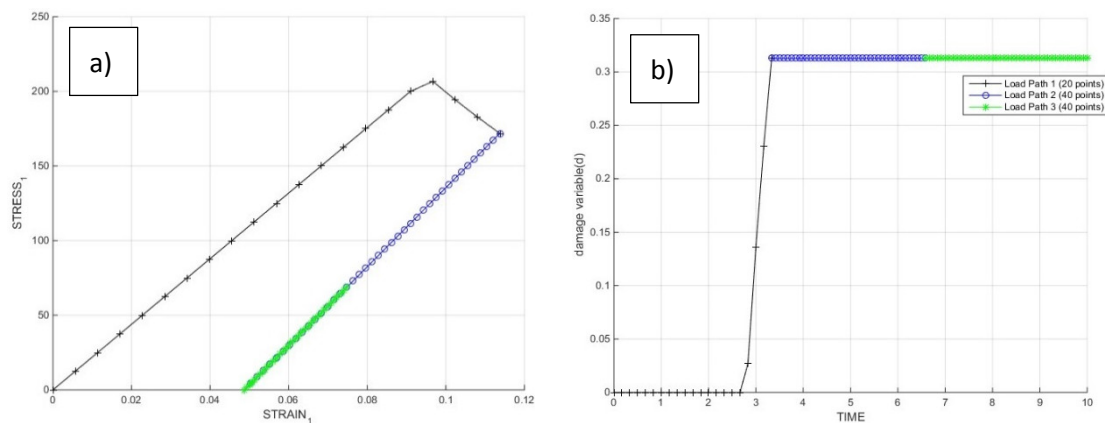


Figure 12: Results for only tension model. a) Strain – Stress curve, b) Damage variable (d) – Time.

As it can be seen the behaviour of this model is almost the same as the behaviour of the model described before. There are no big differences between them.

Non-symmetric model

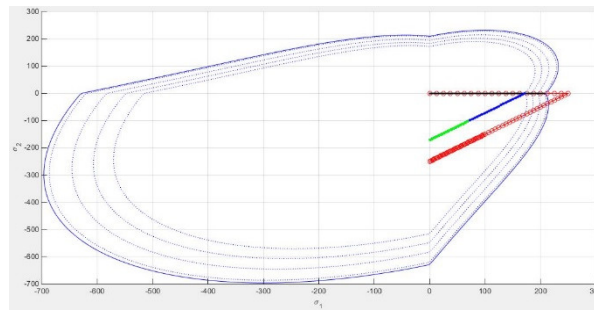


Figure 13: Non-symmetric model behaviour for biaxial loading and unloading.

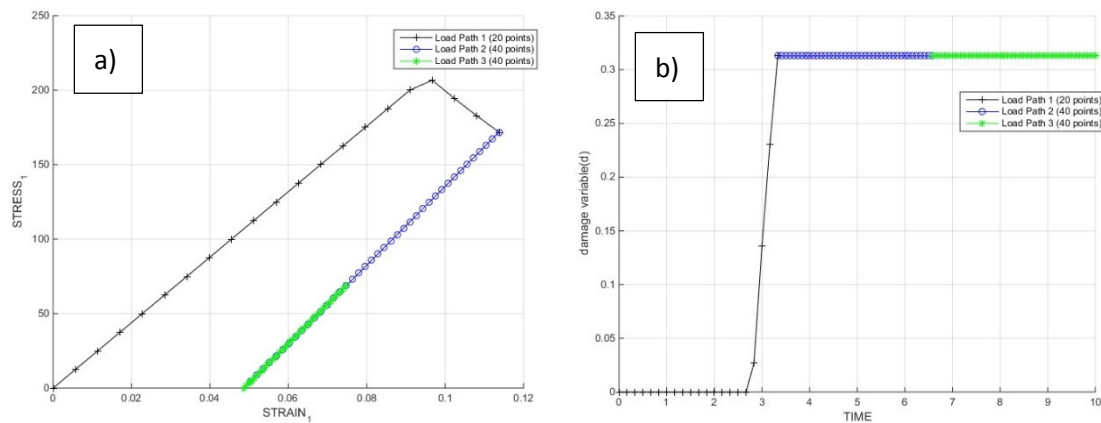


Figure 14: Results for non-symmetric model. a) Strain – Stress curve, b) Damage variable (d) – Time.

