Computational Solid Mechanics

ASSIGNMENT 1

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Introduction:

In this report, the implementation of isotropic damage models will be discussed. To cover the various models based on isotropic damage concept, a base class Isotropic DamageMaterial is defined first, declaring E= 20000 N/m^2 , Poisson's ratio = 0.3, etc. The derived classes then only implement a particular damage-evolution law.

The supplied MATLAB code is being implemented with two damage models, viz.,

a) The continuum isotropic damage "non-symmetric tension-compression damage" model.
b) The "tension-only" damage model

PART I (rate independent models)

Furthermore, for the above damage models linear and exponential hardening/softening (H<0 and H>0) laws are being implemented and discussed for the following cases :

- **Non-symmetric tension-compression damage model**

CASE 1:

$\Delta\sigma_1$	$\Delta\sigma_2$	Type of loading
$\sigma_1^{(1)} = 400$	$\sigma_2^{(1)} = 0$	(uniaxial tensile loading)
$\sigma_1^{(2)} = -600$	$\sigma_2^{(2)} = 0$	(uniaxial tensile unloading/compressive loading)
$\sigma_2^{(3)} = 200$	$\sigma_2^{(3)} = 0$	(uniaxial compressive unloading/ tensile loading)

For the linear hardening case of uniaxial loading, in the stress- strain curve in Figure 1, it can be seen that for non- symmetric tension-compression damage model when the material is under tensile loading initially, it behaves elastically below yield stress of 150 N/m^2. The hardening modulus is 0.1. Once it exceeds the yield stress value, on further tensile loading, the material hardens to undergo deformation. This results in the increase in its elastic domain which gives rise to a new damage surface which has an increased value of yield stress limit shown by the dotted line in stress surface diagram in the figure. When further the material is subject to compressive loading, even though it exceeds the yield stress value, the material doesn't undergo deformation since it is within the elastic domain as seen in Figure 1. The material is further subject to tensile loading, since it is within the new elastic limit, there is no deformation as seen in the stress-strain curve.

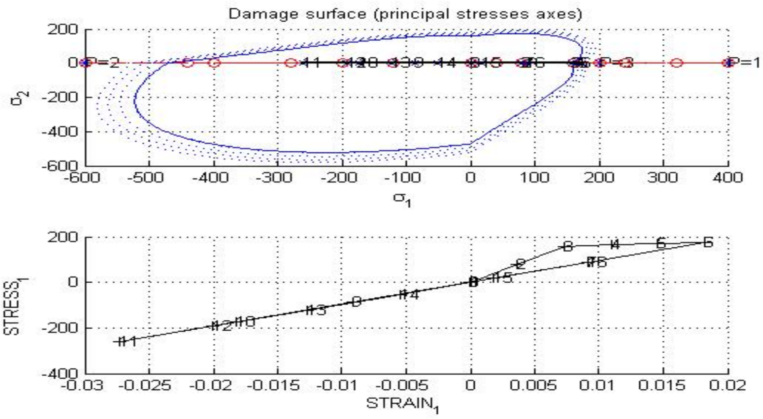

Figure 1. Non-symmetric tension-compression damage model (case 1)

CASE 2:

$\Delta\sigma_1$	$\Delta\sigma_2$	Type of loading
$\sigma_1^{(1)} = 400$	$\sigma_2^{(1)} = 0$	(uniaxial tensile loading)
$\sigma_1^{(2)} = -300$	$\sigma_2^{(2)} = -300$	(biaxial tensile unloading /compressive loading)
$\sigma_2^{(3)} = 500$	$\sigma_2^{(3)} = 500$	(biaxial compressive unloading /tensile loading)

