Computational Solid Mechanics
Assignment 2
Report on Computational Plasticity

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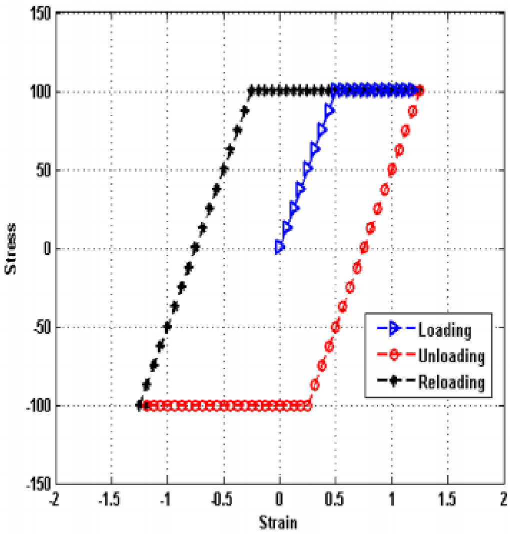
1 1D-PLASTICITY:

1.1 Rate Independent perfect plasticity Models:

In perfect plasticity model there is no hardening allowed i.e., the yield stress level does not depend in any way the degree of plasticisation. And For rate independent case, there is no dependency of η parameter and we set this parameter in small value or zero, but for different loading paths and unloading paths we have different behavior. In our problem, I have considered the uni-axial loading as (250,-250,250) N/m^2 , (300,-300,300) N/m^2 and (350,-350,350) N/m^2 and the yield stress as 100 N/m^2 with other material properties as $\nu = 0.3$, Young's Modulus as E=200 N/m^2 ,Number of time steps = 100. We know that we do not have the presence of K and H here and the study of rate independent model is analysed. For the general statement we can say that in rate independent case loading and unloading is slow.



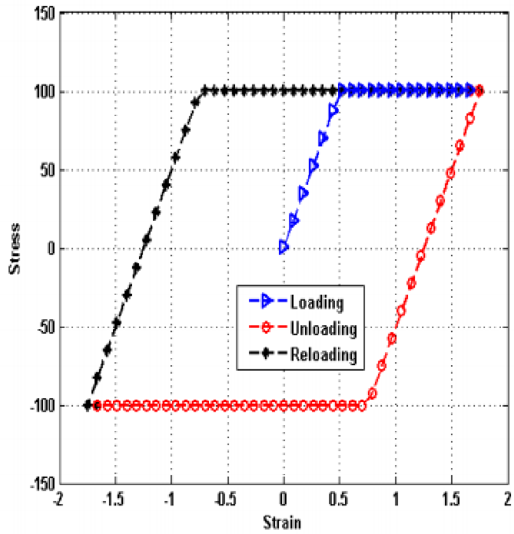


Figure 1: Stress-Strain plots for different loading paths

In the above figure 1 we can see that,the left figure shows the stress-strain plots for loading and unloading values (250,-250,250) N/m^2 and the right figure shows the stress-strain plots for loading and unloading values (350,-350,350) N/m^2. And also from the above figures we can observe that there is linear elastic behavior until the value of stress $100N/m^2$ since it is our yield stress. As and when it passes through the yield stress we have the plastic behavior in loading until certain value of strain and then unloading is done and it behaves the same in unloading and again loading is carried to complete the uni-axial loading and the behavior is close as expected and our implementation is correct.

For uni-axial loading i.e.(250,-250,250)N/m^2 and with η=0,10,20 showed the behavior of rate independent perfect plasticity, but when the loading was increased to (300,-300,300)N/m^2 and (350,-350,350)N/m^2 and with same η values as I mentioned earlier, even though we used rate independent model it showed the behavior of rate independent case which can be seen clearly in Figure 1(right).

1.2 Rate dependent perfect plasticity Models:

For rate dependent case, as η parameter plays its role we are varying it with 10,30,50 and 100 as input and keeping the constant uni-axial loading i.e(450,-450,450)N/m^2. Another test was performed by keeping η=50 and varying the loading with the corresponding values (450,-450,450)N/m^2 and (400,-400,400)N/m^2.

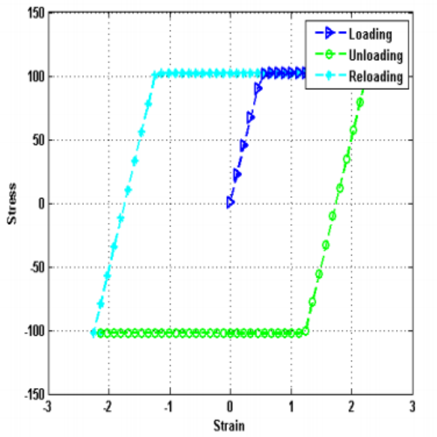


Figure 2: Stress-Strain plots for(450,-450,450)N/m^2 with η=30

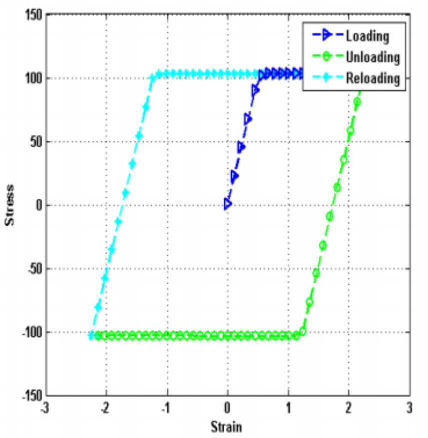


Figure 3: Stress-Strain plots for(450,-450,450)N/m^2 with η=50

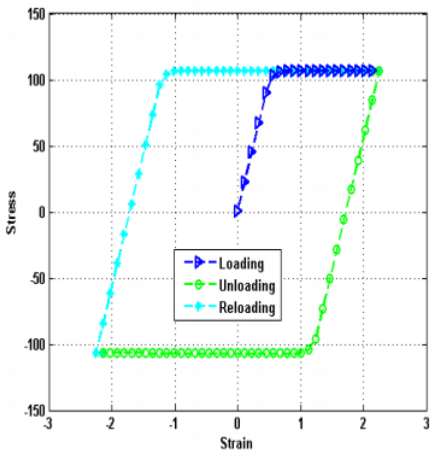


Figure 4: Stress-Strain plots for(450,-450,450)N/m^2 with η=100

Figure 2,3 and 4, shows us the variation of stress by varying the η parameter with respect to time. But as when the value of η is increased we get the variation in stress and strain curve, and we can observe more curvy nature as it passes through the yield stress. In this case the behavior is quite different in the initial loading case. The material behaves as linear elastic until the yield stress but as and when it pass through the yield stress there is some variation in the curve until certain strain value. Because of η and the behaviour is same in the complete unloading and loading cycle. The behavior is that there is a blunt curve after the yield stress is passed.

The variation of the stress with respect to time is like relaxation stepping that is the stress is increased and then decreased and again increased with respect to time for varying loads which can be seen in the plot below in Figure 5.

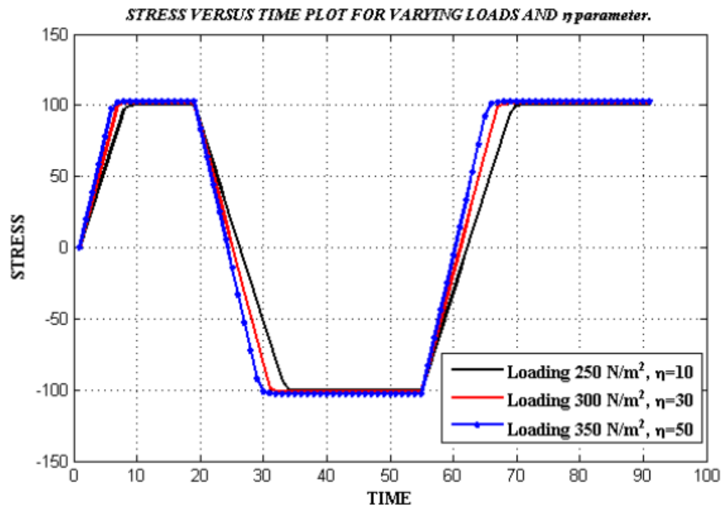


Figure 5: Stress time plot for perfect plasticity with Varying loads and η

1.3 Linear isotropic Hardening:

In isotropic hardening the evolution of the yield surface is such that, at any state of hardening, it corresponds to a uniform expansion of the initial yield surface, without translation. For this model, the elastic domain expands equally in tension and compression during plastic flow.

1.3.1 Rate independent:

In my Code, I have considered an uni-axial loading with (250,-250, 250)N/m^2, yield stress=100 N/m^2 and the Isotropic hardening parameter $\mathbf{K}$ = 100, 150 and 200 for 100 time steps and the study for rate independent model is analysed. For the rate independent case I have considered η=0 or smaller value. We can clearly see from the below figures, the expansion in the yield surface or elastic domain. And as when it passes through the yield stress there is increase this is mainly because of strain in the material increases when subjected to plastic deformation. The symmetry is kept with respect to σ=0 and it has been increased two times the yield surface.

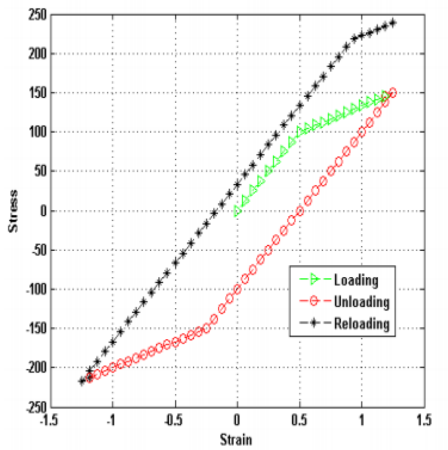


Figure 6: Stress-Strain plots for η=0 and $\mathbf{K}$=100

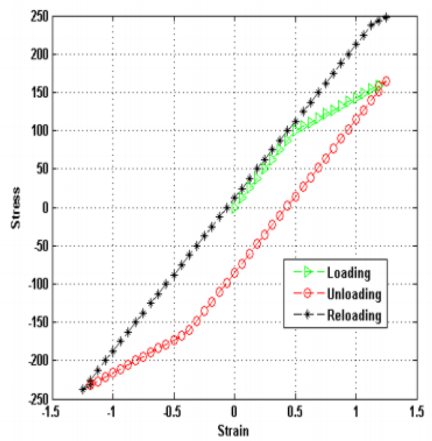


Figure 7: Stress-Strain plots for η=0 and $\mathbf{K}$=150

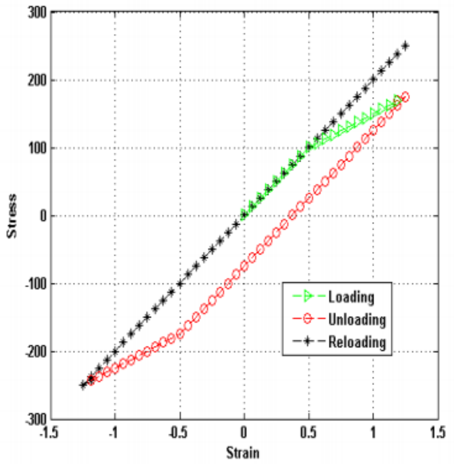


Figure 8: Stress-Strain plots for η=0 and **K**=200

The test was done for different values for isotropic hardening parameter and from the above plots we can say that as and when parameter **K** is increased there are some changes which can be seen from above Figure 6,7 and 8 which is mainly because of strain is increased along with the parameter **K** in the material when it is subjected to plastic deformation.

1.3.2 Rate dependent:

For the rate dependent case I have considered a constant loading of (400,-400,400)N/m^2 and yield stress=250N/m^2, because as the loading value is decreased we get rate independent case even though we increase the value of η. Here we are varying the value of η with values 20, 50 and 100. The parameter **K** is also varied with values 100, 150 and 200 and the behavior of the stress and strain curve is analysed and also the dependence of stress versus time is analysed.

We can observe from the below figures 9,10 and 11 that there is some slant curve as and when the yield stress is crossed after yield surface is increased same as that in the case of rate independent.In the initial loading case the material behaves as linear elastic and as it passes through the yield stress the plastic loading starts until certain value of stress and when unloading cycle starts the material crosses the yield stress not at -200 but in slightly higher value in negative direction which is the behavior of isotropic hardening, where in it shows that there is expansion in the yield surface and the loading cycle is completed.