As it said before, there were not found big differences between the three models used for the same uniaxial – biaxial loading and unloading. But, it can be said that the non-symmetric and only tension models are more conservatives that the symmetric model when are submitted by a biaxial compressive loading.

1.3.3. Biaxial loading and unloading

The Table 3 shows the loading path and material parameters used.

Table 3: Total loading path and material parameters.

Load Path 1		Load Path 2		Load Path 3		Material parameters			
$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	E	σ_{yied}	ν	H
250	250	-600	-600	350	350	2000	200	0.3	-0.1

The loading path and the material parameters, shown above, were used not only for symmetric model, but also for only tension and non-symmetric models. Exponential hardening law was used as well.

Symmetric model

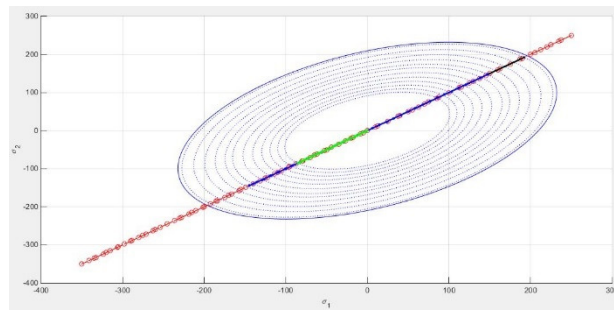


Figure 15: Symmetric model behaviour for biaxial loading and unloading.

The Figure 15 shows the behaviour of the model when it submitted by a tensile loading/unloading path, and a compressive loading/unloading path. It can be observed that both tensile and compressive loading have crossed the damage surface yielding a degradation in the material.

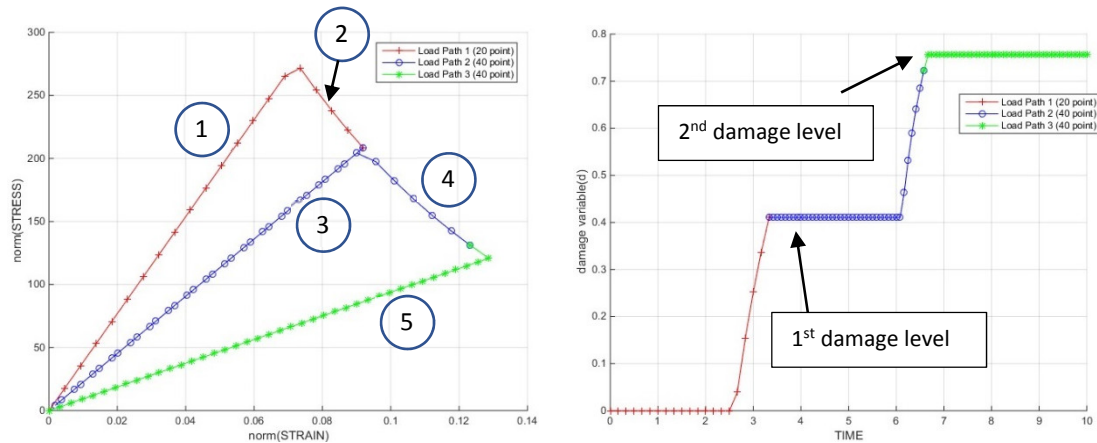


Figure 16: Results for symmetric model. a) Strain – Stress curve, b) Damage variable (d) – Time.

Different events are shown and numbered in the Figure 16a. Each event is described below. The event n° 1 is corresponded with an elastic loading, the second event produced a damage in the material. After that, an elastic tensile unloading and an elastic compressive loading is observed (event n° 3) followed for a second damage in the material (event n° 4). Finally, the event n° 5 shows an elastic compressive unloading.

The Figure 16b shows two level of damage correspond to the tensile and compressive loading. It is in accordance with the Figure 16a and several remarks can be said. First of all, the first damage level was produced by a tensile loading. This damage produces a degradation in the material that match with a reduction in the slope of the straight blue line shown in the Figure 16a. A second damage level was yielded due to a compressive loading. This damage produced a second degradation in the material that can be observed in a second reduction of the slope of the straight green line shown in the Figure 16b.

Remark 1: In order to show different degradation levels of the material, it was plot the norm(Strain) against the norm(stress). In this way, it can be shown the degradation of the material when a compressive loading is applied.