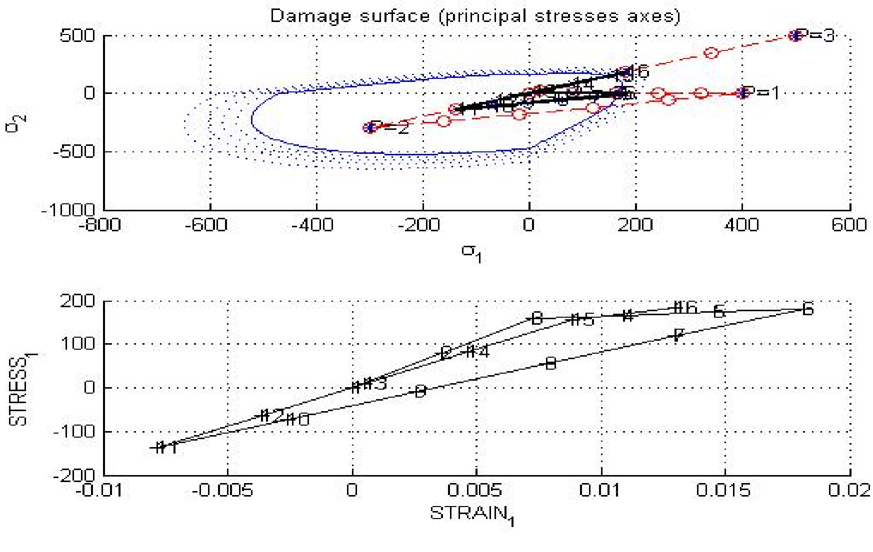

Figure 2. Non-symmetric tension-compression damage model (case 2)

In stress-strain curve of Figure 2., it can be seen that for linear hardening when the uniaxial loading is applied initially, the material behaves elastically until it reaches the yield stress limit of 150 N/m^2 . The hardening/softening modulus is 0.1. When the load is applied further, the material starts deforming and hardening takes place (from point 3 to 6) thus giving rise to new damage surface. On applying biaxial tensile unloading/compressive loading the material doesn't exceed the yield stress value and hence no damage occurs. On application of biaxial tensile loading further it can be seen that even though the loading exceeds the old yield stress value there is no further deformation, since it doesn't exceed the new damage surface. But in some cases it is observed on changing the incremental sigma values, the tensile loading exceeds the new yield surface value and deforms/damages the material further giving rise to new value of damage surface.

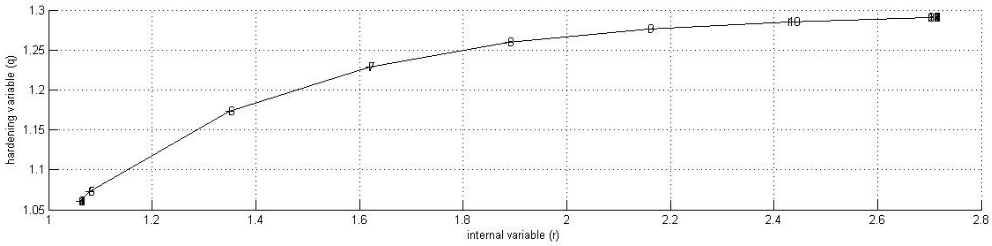

Figure 3 Exponential Hardening for non-symmetric tension-compression damage model (case 2)

Figure 3 is a curve of exponential hardening observed for non-symmetric tension-compression damage model. Here the hardening variable q'(r) >0 and increases exponentially hence hardening of the material has occurred. This behaviour is observed in all the hardening cases irrespective of the loading type.

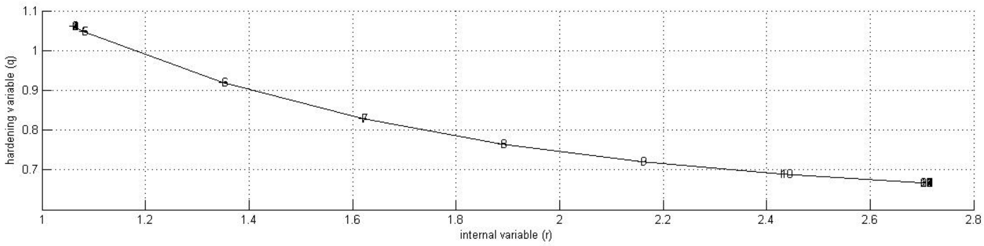

Figure 4 Exponential Softening non-symmetric tension-compression damage model (case 2)

Figure 4, is a curve of exponential hardening observed for non-symmetric tension-compression damage model. The hardening/softening modulus is -0.1. Here the hardening variable q'(r) < 0 and decreases exponentially hence softening of the material has occurred. This behaviour is observed in all the softening cases irrespective of the loading type.