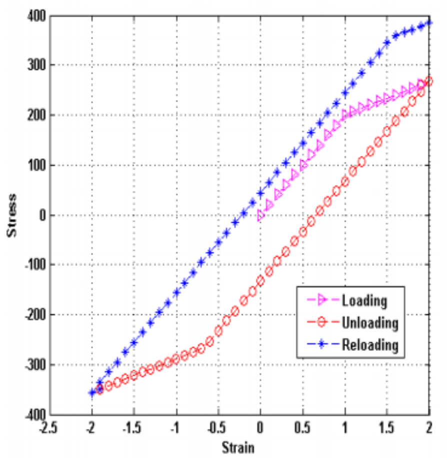


Figure 9: Stress-Strain plots for η=20 and **K**=100

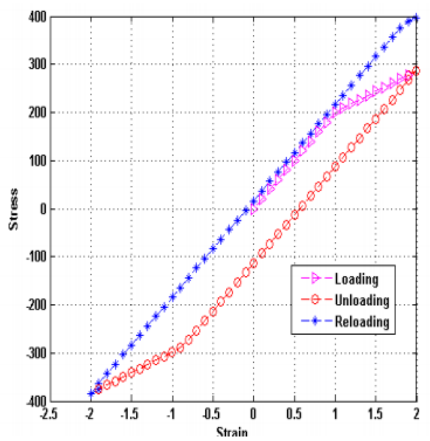


Figure 10: Stress-Strain plots for η=50 and **K**=150

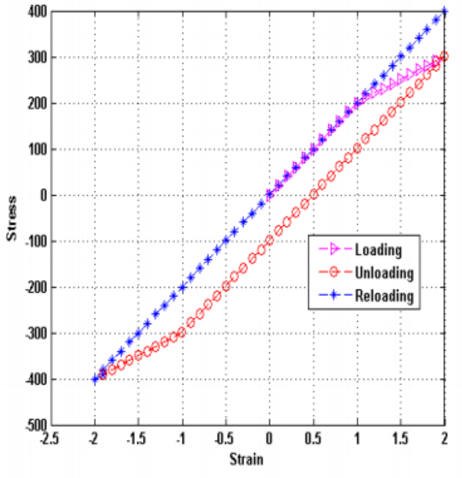


Figure 11: Stress-Strain plots for η=100 and $\mathbf{K}$=200

The variation of stress versus time in rate dependent model is shown in Figure 12 and the variation is explained as the time is increased the stress nature behaves as a manner of relaxing in nature.

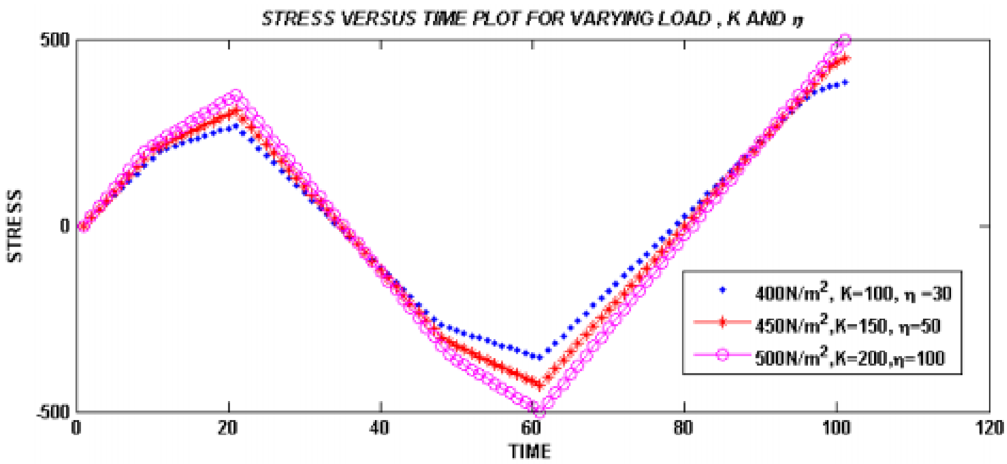


Figure 12: Stress Time plot for Varying loading,η and $\mathbf{K}$ parameter

1.4 Non-Linear isotropic Hardening:

1.4.1 Rate independent:

The behavior of nonlinear isotropic hardening is explained in the following paragraph and Figures depicts the variation of them with constant loading of (16e8,-16e8,16e8)N/m^2 and with E=2.1e11 N/m^2,K=1e11 N/m^2 and η is set to zero. But here we vary the parameter δ with values 5,10,15 and 20. The considered yield Stress σ_y=3e8 N/m^2 and σ_∞ = 5e8 N/m^2.

Figure 13,14 and 15 shows the variation of stress and strain in non-linear isotropic hardening. We can observe that there is expansion in the yield surface which was an expected behavior from isotropic nonlinear model. But as when the value of δ was increased, the rate of the stress was increased asymptotically which can be clearly seen from below Figures 13,14 and 15. But in Figure 15, we can see that there is overlapping of the reloading with the unloading for higher values of δ. This behavior is mainly because of sigma infinity,σ_∞ which was considered to be 5e8 N/m^2. This behaviour can be vanished or reduced by either increasing the yield stress or decreasing the σ_∞. Here I have decreased the value of σ_∞ from 5e8 to 4e8, and the behaviour is checked with increasing the value of δ=20, which can be see in Figure 16 where the behavior is same but there is no overlapping.

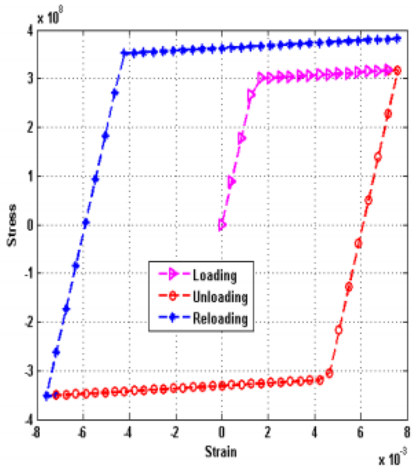


Figure 13: Stress-Strain plots for δ=5

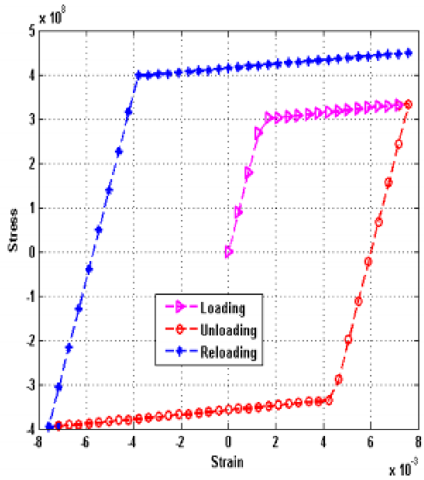


Figure 14: Stress-Strain plots for δ=10

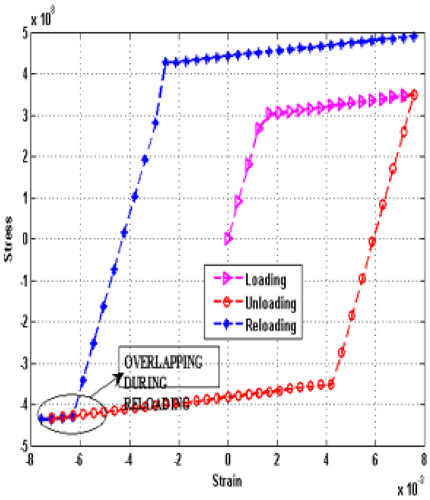


Figure 15: Stress-Strain plots for δ=15

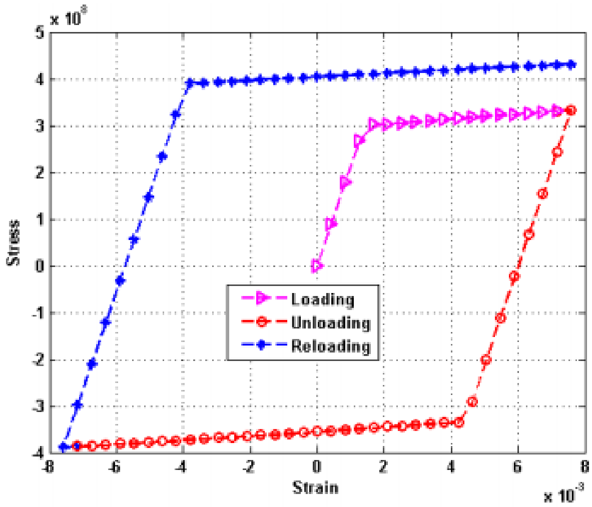


Figure 16: Stress-Strain plots with decreased σ_∞ and δ=20

1.4.2 Rate dependent:

For the rate dependent case we have the influence of the η parameter along with δ which is a must for non-linear isotropic hardening. Here we are varying the values of η and δ. The main difference from the rate independent case is that we can see a slant curve as and when the yield stress is passed and the plastic loading is started, but during unloading the yield stress is not the one which was considered but now it has been increased slightly, which justifies us that the yield surface is expanded which is the basic and main idea of isotropic hardening which can be seen in Figure 17,18 and 19 for values varied in parameters like η and δ. But if we increase the value δ, we can see overlapping of reloading with the unloading . The loading values are considered as same as for rate independent, (16e8,-16e8,16e8)N/m^2.

The below Figure 20 below depicts the behavior of stress with time by varying the parameters η and δ(δ=20, 30, 50 and η=50e12, 10e13, 50e13), where we can see how stress varies with time by presetting the above mentioned parameters. And Figure 21 shows the exponential plot for isotropic nonlinear moduli defined in the stress space.

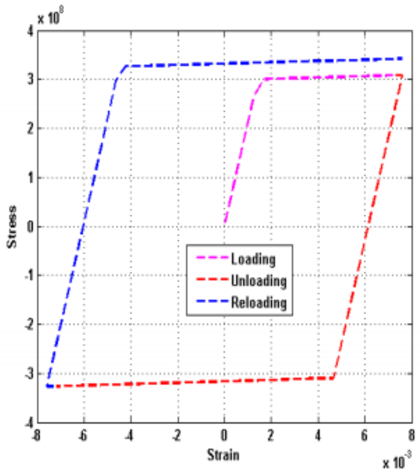


Figure 17: Stress-Strain plots for δ=5 and η=10e12

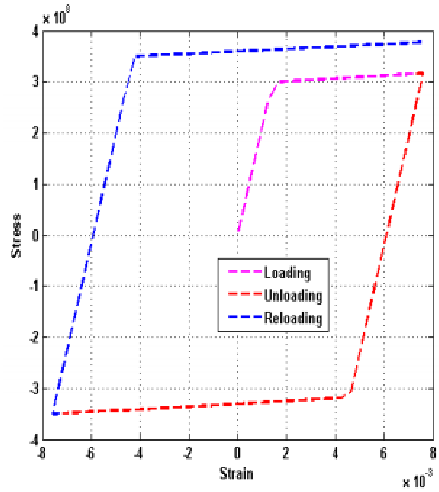


Figure 18: Stress-Strain plots for δ=10 and η=10e13

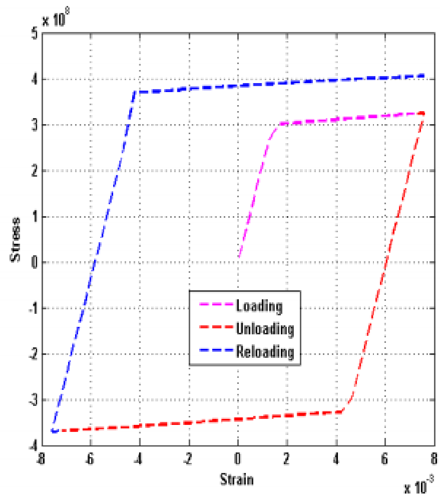


Figure 19: Stress-Strain plots for δ=15 and η=75e13

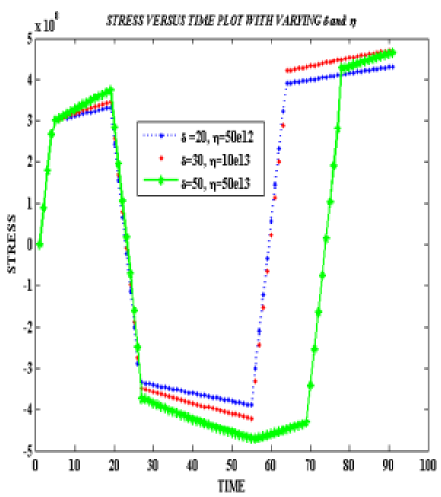


Figure 20: Stress time plot for varying η and δ

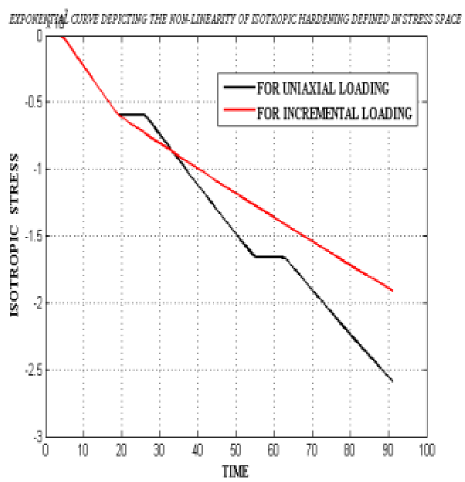


Figure 21: Exponential plot of isotropic hardening moduli defined in stress space .

1.5 Linear Kinematic Hardening:

In kinematic hardening, the yield surface preserve their shape and size but translate in their stress space as a rigid body and size of the yield surface remain constant.

1.5.1 Rate independent:

In my Code, I have considered an uni-axial loading with (250,-250, 250)N/m^2, yield stress=100 N/m^2 and varying the Kinematic hardening parameter **H** for 100 time steps and the study for rate independent model is analysed.