Only tension model

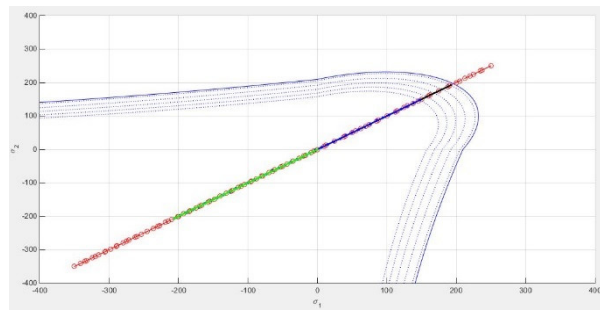


Figure 17: Only tension model behaviour for biaxial loading and unloading.

This model has shown a different behaviour compared with the model described before. In this case, the model did not present a degradation or damage submitted by a compressive loading due to it has an infinite compressive elastic region.

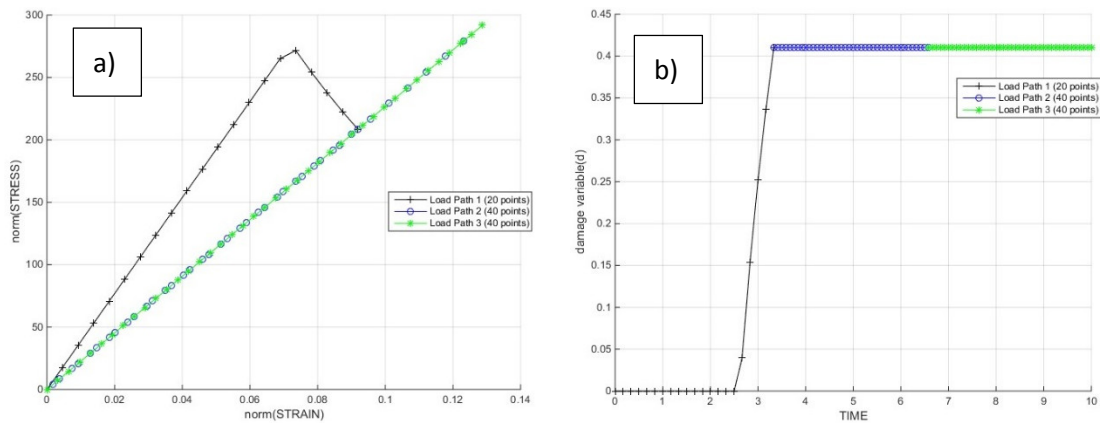


Figure 18: Results for only tension model. a) Strain – Stress curve, b) Damage variable (d) – Time.

The remark written before can be seen in the Figure 18a and b. The Figure 18a presents an elastic stage followed by a corresponding damage followed by an elastic tensile unloading and an elastic compressive loading. According to these observation, the Figure 18b only shows a level of damage.

Non-symmetric model

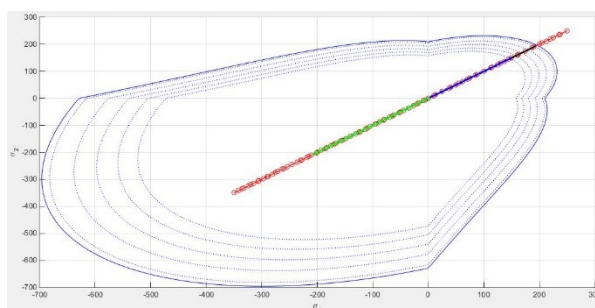


Figure 19: Non-symmetric model behaviour for biaxial loading and unloading.

It can be seen that this model presented a similar behaviour that the model described above. In this case the compressive loading did not cross the damage surface, due to this, there was no damage by applying a compressive loading path.