CASE 3:

$\Delta\sigma_1$	$\Delta\sigma_2$	**Type of loading**
$\sigma_1^{(1)} = 400$	$\sigma_2^{(1)} = 400$	(biaxial tensile loading)
$\sigma_1^{(2)} = -300$	$\sigma_2^{(2)} = -300$	(biaxial tensile unloading /compressive loading)
$\sigma_2^{(3)} = 500$	$\sigma_2^{(3)} = 500$	(biaxial compressive unloading /tensile loading)

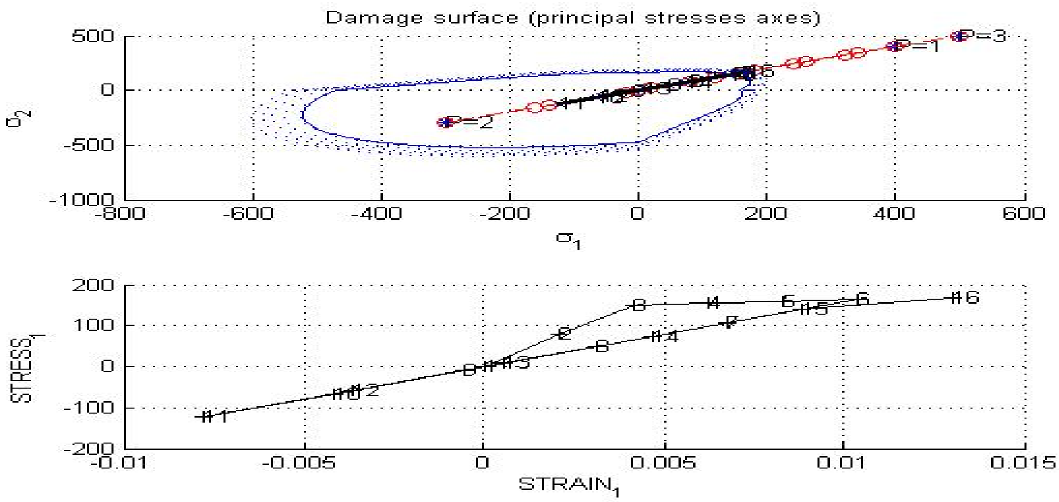

Figure 5. Linear hardening in non-symmetric tension-compression damage model (case 3)

From Figure 5 we can see that, for biaxial tensile loading till yield stress limit of 150 N/m^2, the material behaves elastic, and on further loading, it deforms to expand. The hardening/softening modulus is 0.1. For biaxial compressive loading, the material doesn't deform as it remains in the elastic domain. Further application of biaxial tensile loading makes material stress to exceed slightly with the new elastic domain with new yield stress value, thus giving rise to further damage.

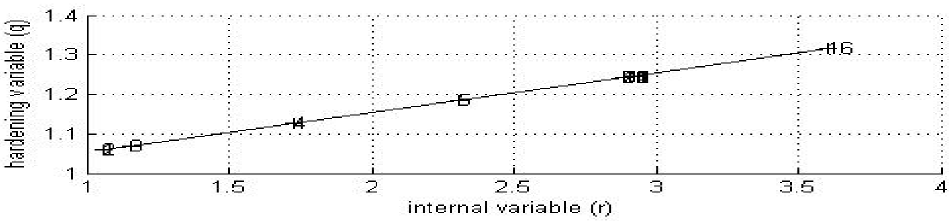

Figure 6 Linear hardening in non-symmetric tension-compression damage model (case 3) q vs r

In the above figure the hardening variable q'(r) < 0 increases linearly hence hardening of the material has occurred**. This behaviour is observed in all the softening cases irrespective of the loading type.**