For the rate independent case η can either be zero or lower value. From Figure 22,23 and 24 we can clearly see the translation of the yield surface that is the movement in the surface. When the traction test is performed as and when the stress is passed through the yield surface we have a straight curve for the rate independent case and this straight curve is also seen when we do the unloading test which can be seen in the Figure. The symmetry is not kept with respect to σ=0, but for **H**=100, it starts at 50, this is mainly because of the behavior of the Kinematic Hardening model and hence we have translation not an expansion here. But for **H**=150 and 200, during unloading. the plastic unloading starts very early which can be seen in Figures 23 and 24.

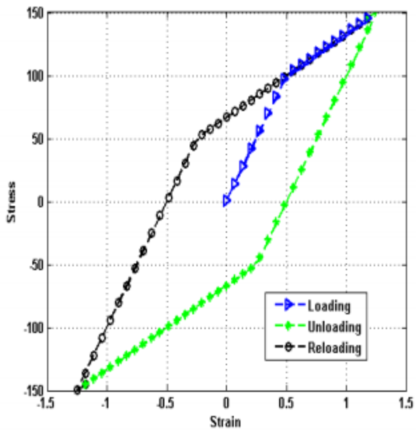


Figure 22: Stress-Strain plots for **H**=100

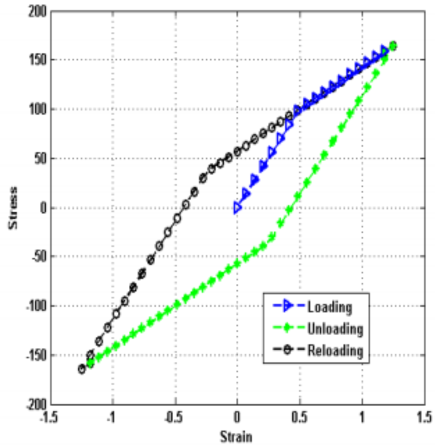


Figure 23: Stress-Strain plots for **H**=150

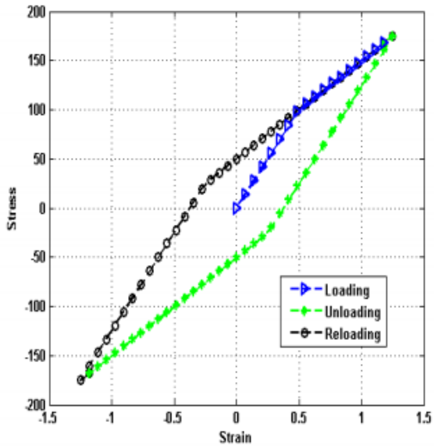


Figure 24: Stress-Strain plots for **H**=200

1.5.2 Rate dependent:

For the rate dependent case I have considered varying η with the values 500,800 and 1000. Even Kinematic hardening variable H is also varied , and the behavior of the yield surface is same as that in the case of rate independent but, the main difference is that there is some slant curve as and when the yield stress is crossed which can be clearly seen in Figures 25,26 and 27 below. In the initial loading case the material behaves as linear elastic and as it passes through the yield stress the plastic loading starts until certain value of stress and when unloading cycle starts the material crosses the yield stress not at -100 but in slightly lower value in negative direction, i.e. -50 N/m^2 , which is the behavior of kinematic hardening, where in it shows that there is translation in the yield surface and the loading cycle is completed. The stress versus time plot is as shown below Figure 28 this is compared with the values obtained for isotropic linear and nonlinear hardening and perfect plasticity as well.

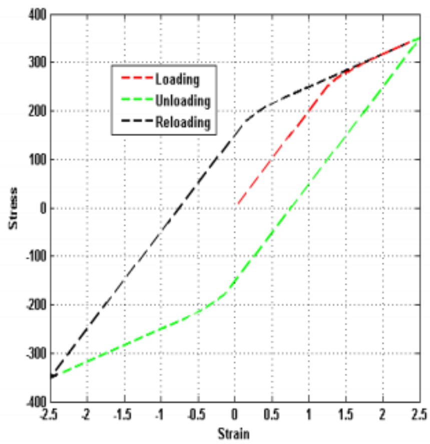


Figure 25: Stress-Strain plots for η=500 and **H**=100

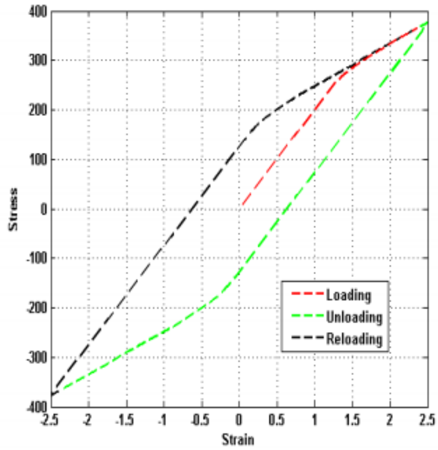


Figure 26: Stress-Strain plots for η=800 and **H**=150

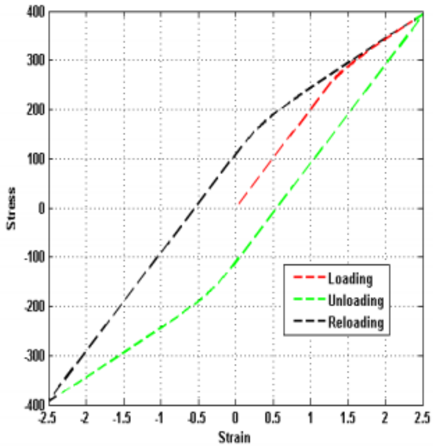


Figure 27: Stress-Strain plots for η=1000 and **H**=200

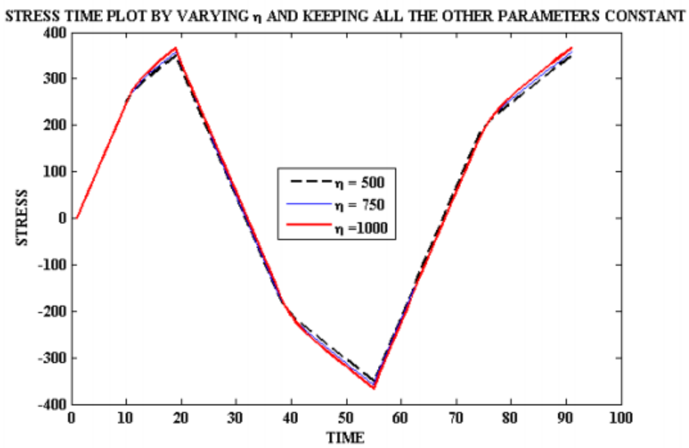


Figure 28: Stress-time plot for varying η

1.6 Non-Linear isotropic and Linear kinematic hardening:

1.6.1 Rate independent:

The behavior of nonlinear isotropic hardening and linear kinematic hardening is explained in the following paragraph and Figures depicts the variation of them with constant loading of (16e8,-16e8, 16e8)N/m^2, E=2.1e11 N/m^2 and with varying **K**,**H**,δ,σ_y and σ_∞. And η is set to zero value. The following Table 1 shows us the different parameters that are varied for the test to be carried.

Parameters	Test 1	Test 2	Test 3
δ	5	10	15
H	0.1e9	1e9	2e9
σ_y	3e8	3.5e8	3e8
K	0.5e9	2e9	3e9
σ_∞	5e8	4.5e8	4e8

Table 1: Table showing input parameters considered for Analysis.

The following Figures 29,30 and 31 shows the variation stress and strain curves for test 1,test 2 and test 3 parameters. From the Figure 29 for test 1, we can see that there is both expansion and translation which is as expected and even it applies the same for test 2 as well shown in Figure 30. Initially for loading the stress is within the linear elastic region until the yield stress, but as it crosses the yield stress we expect plasticity. But, when we consider the unloading, the plasticity doesn't start at after crossing the yield stress but slightly a value below the yield stress considered which is the behavior of Linear kinematic hardening. Again, for reloading we have an expansion that is the plastic loading starts at a greater value than the yield stress which was the behavior of non-linear isotropic hardening.Here the value δ decides the rate at which we get the asymptotic stress. But, when we take the values of test 3 we can see some overlapping (Figure 31) of reloading with unloading which is mainly because of increase in many parameters like **K**,**H**,σ_y and σ_∞. This over lapping of the reloading on unloading can be rectified with change the values or decrease in the values of the above parameters, in order to get the expected stress versus strain curves.

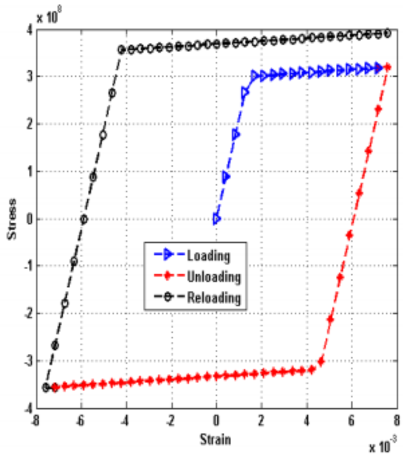


Figure 29: For varying parameters of test 1 from table 1

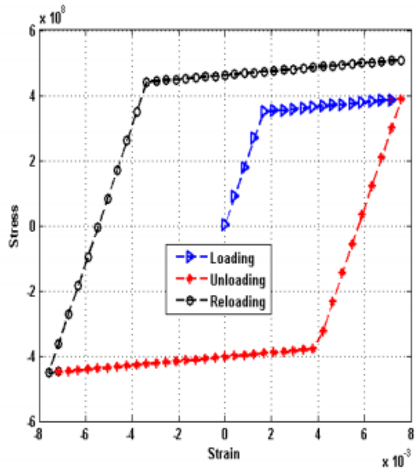


Figure 30: For varying parameters of test 2 from table 1

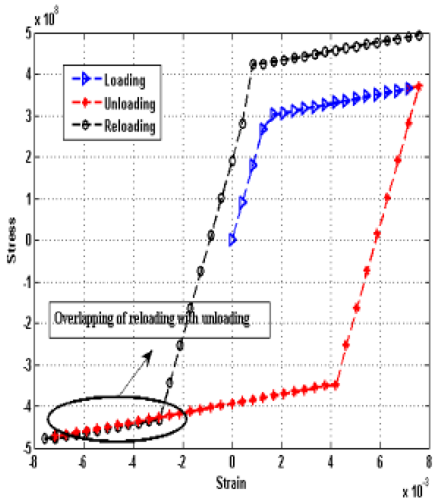


Figure 31: For varying parameters of test 3 from table 1

1.6.2 Rate dependent:

Here the main difference is that the η parameter is varied with values like 10e13,10e14 and 75e14 and the dependence of η is studied. For this case Table 1 was used and considered the same uni-axial loading used in rate independent case. The main difference that we encounter here is because of the exponential hardening the value of δ, σ_y and σ_∞ decides the rate at which we get the asymptotic stress as advised to carry the test for different values the following results were encountered which was very similar to the rate independent. But because of the change in the η parameter we can see a slight curve as and when the yield stress is passed for loading and same for unloading and reloading. There in expansion as well as translation in the boundary of the yield stress which is expected and the code works as required. The following Figure 32 shows the variation of η parameter in stress strain curve (As the behaviour was same as rate independent,only one Figure is shown for this study which depicts the slant variation in the stress and strain curve) and Figure 33 The stress time curve with different parameters and η values.

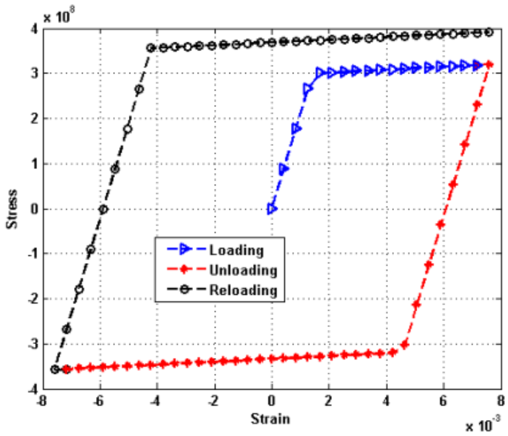


Figure 32: Stress Strain plot for nonlinear isotropic hardening and Linear kinematic hardening with Varying η

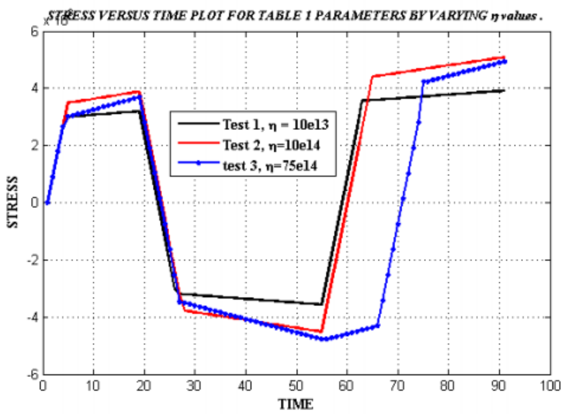


Figure 33: Stress time plot for Nonlinear Isotropic hardening and Linear Kinematic hardening model with Varying η

Note: The matlab code which I have written for 1D Plasticity is attached in the Appendix 1 and 2. In order to change the model from rate dependent to rate independent, we have to change just by simply setting η to smaller value or zero. By this we can retain back the rate independent model.