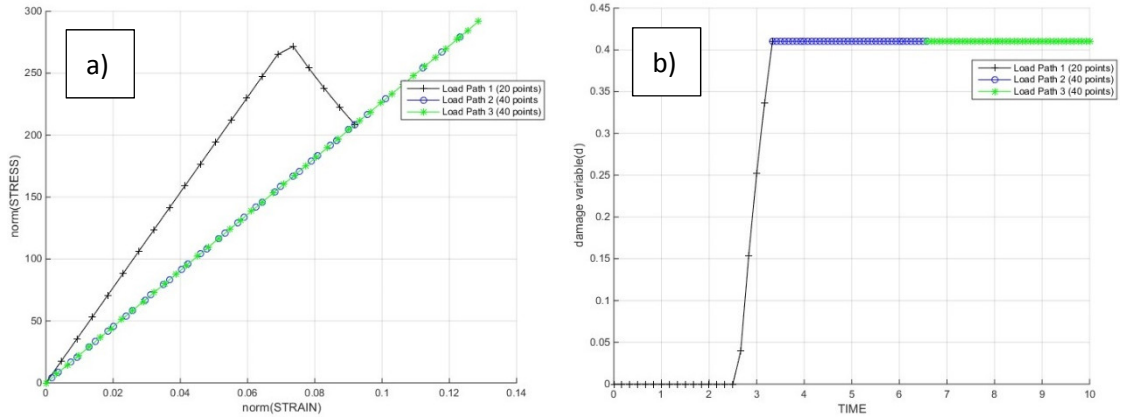


Figure 20: Non-symmetric model behaviour for a biaxial loading and unloading.

The observations written for the only-tension damage model are valid for this model as well.

2. Part II: Rate-dependent

The following part will show different results obtained by variation of several parameters such as viscous parameter, strain ratio, among others.

A uniaxial loading path was used in order to show how the material behaviour is affected by the variation of different parameters. The different results will be shown in several strain-stress curves.

Table 4: Load path.

Load Path 1		Load Path 2		Load Path 3	
$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$	$\Delta\bar{\sigma}_1$	$\Delta\bar{\sigma}_2$
100	0	100	0	300	0

It can be observed that the loading path will cross the damage surface. The reason of why this path was chosen, is in the fact that will be shown the variation of the material damage when the different parameters change. Finally, the linear hardening will be used.

2.1. Variation of the viscous parameter

The Table 5 shows different parameters; material parameters, integration parameters and different values of the viscous parameter to be evaluated.

Table 5: Material, Integration and viscous parameters.

Material Parameters				Integration Parameters		Viscous Parameter			
E	σ_{yied}	ν	H	Total time int.	α	η			
2000	200	0.3	-0.1	10	1	0	0.1	1	10

The Figure 21a shows that while the viscous parameter increases the initial damage threshold ($\epsilon_0 \sigma_{yied}$) increases as well. It can be observed when viscosity takes large values, the curves strain-stress becomes a straight line. On the other hand, the Figure 21b shows the damage in

the material. It can be seen while the viscous parameter increases the material starts to behave as a “rubber”.

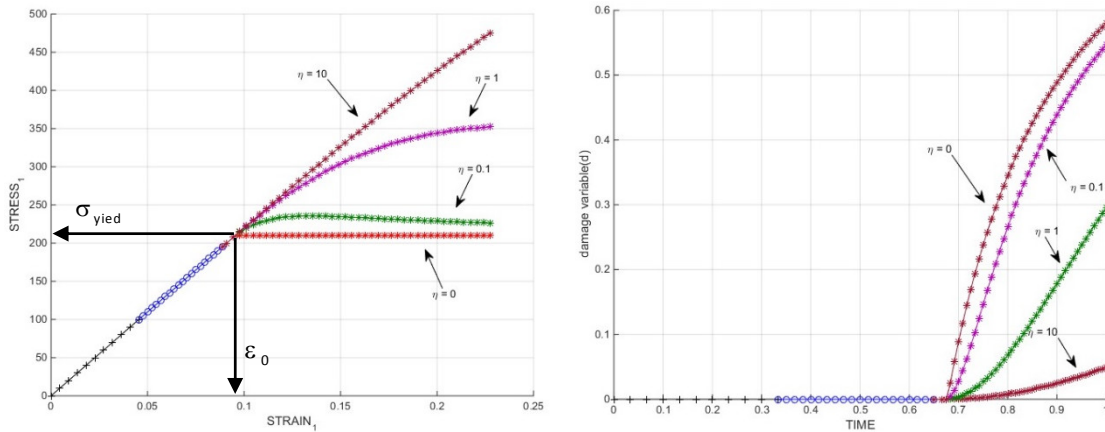


Figure 21: Variation of viscous parameter. a) Strain-Stress curves, b) Damage variable (d) – time.

2.2. Variation of the strain ratio

Different values of the total time integration will be evaluated in order to produce a variation of the strain ration. The Table 6 shows different parameters that will be used; material and integration parameters.

Table 6: Material and Integration parameters.