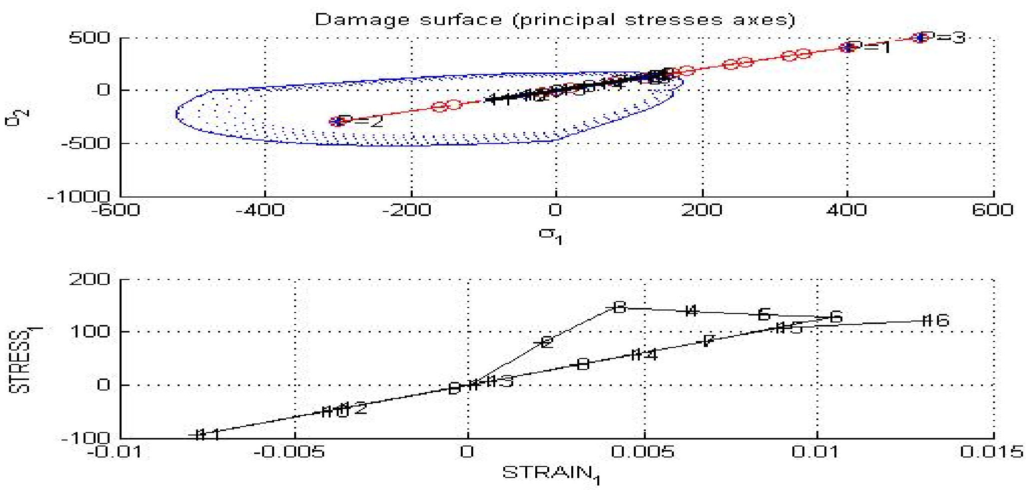

Figure 7 Linear softening in non-symmetric tension-compression damage model (case 3)

Figure 6 is discussed for Linear softening observed in case 3 of non-symmetric tension-compression damage model where the material behaves the same as in case of hardening, except that the deformation leads the body to shrink. The hardening/softening modulus is -0.1.

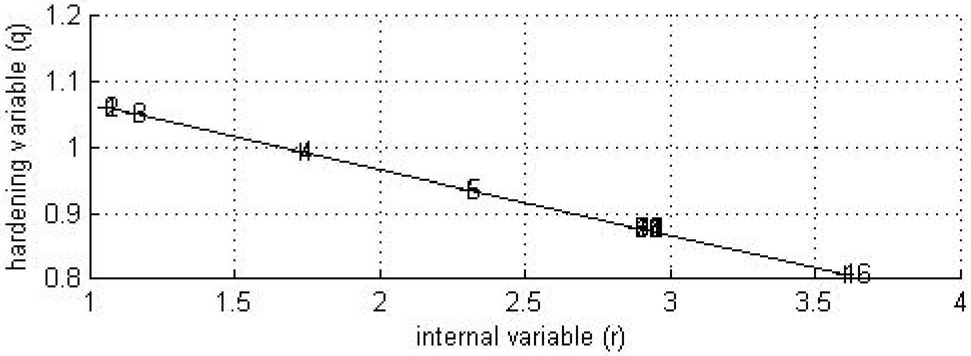

Figure 8 Linear softening in non-symmetric tension-compression damage model (case 3) q vs r

In the above figure the hardening variable q'(r) < 0 decreases linearly hence softening of the material has occurred**. This behaviour is observed in all the softening cases irrespective of the loading type.**

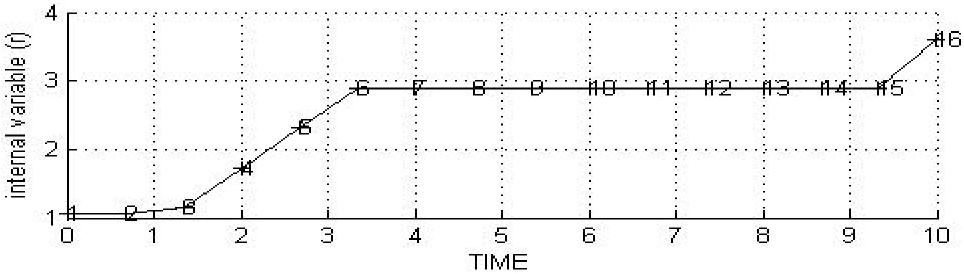

Figure 9 r vs t plot for non-symmetric tension-compression damage model (case 3)

The above plot shows that $\dot{r} \geq 0$, which verifies the correctness of implemented code with theory.

- <u>**Tension only damage model**</u>
  CASE 1:

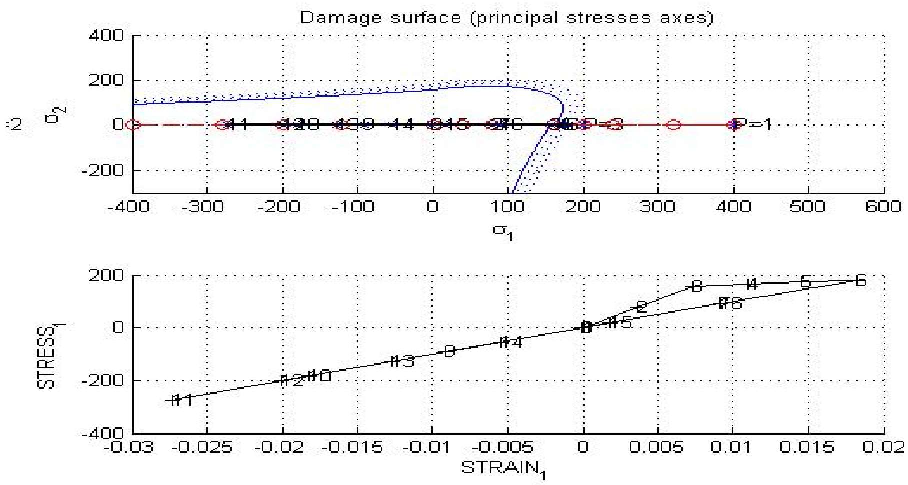

Figure 10 Tension only damage model (case 1)

For the linear hardening case of uniaxial loading, in the stress- strain curve in Figure 1, it can be seen that for tension only damage model when the material is under tensile loading initially, it behaves elastically below yield stress of 150 N/m^2. The hardening/softening modulus is 0.1. Once it exceeds the yield stress value, on further tensile loading, the material hardens to undergo deformation. This results in the increase in its elastic domain which gives rise to a new damage surface which has an increased value of yield stress limit shown by the dotted line in stress surface diagram in the figure. When further the material is subject to compressive loading, even though it exceeds the yield stress value, the material doesn't undergo deformation since it is within the elastic domain as seen in Figure 1. The material is further subject to tensile loading, since it is within the new elastic limit; there is no deformation as seen in the stress-strain curve.

CASE 2:

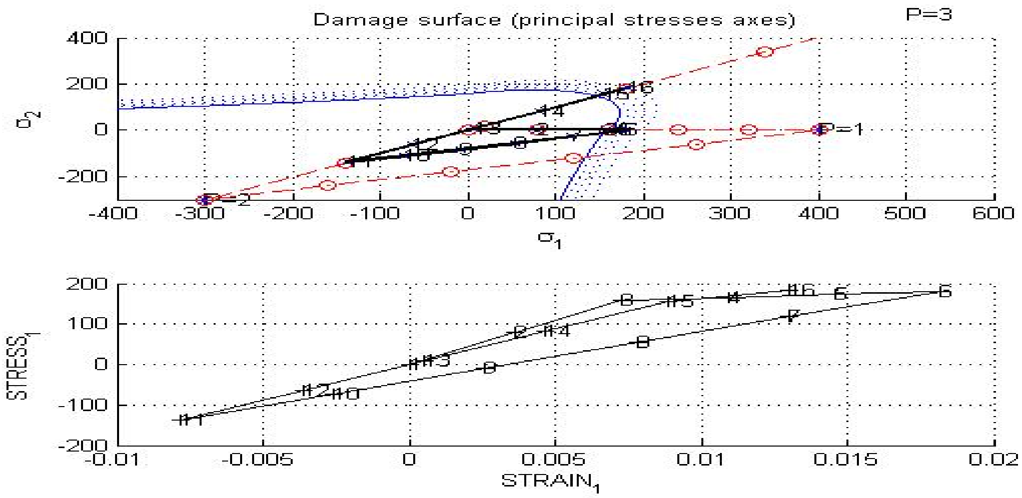

Figure 11 Tension only damage model (case 2)

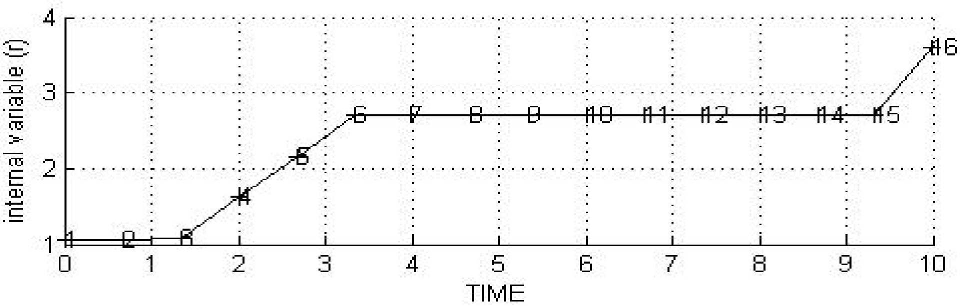

Figure 12 r vs t plot for Tension only damage model (case 2)

The above figure shows internal variable (r) vs time (t) plot for tension only model whose nature is similar to that obtained for non-symmetric tension-compression damage model. Here internal variable remains constant w.r.t. time in elastic regime and results damage linearly after exceeding the yield stress value. It is further constant again in compression loading even after it reaches the yield stress value as it is in elastic regime and on tensile loading it starts damaging linearly again.