2 J2-PLASTICITY:

2.1 Rate Independent perfect plasticity Models:

To get the perfect plasticity models the values of Isotropic hardening parameter, **K** and Kinematic Hardening parameter, **H** is set to Zero and dependence is only on Young's modulus, **E**. And for rate independent case we know that η=0. Here I have considered loading paths as [250,-250,250] N/m^2 given in 3D, and Young's modulus, **E**=200 N/m^2, ν=0.3 and total time steps as 100. And we obtain the plots showing perfect plasticity behavior with normal stress-strain and deviatoric stress-strain as shown in Figures 34 and 35.

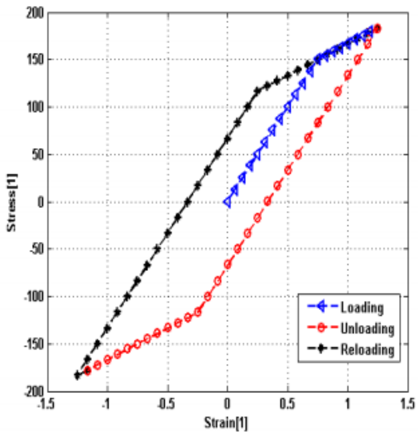


Figure 34: Stress[11]-Strain[11] plots for perfect plasticity

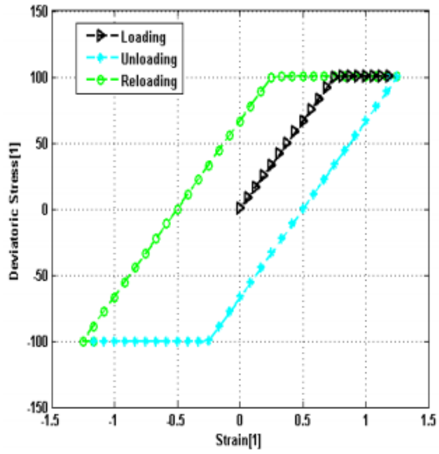


Figure 35: Deviatoric[Stress11]-Strain[11] plot for perfect plasticity

When we compare the above Figures 34 and 35, we can say that the stress and strain behave s normally as expected and even the deviatoric stress versus strain curve as of J2 plasticity model . But, in the stres strain curve we can see that when we are loading as and when we cross the yield stress which is considered to be 150, the plastic loading starts but this was not the case in 1D model since we had a complete instantaneous elastic behavior. But here we can see the plastic loading and same repeats again for the whole Uni-axial loading-unloading cycle. When we see the plot of deviatoric stress and strain we see the instantaneous elastic behavior like in 1D plasticity but the it starts way early than the considered yield stress this is mainly because of the yield function, as the deviatoric stress is $\sqrt{\frac{2}{3}}$ times the yield stress considered. Hence our Instantaneous elasticity state starts at 100 N/m^2.

2.2 Rate dependent perfect plasticity Models:

For this case I have increased the uni-axial loading in order to verify that my code works for all given values which is considered to be [350,-350,350] N/m^2 in 3D, **E**=200 N/m^2, ν=0.3 and the main parameter which depicts rate dependency η=200. And we obtain the plots showing rate dependent perfect plasticity behavior with normal stress-strain and with deviatoric stressas as shown in Figures 36 and 37.

From the Figures 36 and 37, we can see that the behavior is same like in rate independent case but, the only difference here is that we can see a slant curve after the yield stress is passed in normal stress strain curve in Figure 36 which is because of of the η parameter. We can also see slant curve because of the η parameter in Deviatoric stress-strain in curve in Figure 37. Hence we can see that as we increase the value of η parameter we can see even more rate dependency in both the stress strain curve which is plotted below.

Also Figure 38 below shows the stress[11]-time and deviatoric stress[11] and time variation showing the influence of the viscosity parameter and the loading rate. It explains how the stress[11] and deviatoric[stress11] varies with time for perfect plasticity with η=200 and the loading rate.

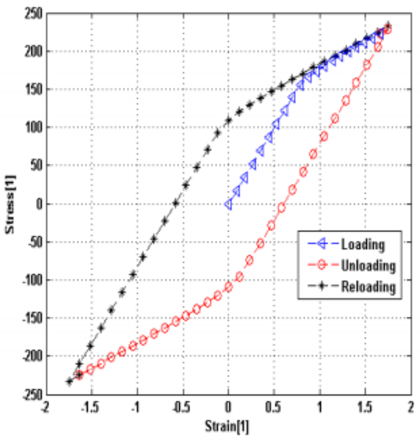


Figure 36: Stress[11]-Strain[11] plots for perfect plasticity

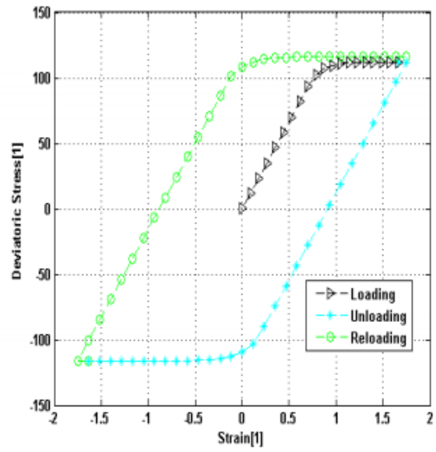


Figure 37: Deviatoric[Stress11]-Strain[11] plot for perfect plasticity

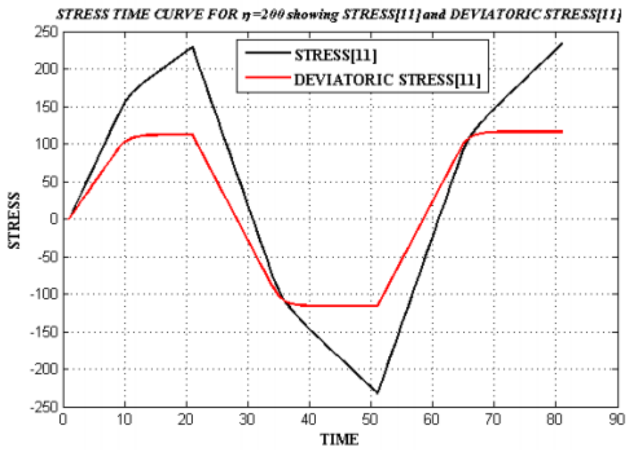


Figure 38: Stress time plot for perfect plasticity with η=200 for stress[11] and dev[stress11]

2.3 Linear isotropic Hardening:

2.3.1 Rate independent:

In this case the considered uni-axial loading is [350,-350,350] N/m^2 in 3D, **K**=150, Young's modulus, **E**=200 N/m^2, ν=0.3 and total time steps as 100. And we obtain the plots showing perfect plasticity behavior with normal stress-strain and with deviatoric stressas as shown in Figures 39 and 40 below. From Figure 39 which shows the plot of normal stress versus strain wherein we can see an expansion of the yield surface and when loading, the plastic loading starts after the yield stress is considered which 150 N/m^2. When we unload we can see that the plastic unloading starts after the yield stress and this is same when we again reload to complete the uni-axial cyclic loading.

But from Figure 40 which shows us the behavior of deviatoric stress versus the strain, we can see that there is an expansion in the yield surface which is mainly because of the Isotropic linear hardening parameter **K**, but we can observe that the plastic loading starts way early because, as the deviatoric stress is $\sqrt{\frac{2}{3}}$ times the yield stress considered, which is 150 N/m^2. Hence our linear hardening starts at 100 N/m^2. And after cyclic loading is completed there is an expansion.

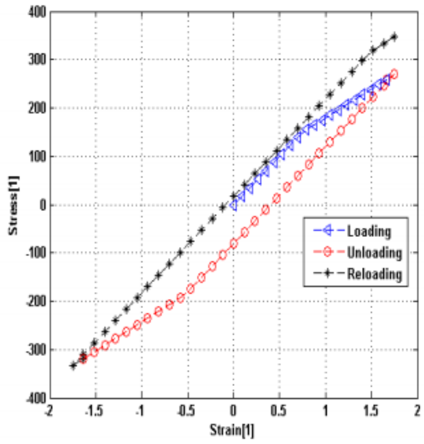


Figure 39: Stress[11]-Strain[11] plots showing Isotropic Linear hardening behavior

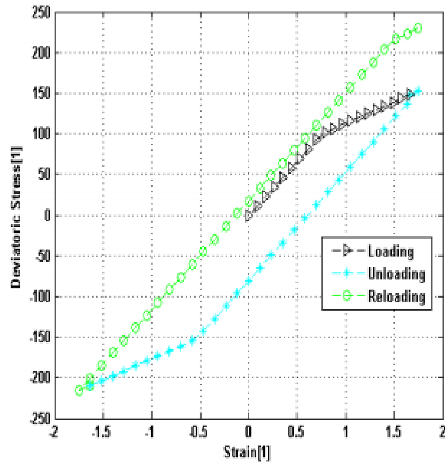


Figure 40: Deviatoric[Stress11]-Strain[11] plots showing Isotropic Linear hardening behavior

2.3.2 Rate dependent:

For the rate dependent case, different loading paths considered to be [400,-400,400] N/m^2. The values of **K** and η parameter is varied and obtained the following plots as shown in Figures 41-44 below.

From the below Figures of rate dependent model for varying the **K** and η parameter, the only difference with rate independent case is that we see a slant curve when the plastic loading starts which is same for both Normal stress and strain as well as deviatoric stress and strain curves.

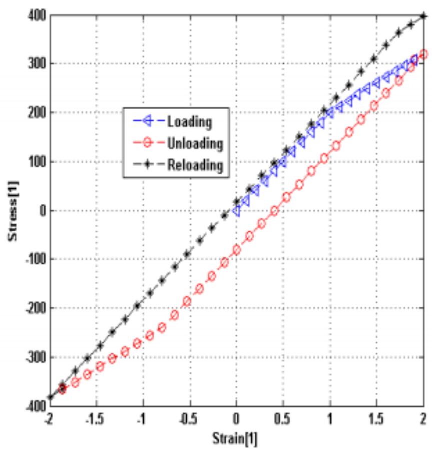


Figure 41: Stress[11]-Strain[11] plots for **K**=150 and η=25

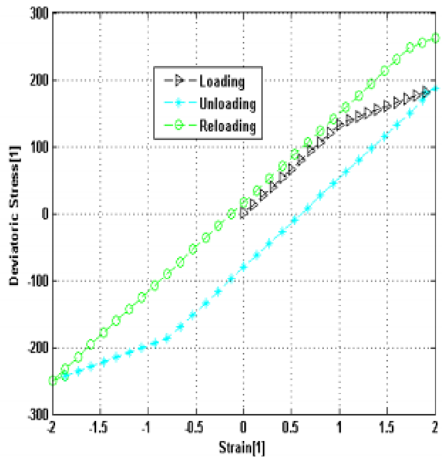


Figure 42: Deviatoric[Stress11]-Strain[11] plot for **K**=150 and η=25

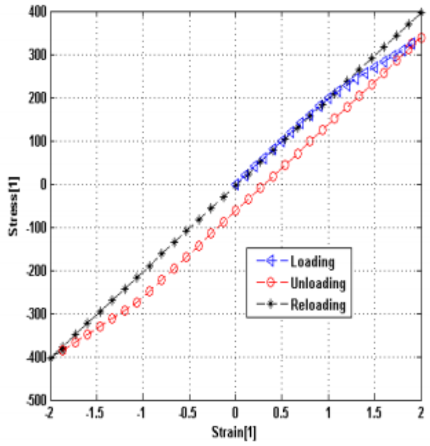


Figure 43: Stress[11]-Strain[11] plots for **K**=250 and η=100

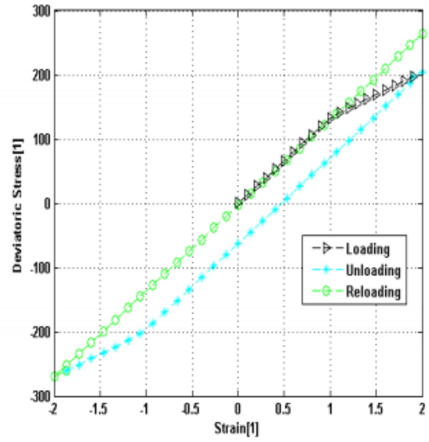


Figure 44: Deviatoric[Stress11]-Strain[11] plot for **K**=250 and η=100

Figure 45 below shows the stress[11]-time and deviatoric stress[11] and time variation showing the influence of the viscosity parameter and the loading rate. It explains how the stress[11] and deviatoric[stress11] varies with time for Isotropic Linear hardening model with varying η and the loading rate.