Material Parameters					Integration Parameters				
E	σ _{yied}	ν	H	η	α	Total time int.			
2000	200	0.3	-0.1	1	1	0.1	1	10	100

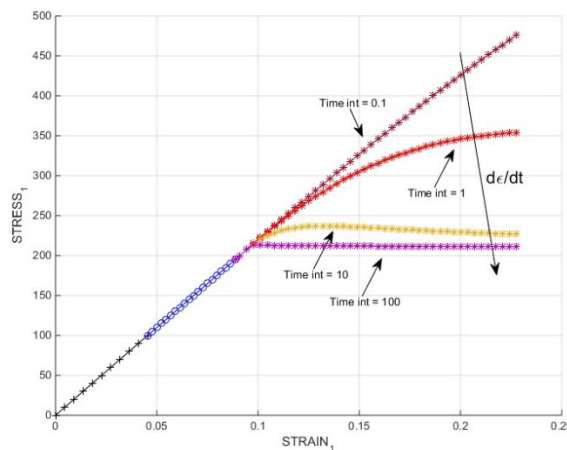


Figure 22: Variation of strain ration.

It can be observed the variation of the strain ratio and the viscosity have similar effect on the strain-stress relation. Another important remark is while the strain ration decreases the system will behave as quasi-static system. In other words, while the strain ration increases the system

is not able to dissipate the energy received, as consequence the material will not be able to resist loads and a total damage of the material could appear.

2.3. Variation of alpha and its influence over strain-stress relation

Different values of alpha are shown in the Table 7. Each of these values will be evaluated. The influence of the variation of this parameter will be studied.

Table 7: Material and Integration parameters.

Material Parameters					Integration Parameters					
E	σ_{yied}	ν	H	η	Total time int.	α				
2000	200	0.3	-0.1	1	100	0	0.25	0.5	0.7	1

As it can be known the alpha method used as time integration method is stable with values of alpha greater or equal than 0.5. The Figure 23 shows that the method becomes unstable and less accurate with alpha values equal to 0 or 0.25. On the other hand, for alpha values greater than 0.5, the method is stable and accurate, in particular, for alpha = 0.5, the method has second order of accuracy.

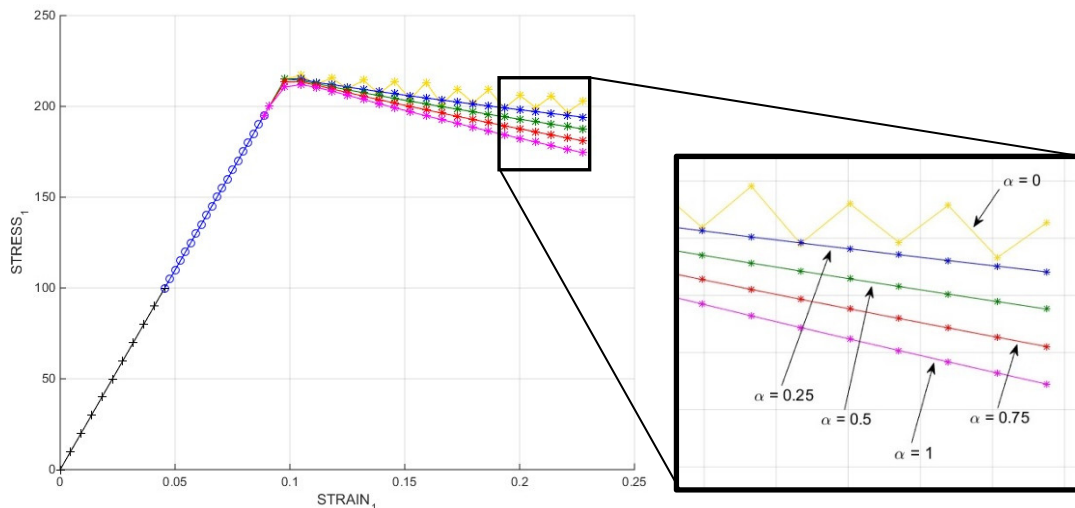


Figure 23: Variation of alpha.

2.4. Variation of alpha and its influence over constitutive matrices

The same parameters will be used. It will be studied the behaviour of the algorithm and tangential constitutive matrices, more precisely the C_{11} component of each matrix.