CASE 3:

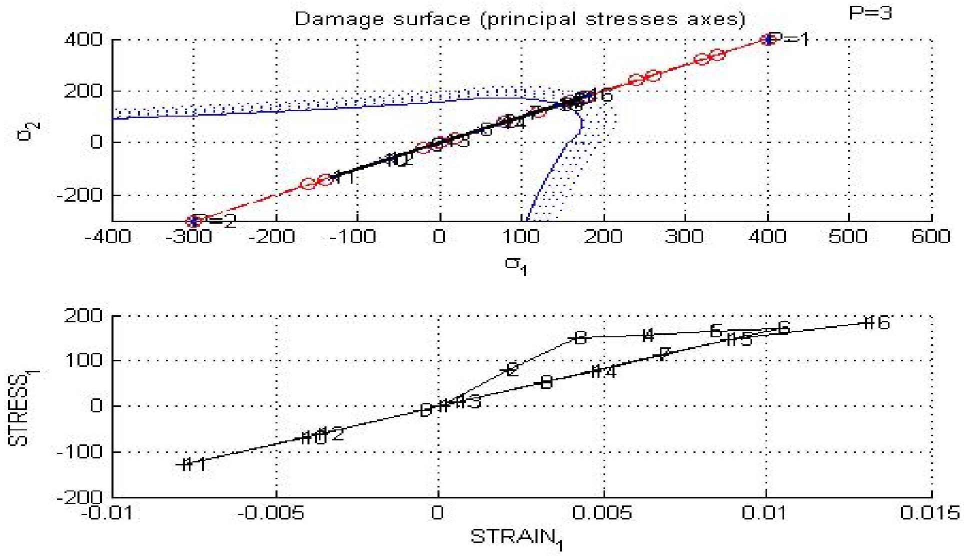

Figure 13 Tension only damage model (case 3)

Figure 11 and 13 show similar behaviour for tension only damage model as those of respective non-symmetric tension-compression damage model.

PART II (Rate dependent models)

The MATLAB code for symmetric tension compression model is being implemented and linear hardening/softening parameter as 0.2 is being solved for following cases.

CASE 1: For variable viscosity parameters

Here, incremental sigma values for uniaxial case are taken as $\sigma_1^{(1)} = 200$, $\sigma_2^{(1)} = 400$ $\sigma_3^{(1)} = 600$ and alpha is taken constant as 1.

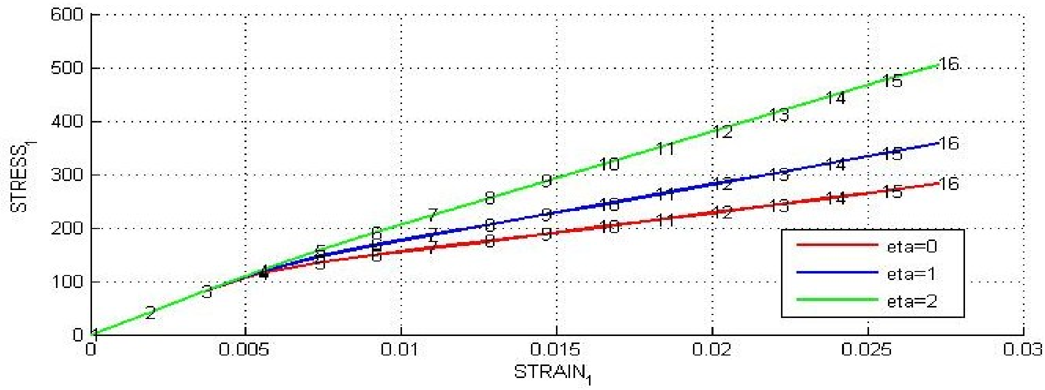

Figure 14 For different viscosity parameters η

The viscosity parameters are taken as $\eta_1 = 0\ \mathrm{N/m^2s}$, $\eta_2 = 1\ \mathrm{N/m^2s}$, $\eta_3 = 2\ \mathrm{N/m^2s}$. As it can be seen from figure 14, as η increases the viscosity of the material increases and hence the damage in less compared to lower values of η. Hence for higher values of η the stress increases faster but the material expands or compresses less.