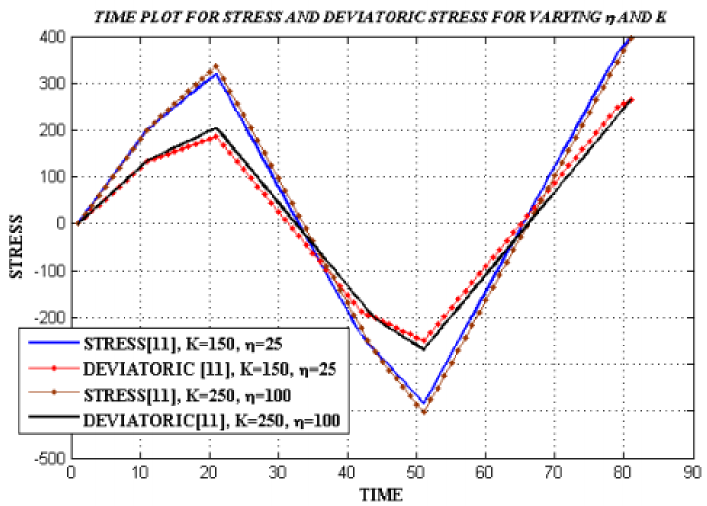


Figure 45: Stress time plot for Isotropic Linear hardening with varying **K** and η for stress[11] and dev[stress11]

2.4 Non-Linear isotropic Hardening:

2.4.1 Rate independent:

For the rate independent case we have considered the following value for the parameters and we are varying the the value of delta δ. The uni-axial loading considered to be [14e8,-14e8,14e8] N/m^2 in 3D, **E**=2.11e11, **K**=2e11, σ_y=6e8 and σ_∞=8e10. The test or simulation was carried out with various values of δ, but for the report I have included the plots for values of δ=20 and 30. Figures 46-49 below shows the plots of Normal Stress-Strain and Deviatoric stress and strain plot for varying δ .

We can see that from Figure 46 and 47, which shows how different stress vary with δ=20 where we can see an expansion in the yield surface and also showing the nonlinear behavior but as we increase the value of δ to 30, we can see that there is an overlapping of the reloading curve over the unloading which mainly signifies the non-linear behavior from Figure 48 and 49 which can be minimized by decreasing the value of δ and σ_∞ which must be very close to σ_y value. Exponential curve for non-linear isotropic hardening curve defined in the stress space for uni-axial loading shown in Figure 50, verifies us that our implementation for non-linear hardening is correct.

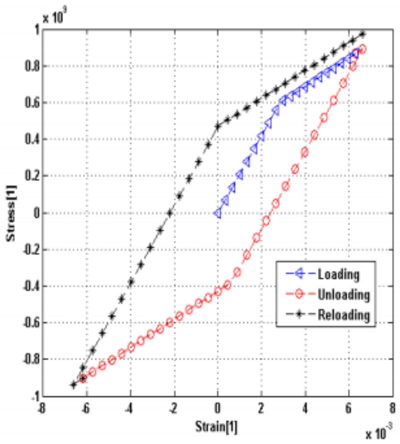


Figure 46: Stress[11]-Strain[11] plots for δ=20

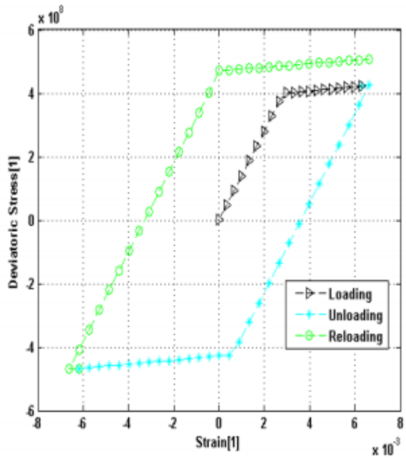


Figure 47: Deviatoric[Stress11]-Strain[11] plot for δ=20

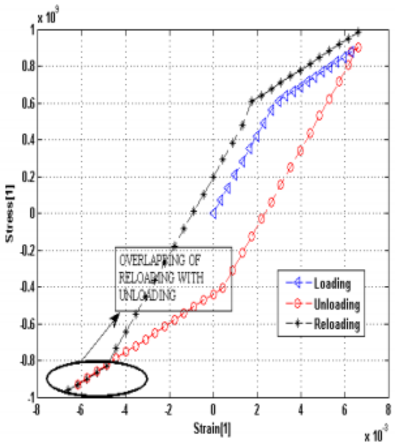


Figure 48: Stress[11]-Strain[11] plots for δ=30

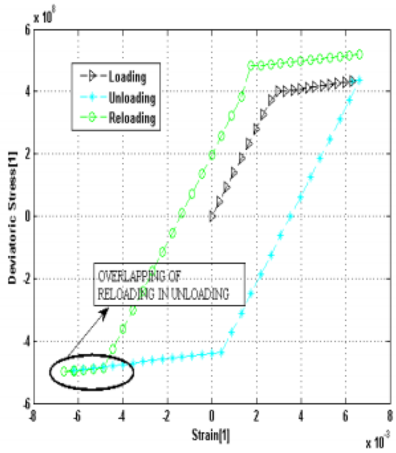


Figure 49: Deviatoric[Stress11]-Strain[11] plot for δ=30

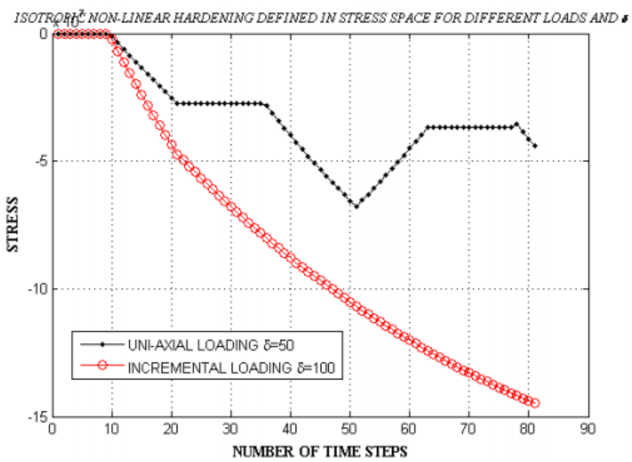


Figure 50: Exponential curve showing for Isotropic Non-Linear hardening curve defined in the stress space for uni-axial loading.

2.4.2 Rate dependent:

For this case I have considered η as 1e13 and all the other parameters are kept constant as in the case of rate independent case and the plots of Stress and deviatoric stress versus strain is analysed, which is shown in below Figures 51 and 52. The main difference is that we can see slant curve in both cases as and when we pass the yield stress which is mainly because of the η parameter. The test was carried out for different values of η but for the convenience for the report the plot for mentioned value of η is considered, and we can also see the expansion in the yield surface.

Figure 53 below shows the stress[11]-time and deviatoric stress[11] and time variation showing the influence of the viscosity parameter and the loading rate. It explains how the stress[11] and deviatoric[stress11] varies with time for Isotropic Non-Linear hardening model with varying η and the loading rate.

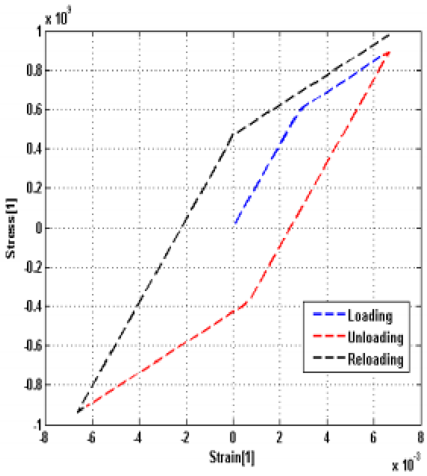


Figure 51: Stress[11]-Strain[11] plots showing Non-Linear isotropic hardening behaviour

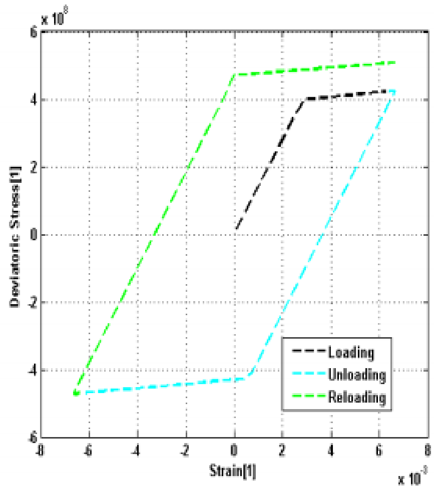


Figure 52: Deviatoric[Stress11]-Strain[11] plot showing Non-Linear isotropic hardening behaviour

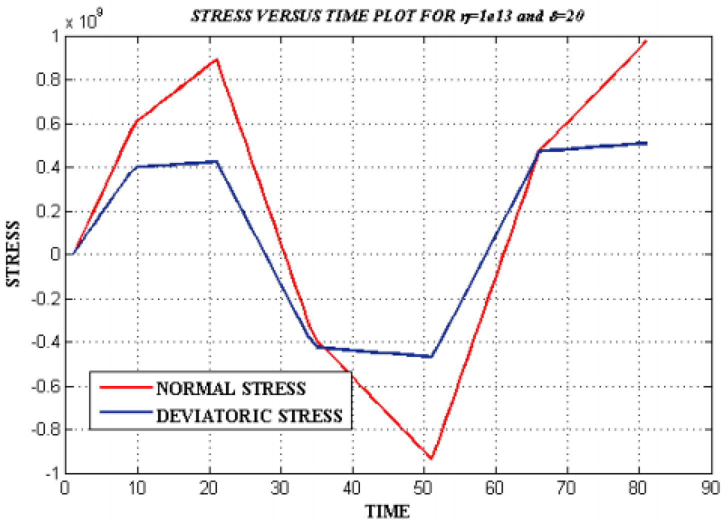


Figure 53: Stress-time plot for Non-Linear isotropic hardening with η for stress[11] and dev[stress11]

2.5 Linear Kinematic Hardening:

2.5.1 Rate independent:

In this case the considered uni-axial loading is [250,-250,250] N/m^2 in 3D, kinematic hardening parameter **H**=150 and the respective parameters considered for the above case. And we obtain the plots showing normal stress-strain and deviatoric stress-strain with kinematic hardening behavior as shown in Figures 54 and 55.

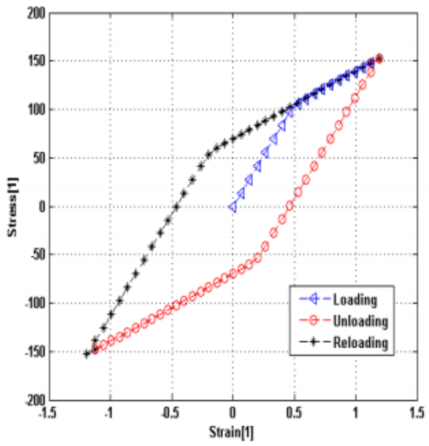


Figure 54: Stress[11]-Strain[11] plots showing Linear Kinematic hardening behaviour

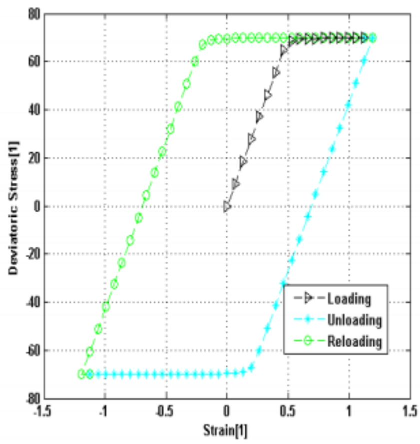


Figure 55: Deviatoric[Stress11]-Strain[11] plot showing Linear Kinematic hardening behaviour

As we can see from above Figures, the behaviour is same as that in the case of 1D kinematic linear hardening as there is translation in the yield surface. And we can see the similar translation in the yield surface in J2 Kinematic linear hardening which is mainly because of the kinematic hardening modulus **H**. As we increase the value of this parameter we get even more refined curves, and the test were carried out for different values of **H**.

2.5.2 Rate dependent:

For the rate dependent case, different loading paths considered to be [400,-400,400] N/m^2 in 3D. The values of **H** and η parameter is varied and obtained the following plots as shown in Figures 56 and 57 below.