It can be observed again the lack of stability and accuracy of the method for alpha values less than 0.5. The Figure 24a shows an oscillatory result for the component C_{11} of the algorithm matrix, correspond to alpha value = 0, although an oscillatory result for alpha value equal to 0.25 was not observed, this result does not guarantee the accuracy of it. On the other hand, stable results were computed with alpha values greater or equal than 0.5, in particular, for alpha = 0.5 the method has second order of accuracy.

Similar remarks can be written for the result obtained of the component C_{11} of the tangent matrix. But, for the alpha value equal to 0.25 an oscillatory result was observed and it can be seen in the Figure 24b.

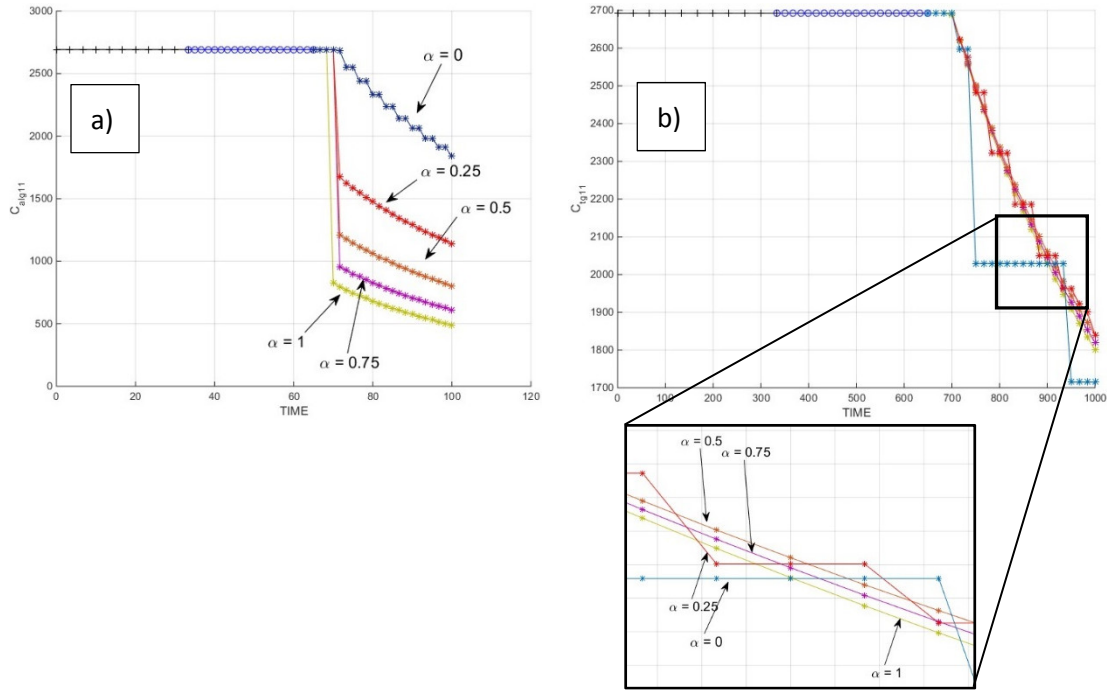


Figure 24: The influence of variation of alpha over: a) Algorithm constitutive matrix and b) Tangent constitutive matrix.

An important remark, between the algorithm and tangent matrices, can be said. First of all, these two matrices were damaged as consequence of the loading path. The computed result made continuous plot for the tangent constitutive matrix (Figure 24b) without a jump between the horizontal line, correspond to the undamaged of the material, and the asymptotic curve, correspond to the damage of the material. On the other hand, it can be seen that the algorithm constitutive matrix showed a discontinued behaviour (Figure 24a). This fact is due to the expression of the algorithm constitutive matrix is a piecewise function, where $C_{alg} = C_{tg}$ in the elastic region and C_{alg} is different than C_{tg} in the damage region.

3. Annex

In this section are included the parts of the code that were modified.