CASE 2: For variable α values

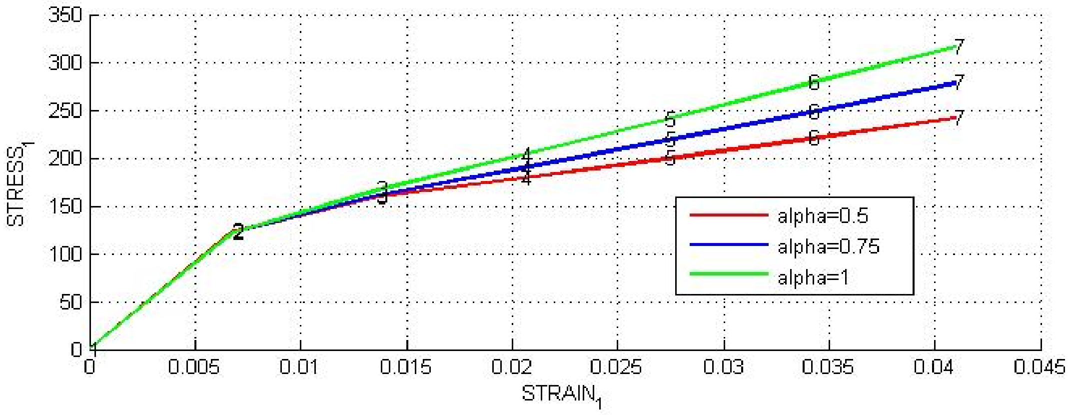

Figure 15 For different values of α

For $\alpha = 0$, the model becomes explicit and hence the stress strain curve oscillates and gives bad prediction of results. Same is the case with $\alpha = 0.25$ where model follows Forward Euler method. It is observed that for alpha= 0 to 0.5 the results are not in good prediction and oscillate. For $\alpha = 0.5$ the model becomes Crank-Nicolson and gives good prediction of results, where even though the stress increases, damage surface doesn't increase proportionally.

CASE 3: For variable strain rates

As shown in $\alpha = 1$ the equation becomes Backward Euler and gives less increase in damage surface on linear increase in stress surface compared to rate independent case.

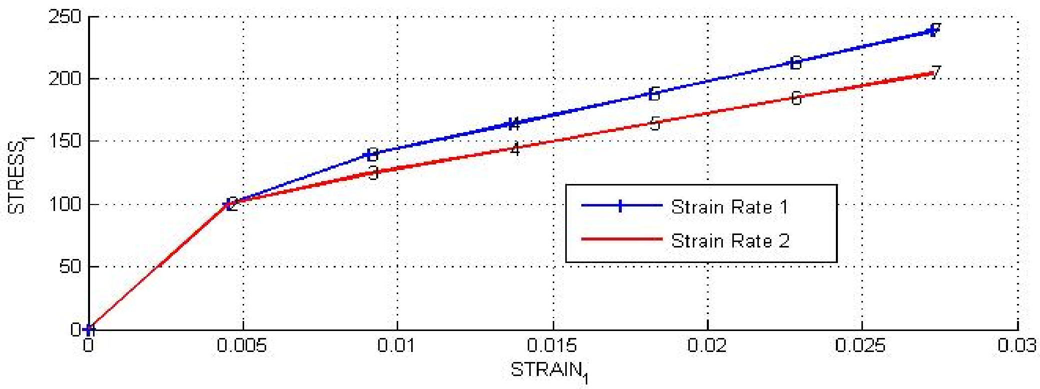

Figure 16 For Variable Strain Rate

The strain rate is changed by changing the TimeTotal in the code. Strain rate is inversely proportional to Total Time. In figure 16, for Time Total = 10 the curve in blue is shown and for TimeTotal = 1000 the red curve is shown. It can be seen that there is less difference in both curves for large change in Total Time. Hence, we can conclude that for large change in Time Total the change in strain rate is very less. This shows that low strain rates—events that occur over a longer time frame—favour the viscous or energy-damping aspects of material behaviour.