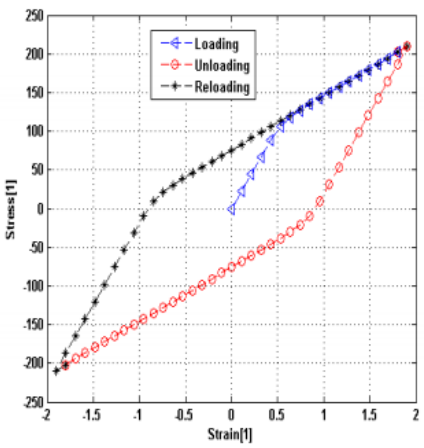


Figure 56: Stress[11]-Strain[11] plots showing Linear Kinematic hardening behaviour

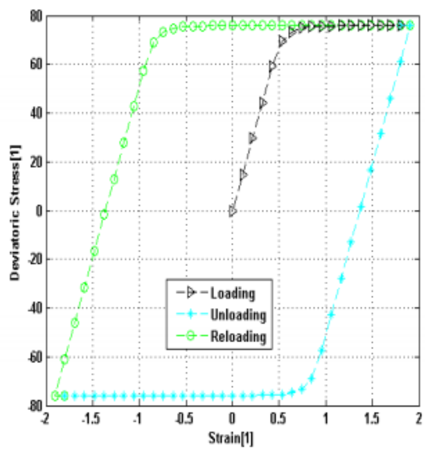


Figure 57: Deviatoric[Stress11]-Strain[11] plot showing Linear Kinematic hardening behaviour

Figures 56 and 57 shows the variation of stress and deviatoric stress versus strain. We can see that there is a translation in the yield surface the test was carried out by varying the values of η and **H**. And also we can observe a slant curve which is mainly because of the η parameter.

Figure 58 below shows the stress[11]-time and deviatoric stress[11] and time variation showing the influence of the viscosity parameter and the loading rate. It explains how the stress[11] and deviatoric[stress11] varies with time for Linear Kinematic hardening model with varying η and the loading rate.

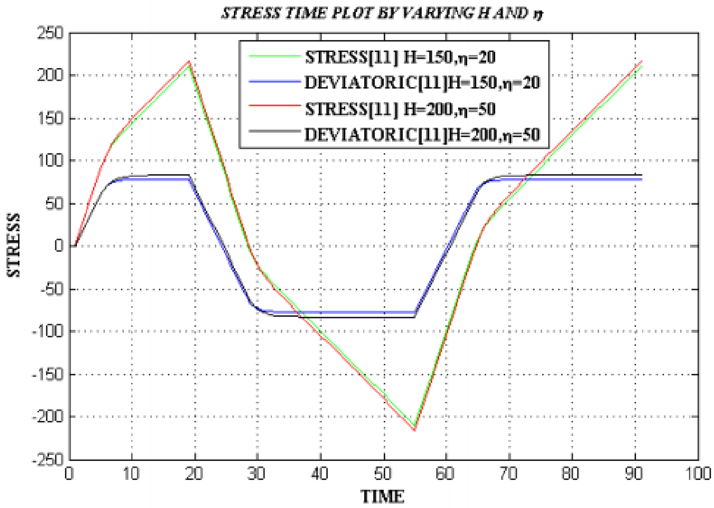


Figure 58: Stress time plot for Linear Kinematic hardening with varying **H** and η for stress[11] and dev[stress11]

2.6 Non-Linear isotropic and Linear kinematic hardening:

2.6.1 Rate independent:

For this case I have considered uni-axial loading to be [14e8,-14e8,14e8] N/m^2 in 3D, **E**=2.1e11, **K**=0.9e8, **H**=0.5e8, σ_y=6e8 and σ_∞=8e10. Here we are varying the alue of Delta δ, hence I have performed simulation for various values of δ, but for report I have included the plots for values of δ=10 and 50. Figures 59-62 below shows the plots of Normal Stress-Strain and Deviatoric stress and strain plot for varying δ .

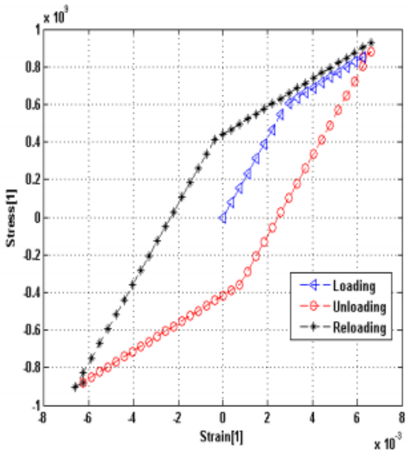


Figure 59: Stress[11]-Strain[11] plots for δ=10

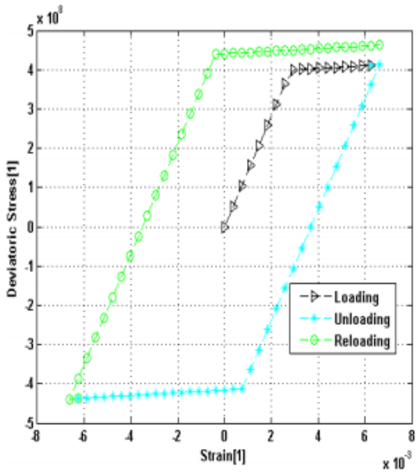


Figure 60: Deviatoric[Stress11]-Strain[11] plot for δ=10

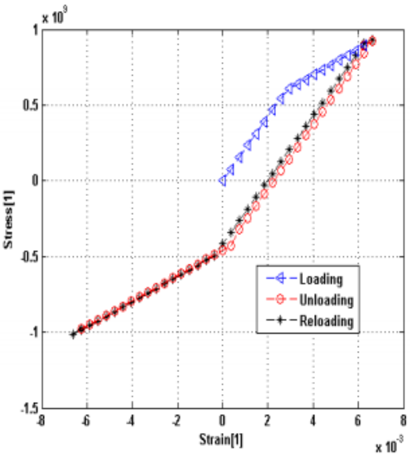


Figure 61: Stress[11]-Strain[11] plots for δ=50

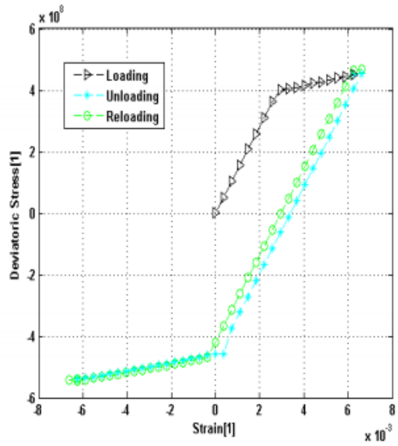


Figure 62: Deviatoric[Stress11]-Strain[11] plot for δ=50

Figure 59 and 60 shows us the variation of normal stress-strain and deviatoric stress-strain with δ=10, we can see that there is both translation and expansion in the yield surface and the nonlinear behaviour in terms of isotropic hardening as well. But when we increase the value of δ further to 50 we can observe the overlapping of reloading with unloading which is because of nonlinear isotropic hardening which can be seen in Figures 61 and 62. But this can be reduced by either decreasing the value of δ or setting the value of σ_∞ close to σ_y.

2.6.2 Rate dependent:

For this case I have considered η=1e13 and all the other parameters are kept constant as in the case of rate independent case and the plots of stress-strain and deviatoric stress-strain is analysed. The test was carried out for different values of η but for the convenience of the report, the plot for mentioned value of η is shown. The main difference is that we can see a slant curve in both cases as and when we pass the yield stress this is mainly because of the η parameter. Obtained plots for rate dependent case is shown in Figures 63 and 64 below. We can observe the expansion and translation in the yield surface.

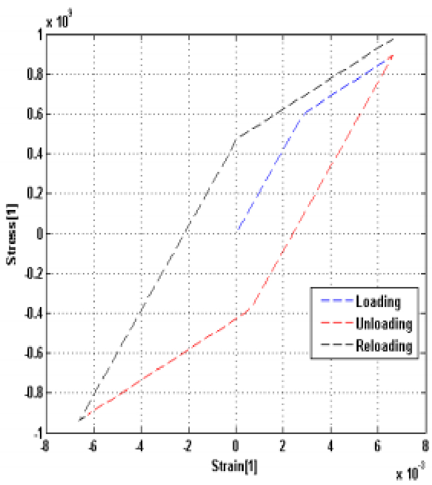


Figure 63: Stress[11]-Strain[11] plots for η=1e13

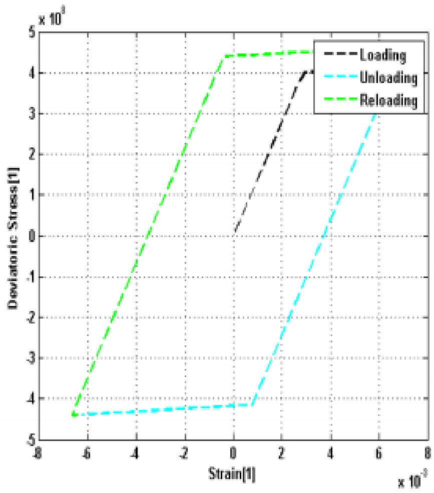


Figure 64: Deviatoric[Stress11]-Strain[11] plot for η=1e13

The stress variation with respect to time and the exponential curve verifying the non-linear behavior of isotropic hardening in the stress space can be seen in Figure 65 and 66 below, where we can see how the stress[11] and deviatoric[stress11] varies with time with varying η and the loading rate.

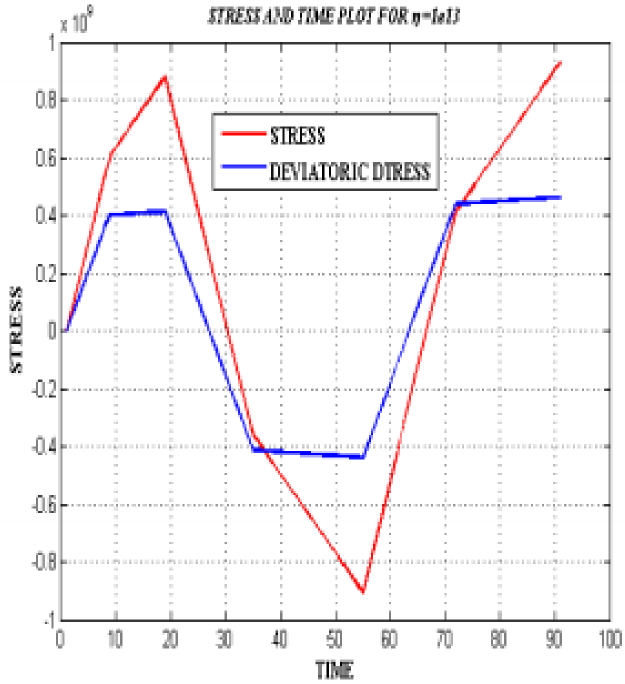


Figure 65: Stress-time plot for Non-Linear isotropic and Linear kinematic hardening with η for stress[11] and dev[stress11]

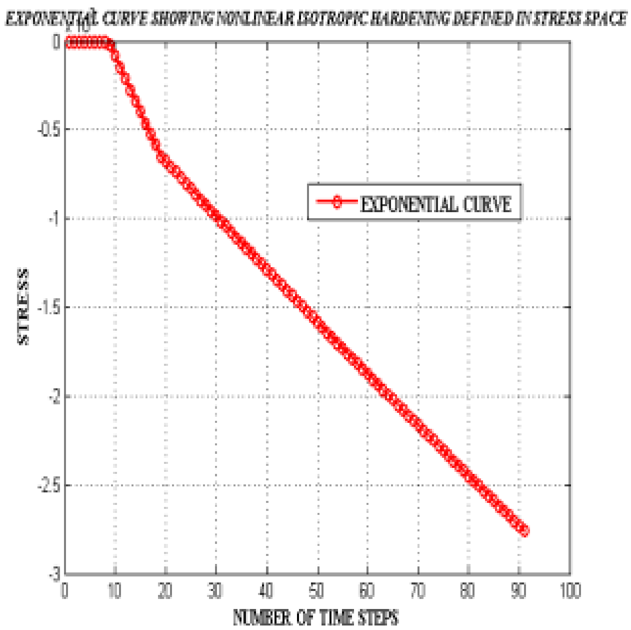


Figure 66: Exponential curve showing for Isotropic Non-Linear hardening curve defined in the stress space for uni-axial loading.

Note: The matlab code which I have written for J2 Plasticity is attached in the Appendix 3,4 and 5. In order to change the model from rate dependent to rate independent, we have to change just by simply setting η to smaller value or zero. By this we can retain back the rate independent model.