Dibujar_criterio_dano1.m (function)

```

elseif MDtype==2
    tetha=[-pi/2+0.01:0.01:pi-0.01];
    D=size(tetha);
    m1=cos(tetha);
    m2=sin(tetha);
    Contador=D(1,2);

    radio = zeros(1,Contador) ;
    s1 = zeros(1,Contador) ;
    s2 = zeros(1,Contador) ;

    for i=1:Contador
        % McAuly Braket x*(x>0) if x>0 I obtain 1, Otherwise I obtain a 0
        A = m1(i)*(m1(i)>0);
        B = m2(i)*(m2(i)>0);

        radio(i)= q/sqrt([A B 0 nu*(A+B)]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]');

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);

elseif MDtype==3
    tetha=[0:0.01:2*pi];
    %
    %* RADIUS
    D=size(tetha);
    m1=cos(tetha);
    m2=sin(tetha);
    Contador=D(1,2);

    radio = zeros(1,Contador) ;
    s1 = zeros(1,Contador) ;
    s2 = zeros(1,Contador) ;

    alpha_N=0;
    alpha_D=0;
    for i=1:Contador
        % McAuly Braket x*(x>0) if x>0 I obtain 1, Otherwise I obtain a 0
        A = m1(i)*(m1(i)>0);
        B = m2(i)*(m2(i)>0);
        alpha_N =A+B;
        alpha_D =abs(m1(i))+abs(m2(i));
        alpha = alpha_N/alpha_D;
        radio(i)= q/((alpha+(1-alpha)/n)*(sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))] *ce_inv*
            [m1(i) m2(i) 0 ...
                nu*(m1(i)+m2(i))]')));

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);
end

```

Only Tension

Non-symmetric

Non-symmetric

Damage main (function): These modifications were made in order to plot different parameters, such as C_{11} tangent or algorithm, among others.

```

LABELPLOT = {'hardening variable (q)', 'internal variable', 'damage variable'
(d)', 'C_a_l_g_l_1', 'C_t_g_l_1'}

% INITIALIZING (i = 1) !!!!
% *****i*
i = 1 ;
r0 = sigma_u/sqrt(E);
hvar_n(5) = r0; % r_n
hvar_n(6) = r0; % q_n
hvar_n(7) = 0; % New!!! added 16/03/2018 damage at t = 0
hvar_n(8) = ce(1,1); % C_alg_11
hvar_n(9) = ce(1,1); % C_t_11

eps_n1 = strain(i,:);
sigma_n1 = ce*eps_n1; % Elastic (is effective sigma)
sigma_v{i} = [sigma_n1(1) sigma_n1(3) 0; sigma_n1(3) sigma_n1(2) 0 ; 0 0 sigma_n1(4)];

nplot = 5 ; % New
vartoplot = cell(1,totalstep+1) ;
vartoplot{i}(1) = hvar_n(6) ; % Hardening variable (q)
vartoplot{i}(2) = hvar_n(5) ; % Internal variable (r)
vartoplot{i}(3) = hvar_n(7) ; % Damage variable (d)
vartoplot{i}(4) = hvar_n(8) ; % Component 11 Constitutive alg matrix for viscous
vartoplot{i}(5) = hvar_n(9) ; % Component 11 Constitutive tang matrix for viscous

% PLOTTING DAMAGE SURFACE
if viscpr == 0
    if(aux_var(1)>0)
        hplotSURF(i) = dibujar_criterio_danol(ce, nu, hvar_n(6), 'r:', MDtype, n
        set(hplotSURF(i), 'Color', [0 0 1], 'LineWidth', 1);
    elseif (aux_var(1)<=0)
    end
else
end

% VARIABLES TO PLOT (set label on cell array LABELPLOT)
% -----
vartoplot{i}(1) = hvar_n(6); % Hardening variable (q)
vartoplot{i}(2) = hvar_n(5); % Internal variable (r)
vartoplot{i}(3) = hvar_n(7); % Damage variable (d)
vartoplot{i}(4) = hvar_n(8);
vartoplot{i}(5) = hvar_n(9);