APPENDIX

- **For inviscid case:**

Modifications in the routine Modelos_de_dano1.m

```
if (MDtype==1)       %* Symmetric
rtrial= sqrt(eps_n1*ce*eps_n1');

elseif (MDtype==2)  %* Only tension
stress=ce*eps_n1';
stress(stress<0)=0;
rtrial=sqrt(eps_n1*stress);

elseif (MDtype==3)  %*Non-symmetric
    stress=ce*eps_n1';
    stress_plus=stress;
    stress_plus(stress_plus<0)=0;
    num = sum((stress_plus));
    den = sum((abs(stress)));
    theta = num/den;
    rtrial = (theta + (1-theta)/n)* sqrt(eps_n1 * ce*eps_n1');

end
```

Modifications in the routine rmap_dano1.m

```
..........
if (viscrp == 0 )
 if(rtrial > r_n)
   %*   Loading

    fload=1;
    delta_r=rtrial-r_n;
    r_n1= rtrial  ;
    if hard_type == 0
        %  Linear
        q_n1= q_n+ H*delta_r;
    else
         % *******************************************************
%          Hardening/Softening exponential law'
         if H>0
             q_infi=r0*1.3;
         A = (H*r0)/(q_infi-r0);
         H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
         q_n1= q_n + H_new*delta_r;
         else
              q_infi=r0*0.5;
         A = (H*r0)/(q_infi-r0);
         H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
         q_n1= q_n + H_new*delta_r;
         end
    end

    if(q_n1<zero_q)
        q_n1=zero_q;
    end
```

Modifications in the routine dibujar_criterio_dano1.m for Tension only model

```
elseif MDtype==2
    tetha=[0:0.01:2*pi];
    %*******************************************************************************
    %* RADIUS
    D=size(tetha);                      %*  Range
    m1=cos(tetha);                      %*
    n1=m1;
    n1(n1<0)=0;
    m2=sin(tetha);                      %*
    n2=m2;
    n2(n2<0)=0;
    Contador=D(1,2);                    %*


    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;

    for i=1:Contador
        radio(i)= q/sqrt([n1(i) n2(i) 0 nu*(n1(i)+n2(i))]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]');

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);
     axis([-400 600 -300 400])
```

Modifications in the routine dibujar_criterio_dano1.m for Non symmetric tension compression

```
elseif MDtype==3
 tetha=[0:0.01:2*pi];
 %* RADIUS
     D=size(tetha);                       %*  Range
     m1=cos(tetha);
     m2=sin(tetha);
     Contador=D(1,2);
     radio = zeros(1,Contador) ;
     s1    = zeros(1,Contador) ;
     s2    = zeros(1,Contador) ;

    for i = 1:Contador
        den = abs(m1(i))+abs(m2(i));
        n1 = m1(i);
        n2 = m2(i);

        if n1<0
             n1 = 0;
        end
        if n2<0
             n2 = 0;
        end
        num = n1+n2;

        radio(i)= q/(((num/den)+(1-(num/den))/n)*sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*ce_inv*[m1(i) m2(i) 0 ...
        nu*(m1(i)+m2(i))]'));

         s1(i)=radio(i)*m1(i);
         s2(i)=radio(i)*m2(i);
    end
    hplot =plot(s1,s2,tipo_linea);
```

- **For Viscous case:**

Modifications in the routine Modelos_de_dano1.m

```
viscpr = Eprop(6);
if Eprop(6)                          % Viscous
    alpha=Eprop(8);
    if (MDtype==1)                   % Symmetric
        Tn=sqrt(eps_n*ce*eps_n');
        Tn1=sqrt(eps_n1*ce*eps_n1');
        T_alpha=(1-alpha)*Tn+alpha*Tn1;
       rtrial=T_alpha;
    end
end
```

Modifications in the routine rmap_dano1.m

```
if(rtrial > r_n)
%*   Loading

fload=1;
delta_r=rtrial-r_n;
r_n1= (((eta-delta_t*(1-alpha))/(eta+alpha*delta_t))*r_n) + ....
    ((delta_t/(eta+alpha*delta_t))*rtrial);
if hard_type == 0
    %  Linear
    q_n1= q_n+ H*delta_r;
else
    % *******************************************************
    %'Hardening/Softening exponential law)
     q_infi=r0*1.3;
     A = (H*r0)/(q_infi-r0);
     H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
     q_n1= q_n + H_new*delta_r;
end

if(q_n1<zero_q)
    q_n1=zero_q;
end
```