APPENDIX 1: MATLAB Subroutines 1D Plasticity – ‘Plasticity_main_1d.m’

```
%%%%%%%%%%%%%%%%%%  $$ ANIL BETTADAHALLI CHANNAKESHAVA $$  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all

%MATERIAL PROPERTIES
sigma_y = 6e8 ; %yield stress
E = 2.1e11 ; %Young modulus
K = 0.9e9 ; %Isotropic hardening Parameter
H = 0.5e9 ; %kinematic hardening parameter
eta = 1e8 ; %viscocity coefficient for rate independent case
delta = 5 ; %paramter for non-linear isotropic hardening
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Rate dependence and independence
disp( ' Please choose model type : ' )
disp( '     [ 1 ] = Rate Independent ' );
disp( '     [ 2 ] = Rate Dependent ');
temp= input('Model Type=');
if temp==1
  Visco =2;
else
  Visco =1;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Linear or Non-Linear Isotropic hardening
disp( ' Please choose model type : ' )
disp( '     [ 1 ] = Linear ' );
disp( '     [ 2 ] = Non-Linear ');
hardening= input('Iso_hardening type=');
if hardening==1
  hardtype =1;
else
  hardtype =2;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%LOADING RANGE- Stress is specified.This is converted to strain which is
%given as input to the model
sigma1 =4000;
sigma_tri1 = 1*sigma1;
sigma_tri2 = -1*sigma_tri1;
sigma_tri3 = 1*sigma1;
%strain correspond to the specified stress
eps(1) = sigma_tri1/E;
eps(2) = sigma_tri2/E;
eps(3) = sigma_tri3/E;
steps = [18 36 36];
eps_1 = linspace(0,eps(1),21);
eps_2 = linspace(eps(1),eps(2),41);
eps_2 = eps_2(2:end);
eps_3 = linspace(eps(2),eps(3),41);
```

```
eps_3 = eps_3(2:end);
eps_input = [eps_1 eps_2 eps_3 ]; % array with input strains at each step
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Timetotal = 100;
timeincrement1=Timetotal/(sum(steps));
timeincrement2=Timetotal/(sum(steps));
timeincrement3=Timetotal/(sum(steps));
totalstep = sum(steps);
delta_t = [timeincrement1 timeincrement2 timeincrement3];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
i=1;
epsilon_pn = [0 0 0];%i n p u t
sigma_niso=0;
sigma_nkin=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for iload = 1:3
    for iman = 1:steps(iload)
    e_n1 = eps_input(i);
    epsilon_n1 = [e_n1 0 0]; %strain vector at n+1
    epsilon_pn1tr = epsilon_pn; %Plastic strain trial vector at n+1
    epsilon_en1tr = epsilon_n1 - epsilon_pn1tr ; %Elastic strain trial vector at n+1
    sigma_n1tr = [(E*epsilon_en1tr(1)) (K*epsilon_en1tr(2)) (H*epsilon_en1tr(3))] ; %stress-
    %-trial vector at n+1
    f_n1tr = abs(sigma_n1tr(1)-sigma_n1tr(3)) - sigma_y + sigma_n1tr(2);%Trial yield function
    if f_n1tr <=0 %if stress due to strain at n+1 within elastic domain
       epsilon_pn1 = epsilon_pn1tr; %Plastic strain at n+1 is Plastic trial vector at n+1
       epsilon_en1 = epsilon_en1tr; %Elastic strain at n+1 is Elastic trial vector at n+1
       sigma_n1 = sigma_n1tr;       %Stress vector at n+1 is Stress trial vector at n+1
       sigma_n1_iso = sigma_n1(2);
    else                            % Stress due to strain at n+1 is out of elastic domain
    % Return mapping algorithm is used for updating stress and strain
    if Visco==1                     %for rate dependent model
        if hardtype==1              %for linear isotropic hardening
    % gama_n1 = (1/ (E + K + H +(eta/delta_t(iload)))*delta_t(iload))* f_n1tr ;
    epsilon_epn1 = epsilon_pn(1)+(E + K + H +(eta/delta_t(iload)))^-1*f_n1tr*sign(sigma_n1tr(1)-
sigma_n1tr(3));
    epsilon_ison1 = epsilon_pn(2)+(E + K + H +(eta/delta_t(iload)))^-1*f_n1tr ;
    epsilon_kinn1 = epsilon_pn(3)-(E + K + H +(eta/delta_t(iload)))^-1*f_n1tr*sign(sigma_n1tr(1)-
sigma_n1tr(3));
    epsilon_pn1 =[epsilon_epn1  epsilon_ison1  epsilon_kinn1] ;
    %Computing Plastic strain at n+1 from Plastic strain at n
    sigma_n1(1) = sigma_n1tr(1) - ( E + K + H +(eta/delta_t(iload)))^-1*f_n1tr*E*(
abs(sigma_n1tr(1) - sigma_n1tr(3))/(sigma_n1tr(1)- sigma_n1tr(3))) ;
    sigma_n1(2) = sigma_n1tr(2) - ( E + K + H +(eta/delta_t(iload)))^-1*f_n1tr * K;
    sigma_n1(3) = sigma_n1tr(3) - ( E + K + H +(eta/delta_t(iload)))^-1*f_n1tr*H*(
abs(sigma_n1tr(1) - sigma_n1tr(3))/(sigma_n1tr(1)- sigma_n1tr(3))) ; % E_ep = E
% *(1/(1-E*(E+K+H+(eta/dtime))));
             %Computing Stress vector at n+1 from Stress trail vector at n+1
        else     % for non linear isotropic hardening case
              %interative method for finding the value of gamma
              [sigma_n1_iso,gam_n1_k1] =
```

```
nonlinear_1d(f_n1tr,epsilon_pn(2),sigma_niso,sigma_y,E,H,delta,Visco,delta_t(iload));
               sigma_n1(1) = sigma_n1tr(1)-gam_n1_k1*delta_t(iload)*E*sign(sigma_n1tr(1)-
sigma_n1tr(3));
               sigma_n1(2)=sigma_n1_iso;
               sigma_n1(3)= sigma_n1tr(3)+gam_n1_k1*delta_t(iload)*H*sign(sigma_n1tr(1)-
sigma_n1tr(3));
               %Computing Stress vector at n+1 from Stress trail vector at n+1
               epsilon_epn1 = epsilon_pn(1)+gam_n1_k1*delta_t(iload)*sign(sigma_n1tr(1)-
sigma_n1tr(3));
               epsilon_ison1 = epsilon_pn(2)+gam_n1_k1*delta_t(iload);
               epsilon_kinn1 = epsilon_pn(3)-gam_n1_k1*delta_t(iload)*sign(sigma_n1tr(1)-
sigma_n1tr(3));
               epsilon_pn1 =[epsilon_epn1 epsilon_ison1 epsilon_kinn1];
               %Computing Plastic strain at n+1 from Plastic strain at n
        end
    else                        % for rate independent model
        if hardtype==1          % for linear isotropic hardening
               gama_n1 = (f_n1tr/(E+K+H)) ;
               epsilon_epn1 = epsilon_pn(1)+(E+K+H)^-1*f_n1tr*sign(sigma_n1tr(1)-sigma_n1tr(3));
               epsilon_ison1 = epsilon_pn(2)+(E+K+H )^-1*f_n1tr ;
               epsilon_kinn1 = epsilon_pn(3)-(E+K+H )^-1*f_n1tr*sign(sigma_n1tr(1)-
sigma_n1tr(3));
               epsilon_pn1=[epsilon_epn1 epsilon_ison1 epsilon_kinn1];
               %Computing Plastic strain at n+1 from Plastic strain at n
               sigma_n1(1)= sigma_n1tr(1)-(E+K+H)^-1*f_n1tr*E*sign(sigma_n1tr(1)-sigma_n1tr(3)) ;
               sigma_n1(2)= sigma_n1tr(2)-(E+K+H )^-1*f_n1tr*K;
               sigma_n1(3)= sigma_n1tr(3)-(E+K+H)^-1*f_n1tr*H*sign(sigma_n1tr(1)-sigma_n1tr(3)) ;
               %Computing Stress vector at n+1 from Stress trail vector at n+1
        else                    % for non linear isotropic hardening case
               %interative method for finding the value of gama
               [sigma_n1_iso,gam_n1_k1] =
nonlinear_1d(f_n1tr,epsilon_pn(2),sigma_niso,sigma_y,E,H,delta,Visco,delta_t(iload));
               sigma_n1(1) = sigma_n1tr(1)-gam_n1_k1*E*sign(sigma_n1tr(1)-sigma_n1tr(3));
               sigma_n1(2) = sigma_n1_iso ;
               sigma_n1(3) = sigma_n1tr(3)+gam_n1_k1*H*sign(sigma_n1tr(1)-sigma_n1tr(3));
               %Computing Stress vector at n+1 from Stress trail vector at n+1
               epsilon_epn1 = epsilon_pn(1)+gam_n1_k1*sign(sigma_n1tr(1)-sigma_n1tr(3));
               epsilon_ison1 = epsilon_pn(2)+gam_n1_k1 ;
               epsilon_kinn1 = epsilon_pn(3)-gam_n1_k1*sign(sigma_n1tr(1)-sigma_n1tr(3));
               epsilon_pn1 =[epsilon_epn1 epsilon_ison1 epsilon_kinn1] ;
               %Computing Plastic strain at n+1 from Plastic strain at n
        end
    end
    end
    epsilon_pn=epsilon_pn1;
    sigma_n=sigma_n1;
    sigma_niso=sigma_n1(2);
    sigma_nkin=sigma_n1(3);
    plot_sigma(i)=sigma_n1(1);
    plot_epsilon(i)=epsilon_n1(1);
    plot_sigmaiso(i)=sigma_n1(2);
    end
end
```

```
figure(1);      %%%% Plot for STRESS VERSUS STRAIN %%%%%%
ep1=plot_epsilon(1:21);
si1=plot_sigma(1:21);
ep2=plot_epsilon(21:61);
si2=plot_sigma(21:61);
ep3=plot_epsilon(61:101);
si3=plot_sigma(61:101);
hline3=plot(ep3,si3,' b ',' LineWidth ',2);
hold on
hline2=plot(ep2,si2,' r ',' LineWidth ',2);
hold on
hline1=plot(ep1,si1,' g ',' LineWidth ',2);
hold on
xlabel('Strain','FontSize',12,'FontWeight','bold');
ylabel('Stress','FontSize',12,'FontWeight','bold');
set(gca,'linewidth',1.25,'FontWeight','bold');
grid on
```

APPENDIX 2 : MATLAB Subroutines for 1d Plasticity (function Non-Linear Isotropic hardening) 'nonlinear_1d.m' called in 'Plasticity_main_1d.m'

```
function [sigmaiso_n1,gam_n1_k1] =
nonlinear_1d(fn1_trial,ehiso_n,sigmaiso_n,sigma_y,E,H,delta,Visco,delta_t)

gam_n1_k1=0;
sigma_zero=sigma_y;   %Yield stress
sigma_inf=8e8;        %Stress at infinity

flag=0;
if Visco==1
    while flag==0
        gam_n1=gam_n1_k1;
        g_fun = fn1_trial-(E+H)*delta_t*gam_n1-((sigma_inf-sigma_zero)*(1-exp(-
delta*(ehiso_n+delta_t*gam_n1)))+H*(ehiso_n+delta_t*gam_n1))+sigmaiso_n ;

        der_g_fun = -(E+H)*delta_t+((sigma_inf-sigma_zero)*delta_t*(-delta)*exp(-
delta*ehiso_n)*exp(-delta*delta_t*gam_n1)-delta_t*H) ;
        gam_n1_k1=gam_n1-(g_fun/der_g_fun);
        if abs(g_fun)<1e-6 || abs(gam_n1_k1-gam_n1)<1e-6
            flag=1;
        end
    end
    sigmaiso_n1 = -(sigma_inf-sigma_zero)*(1-exp((-delta)*(ehiso_n+delta_t*gam_n1)))-
(ehiso_n+delta_t*gam_n1)*H;
else
    while flag==0
     gam_n1=gam_n1_k1;
     g_fun = fn1_trial-(E+H)*gam_n1-((sigma_inf-sigma_zero)*(1-exp(-
delta*(ehiso_n+gam_n1)))+H*(ehiso_n+gam_n1))+sigmaiso_n ;
     der_g_fun = -(E+H)+((sigma_inf-sigma_zero)*(-delta)*exp(-delta*ehiso_n)*exp(-delta*gam_n1)-
H) ;
     gam_n1_k1=gam_n1-(g_fun/der_g_fun) ;
     if abs(g_fun)<1e-6 || abs(gam_n1_k1-gam_n1)<1e-6
            flag=1;
     end
    end
    sigmaiso_n1 = -(sigma_inf-sigma_zero)*(1-exp((-delta)*(ehiso_n+gam_n1)))-(ehiso_n+gam_n1)*H;
end
end
```