```

Rmap_dano1 (function): The modification made on this function were implemented for calculating Algorithm and tangent constitutive matrices, exponential hardening and viscous model.

```

hvar_n1 = hvar_n;
r_n     = hvar_n(5);
q_n     = hvar_n(6);
E       = Eprop(1);
nu      = Eprop(2);
H       = Eprop(3);
sigma_u = Eprop(4);
hard_type = Eprop(5) ;
%viscpr = Eprop(6) ;
eta     = Eprop(7);
alpha   = Eprop(8);
%*****

%*****
%*       initializing                               %*
r0 = sigma_u/sqrt(E);
zero_q=1.d-6*r0; %(1x10-6*ro)
% if(r_n<=0.d0)
%   r_n=r0;
%   q_n=r0;
% end
%*****

%*       Damage surface
%*
[rtrial_prev] = Modelos_de_dano1 (MDtype,ce,eps_n,n); % it's the r on n time step
[rtrial] = Modelos_de_dano1(MDtype,ce,eps_n1,n); % it's the r on n+1 time step
rtrial_n_alpha = rtrial_prev*(1-alpha)+rtrial*alpha; % damage surface at n+alpha step

fload=0;

if viscpr == 0 %inviscid model
    if(rtrial > r_n)
        %Loading
        fload=1;
        delta_r=rtrial-r_n;
        r_n1= rtrial;
        if hard_type == 0
            % Linear
            q_n1= q_n+ H*delta_r;
        elseif hard_type == 1
            % Exponential
            q_inf = r0 + (r0-zero_q); %calculation of q_infinity
            if H > 0
                H_n1 = H*((q_inf-r0)/r0)*exp(H*(1-rtrial_n_alpha/r0)); %calculation of
tangent hard modulus
            else
                H_n1 = H*((q_inf-r0)/r0)*1/(exp(H*(1-rtrial_n_alpha/r0)));%calculation
of tangent soft modulus
            end
            q_n1=q_n+H_n1*(delta_r);
        end
    end
else % Viscous Modelo
    if (rtrial_n_alpha > r_n)
        % loading

```



```

        fload=1;
        delta_r=rtrial_n_alpha-r_n;
        % computation of r at the step n+1
        r_n1 = (eta - delta_t*(1-alpha))/(eta + alpha*delta_t)*r_n + (delta_t/(eta +
alpha*delta_t))*rtrial_n_alpha;

        if hard_type == 0
            % Linear
            H_n1 = H;
            q_n1= q_n+ H_n1*delta_r;
        else
            %Hardening/Softening exponential law
            q_inf = r0 + (r0-zero_q); %calculation of q_infinity
            if H > 0
                H_n1 = H*((q_inf-r0)/r0)*exp(H*(1-rtrial_n_alpha/r0)); %calculation of
tangent hard modulus
            else
                H_n1 = H*((q_inf-r0)/r0)*1/(exp(H*(1-rtrial_n_alpha/r0)));%calculation
of tangent soft modulus
            end
            q_n1 = q_n + H_n1*delta_r; %calculation of q(n+1)
        end
        if(q_n1<zero_q)
            q_n1=zero_q;
        end
    else
% calculation of the Ce_tang_n1
if viscpr == 1
    if rtrial_n_alpha > r_n
        %Algorithm Constitutive Tangent Matrix
        Ce_alg_n1 = (1.d0-dano_n1)*ce+((alpha*delta_t)/(eta+alpha*delta_t))*...
(1/rtrial_n_alpha)*((H_n1*r_n1-q_n1)/(r_n1^2))*((ce*eps_n1')'*
(Ce*eps_n1'));
        C_alg = Ce_alg_n1(1,1);
        %Constitutive Tangent Matrix
        Ce_tan_n1=(1.d0-dano_n1)*ce;
        C_tan = Ce_tan_n1(1,1);
    else rtrial_n_alpha <= r_n
        %Algorithm Constitutive Tangent Matrix
        Ce_alg_n1 = (1.d0-dano_n1)*ce;
        C_alg = Ce_alg_n1(1,1);
        %Constitutive Tangent Matrix
        Ce_tan_n1 = Ce_alg_n1;
        C_tan = C_alg;
    end
end
hvar_n1(5)= r_n1;
hvar_n1(6)= q_n1 ;
hvar_n1(7)=dano_n1;
if viscpr == 1
    hvar_n1(8)= C_alg;
    hvar_n1(9)= C_tan;
end

```

modelo_de_dano1 (function): This implementation was made in order to calculate the different damages region correspond to the different models

```
elseif MDtype==2 %* Only tension
sigma_e = ce*eps_n1';
for i=1:length(sigma_e)
sigma_e_plus(i) = sigma_e(i)*(sigma_e(i)>0);
end
rtrial= sqrt(sigma_e_plus*eps_n1');

elseif MDtype==3 %*Non-symmetric
theta_N=0;
theta_D=0;
sigma_e = ce*eps_n1';
for i=1:2 %length(sigma_e)
sigma_e_plus(i) = sigma_e(i)*(sigma_e(i)>0);
theta_N = theta_N + sigma_e_plus(i);
theta_D = theta_D + abs(sigma_e(i));
end
theta = theta_N/theta_D;
rtrial= (theta+(1-theta)/n)*sqrt(eps_n1*ce*eps_n1');
end
%*****
return
```