APPENDIX 3: MATLAB Subroutines j2 Plasticity - 'Plasticity_j2_main.m'

```
%%%%%%%%%%%%%%%%%%% ANIL BETTADAHALLI CHANNAKESHAVA %%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%This code solves the J2 Model for plasticity. There are two functions
%%which has been called in this which are 'dev.m'and 'nonlinear_j2.m'
clc
clear all
%%%%%%%%%%% MATERIAL PROPERTIES %%%%%%%%%%%%%%%%%%%%%%%%%%%
E = 2.11e11 ; % Youngs Modulus
nu = 0.3 ; % Poisson's ratio
G=E/(3*(1-2*nu)) ; % bulk modulus
lambda=E*nu /((1+nu)*(1-2*nu)) ; % lame const
mu=(3/2)*(G-lambda ) ; % lame const
sigma_y= 6e8 ; % Yield S t r e s s
kappa=G; % lame constant
eta = 1e20 ; % Viscosity Parameter
TimeTotal = 100 ; % Total time
delta=10; % Non-linear delta Parameter
K=0.9e8 ; % Isotropic Hardening parameter defined in strain space
H=0.5e8 ; % Kinematic Hardening parameter defined in strain space
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%% LOADING PATHS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
LOADPATH = zeros(3,3) ;
    LOADPATH(1,:)=[14e8 0 0 ] ; % Loading
    LOADPATH(2,:)=[28e8 0 0 ] ; % Unloading
    LOADPATH(3,:)=[42e8 0 0 ] ; % Reloading
    istep = [20 40 40] ;        % Step size for loading path
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%Selection of Rate Independent and rate dependent Models%%%%%%%%
disp('Please choose Model Type : ' )
disp('[1] = Rate Independent ') ;
disp('[2] = Rate Dependent ') ;
temp= input('Model Type= ') ;
if temp==1
    Visco=2;
else
    Visco=1;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%Selection of Linear or Non-linear Hardening %%%%%%%%%%%%%%%%%%%%%%%%%
disp('Please choose Model Type : ' )
disp('[1] = Linear ') ;
disp('[2] = Non-l i n e a r ') ;
hardening=input('Iso hardening type= ') ;
if hardening ==1
      hardtype =1;
else
      hardtype =2;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%% COMPUTATION OF ELASTIC CONSTITUTIVE TENSOR%%%%%%%%%%%%%%%%%%%%%%%
C1=1;
C2=nu/(1-nu ) ;
```

```
C3=(1-2*nu )/(2*(1-nu)) ;
C4=(E*(1-nu))/((1+nu)*(1-2*nu)) ;
ce= zeros(6,6) ;     % Initializing
ce(1,1)=C1 ;
ce(2,2)=C1 ;
ce(3,3)=C1 ;
ce(1,2)=C2 ;
ce(1,3)=C2 ;
ce(2,1)=C2 ;
ce(2,3)=C2 ;
ce(3,1)=C2 ;
ce(3,2)=C2 ;
ce(4,4)=C3 ;
ce(5,5)=C3 ;
ce(6,6)=C3 ;
fce=C4*ce ;
%%%%%COMPUTATION OF STRAIN AT CORRESPONDING TO LOAD PATHS CONSIDERED%%%%%%
STRAIN = zeros(size(LOADPATH,1),6) ;
for iloc = 1:size(LOADPATH,1)
      SSS =LOADPATH(iloc,:) ;
    sigma_0 =[SSS(1) SSS(2) SSS(3) 0 0 0 ] ;
    strain_di =(inv(fce)*sigma_0')' ;
    STRAIN(iloc+1,:) = strain_di ;
end
mstrain = size(STRAIN,2) ;
strain = zeros(sum(istep)+1 , mstrain) ;
acum = 0 ;
PNT = STRAIN(1,:) ;
for iloc = 1:length(istep)
    INCSTRAIN = STRAIN(iloc+1,:)-STRAIN(iloc,:) ;
  for i = 1:istep(iloc)
    acum = acum+1 ;
    PNT = PNT + INCSTRAIN/istep(iloc) ;
    strain(acum+1,:) = PNT ;
  end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%TIME VECTOR %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
delta_t = TimeTotal/(sum(istep)) ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
i=1;
e_n1_vect = strain(i,:) ;
e_n1 =[e_n1_vect(1) e_n1_vect(4) e_n1_vect(5) ; e_n1_vect(4) e_n1_vect(2) e_n1_vect(6) ;
e_n1_vect(5) e_n1_vect(6) e_n1_vect(3)] ;

ep_n = zeros(size(e_n1)) ;
ehiso_n =0;
ehkin_n = zeros(size(e_n1)) ;
sigmaiso_n =0;
sigma_n = zeros(size(e_n1)) ;
sigmakin_n = zeros(size(e_n1)) ;

pstrain_hist=0;
```

```
stress_hist=0;
strain_hist=0;
devstress_hist=0;
for iload = 1:length(istep)
for iloc = 1:istep(iload)
     i = i+1;
     e_n1_vect = strain(i,:) ;
     e_n1 =[e_n1_vect(1) 0.5*e_n1_vect(4) 0.5*e_n1_vect(5) ;
           0.5*e_n1_vect(4) e_n1_vect(2) 0.5*e_n1_vect(6) ;
           0.5*e_n1_vect(5) 0.5*e_n1_vect(6) e_n1_vect(3)] ;
       %%%%% COMPUTATION OF DEVIATORIC TENSOR %%%
     devi = e_n1-(1/3)*trace(e_n1)*eye(3) ;
           %%%%% TRIAL STATES %%%%%
sigma_n1_trial = kappa*trace(e_n1)*eye(3)+2*mu*(devi-ep_n) ;
sigmaiso_n1_trial = sigmaiso_n ;
sigmakin_n1_trial = sigmakin_n ;
        %%%%% TRIAL FUNCTION %%%%
DEV=(dev(sigma_n1_trial))-sigmakin_n1_trial ;
fn1_trial = sqrt(sum(dot(DEV,DEV)))-((sqrt(2/3))*(sigma_y-sigmaiso_n1_trial)) ;
q_n1_trial = sigmaiso_n1_trial ;
qbar_n1_trial = sigmakin_n1_trial ;
   %%Return mapping algorithm is used for updating stress and strain%%
 if fn1_trial>0
    N_Tens =(dev(sigma_n1_trial)-qbar_n1_trial)/sqrt(sum(dot((dev(sigma_n1_trial)-
qbar_n1_trial),...
        (dev(sigma_n1_trial)-qbar_n1_trial)))) ;
    if Visco==1
         if hardtype==1 % Computation of rate dependent Linear isotropic and kinematic
                %Hardening at N+1 time step
                ep_n1 = ep_n +(fn1_trial/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)))*N_Tens ;
                ehiso_n1 = ehiso_n+(sqrt(2/3))*fn1_trial/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)) ;
                sigmaiso_n1 = q_n1_trial-
(sqrt(2/3)*K*fn1_trial/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)));
                ehkin_n1 = ehkin_n-
(sqrt(2/3)*fn1_trial/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)))*N_Tens;
                sigma_n1 = sigma_n1_trial-
(fn1_trial*2*mu/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)))*N_Tens ;
                sigmakin_n1 =
qbar_n1_trial+(fn1_trial*(2/3)*H/(2*mu+(2*K/3)+(2*H/3)+(eta/delta_t)))*N_Tens ;

         else %%%%% Computation of  rate dependent Non-Linear isotropic Hardening
           [sigmaiso_n1,gam_n1]=nonlinear_J2(fn1_trial,ehiso_n,sigmaiso_n,sigma_y,delta,...
               Visco,delta_t,H,mu); %% NON-LINEAR FUNCTION CALLED

           ep_n1 = ep_n+(gam_n1*delta_t*N_Tens) ;
           ehiso_n1 = ehiso_n+(gam_n1*delta_t*(sqrt(2/3))) ;
           ehkin_n1 = ehkin_n-(gam_n1*delta_t*N_Tens) ;
           sigma_n1 = sigma_n1_trial-(gam_n1*delta_t*2*mu*N_Tens) ;
           sigmakin_n1 = qbar_n1_trial+(gam_n1*delta_t*(2/3)*H*N_Tens) ;
         end
    else
        if hardtype==1 % Computation of rate independent Linear isotropic and kinematic
Hardening%
```

```
            ep_n1 = ep_n+(fn1_trial/(2*mu+(2*K/3)+(2*H/3)))*N_Tens ;
            ehiso_n1 = ehiso_n+(sqrt(2/3))*fn1_trial/(2*mu+(2*K/3)+(2*H/3)) ;
            sigmaiso_n1 = q_n1_trial-(sqrt(2/3)*K*fn1_trial/(2*mu+(2*K/3)+(2*H/3))) ;
            ehkin_n1 = ehkin_n-(sqrt(2/3)*fn1_trial/(2*mu+(2*K/3)+(2*H/3)))* N_Tens ;
            sigma_n1 = sigma_n1_trial-(fn1_trial*2*mu/(2*mu+(2*K/3)+(2*H/3)))*N_Tens ;
            sigmakin_n1 = qbar_n1_trial+(fn1_trial*(2/3)*H/(2*mu+(2*K/3)+(2*H/3)))*N_Tens ;
        else %%Computation of rate independent Non-Linear isotropic Hardening
            [sigmaiso_n1,gam_n1]=nonlinear_J2(fn1_trial,ehiso_n,sigmaiso_n,sigma_y,delta,...
                Visco,delta_t,H,mu); %% NON-LINEAR FUNCTION CALLED

            ep_n1 = ep_n+(gam_n1*N_Tens) ;
            ehiso_n1 = ehiso_n+(gam_n1*(sqrt(2/3))) ;
            ehkin_n1 = ehkin_n-(gam_n1*N_Tens) ;
            sigma_n1 = sigma_n1_trial-(gam_n1*2*mu*N_Tens) ;
            sigmakin_n1 = qbar_n1_trial+(gam_n1*(2/3)*H*N_Tens) ;
         end
     end
  else
      ep_n1 = ep_n ; %%%% FOR PERFECT PLASTICITY
      ehiso_n1 = ehiso_n ;
      ehkin_n1 = ehkin_n ;
      sigma_n1 = sigma_n1_trial ;
      sigmaiso_n1 = q_n1_trial ;
      sigmakin_n1 = qbar_n1_trial ;
  end
      ep_n = ep_n1 ;
      ehiso_n = ehiso_n1 ;
      ehkin_n = ehkin_n1 ;
      sigmaiso_n = sigmaiso_n1 ;
      sigma_n1=sigma_n1 ;
      sigmakin_n1=sigmakin_n1 ;
      x(i)=sigmaiso_n1 ;
      devsigma_n1 = dev(sigma_n1 ) ;
      pstrain_hist=[pstrain_hist , ep_n(1)] ;
      stress_hist=[stress_hist , sigma_n1(1)] ;
      strain_hist=[strain_hist , e_n1(1)] ;
      devstress_hist=[devstress_hist , devsigma_n1(1)] ;
      plot_devsigma(i)=devsigma_n1(1) ;
      plot_sigma(i)=sigma_n1(1) ;
      plot_epsilon(i)=e_n1(1) ;
end
end

figure(1) ;   %%%% Plot for STRESS VERSUS STRAIN %%%%%%
ep1 = plot_epsilon(1:istep(1)) ;
si1 = plot_sigma(1:istep(1)) ;
ep2 = plot_epsilon(istep(1):istep(1)+istep(2)) ;
si2 = plot_sigma(istep(1):istep(1)+istep(2)) ;
ep3 = plot_epsilon(istep(1)+istep(2):end) ;
si3 = plot_sigma(istep(1)+istep(2):end) ;
hline3 = plot(ep3,si3,'k--','LineWidth',1.5) ;
hold on
```

```
hline2 = plot(ep2,si2,'r--','LineWidth',1.5) ;
hold on
hline1 = plot(ep1,si1,'b--','LineWidth',1.5) ;
hold on
legend([hline1,hline2,hline3],'Loading','Unloading','Reloading')
xlabel('Strain[1]','FontSize',10,'FontWeight','bold') ;
ylabel('Stress[1]','FontSize',10,'FontWeight','bold') ;
set(gca,'linewidth',1.25,'FontWeight','bold')
grid on

figure(2) ;   %%%% Plot for DEVIATORIC STRESS VERSUS STRAIN %%%%%%
ep1 = plot_epsilon(1:istep(1)) ;
si1 = plot_devsigma(1:istep(1)) ;
ep2 = plot_epsilon(istep(1):istep(1)+istep(2)) ;
si2 = plot_devsigma(istep(1):istep(1)+istep(2)) ;
ep3 = plot_epsilon(istep(1)+istep(2):end) ;
si3 = plot_devsigma(istep(1)+istep(2):end) ;
hline3 = plot(ep3,si3,'g--','LineWidth',1.5) ;
hold on
hline2 = plot(ep2,si2,'c--','LineWidth',1.5) ;
hold on
hline1 = plot(ep1,si1,'k--','LineWidth',1.5) ;
hold on
legend([hline1,hline2,hline3],'Loading','Unloading','Reloading')
xlabel('Strain[1]','FontSize',10,'FontWeight','bold') ;
ylabel('Deviatoric Stress[1]','FontSize',10,'FontWeight','bold') ;
set(gca,'linewidth',1.25,'FontWeight','bold')
grid on

figure(3) ;
plot(plot_sigma) ;
hold on
plot(plot_devsigma) ;
```

APPENDIX 4 : MATLAB Subroutines for j2 Plasticity (function Non-Linear Isotropic hardening) 'nonlinear_j2.m' called in 'Plasticity_j2_main.m'

```
%%%% This funtion is called in 'Plasticity_j2_main.m' %%%%
function [sigmaiso_n1,gam_n1]=nonlinear_j2(fn1_trial,ehiso_n,sigmaiso_n,sigma_y,delta,...
               Visco,delta_t,H,mu)

gam_n1_k1=0;
sigma_zero=sigma_y;
sigma_inf=8e8;

flag=0;
if Visco==1
    while flag==0
        gam_n1=gam_n1_k1;
        g_fun = fn1_trial-(2*mu+2*H/3)*delta_t*gam_n1-sqrt(2/3)*((sigma_inf-sigma_zero)*(1-exp...
(-delta*(ehiso_n+sqrt(2/3)*delta_t*gam_n1)))+H*(ehiso_n+sqrt(2/3)*delta_t*gam_n1))+sigmaiso_n ;
    if abs(g_fun)<1e-6
         return
    else
        der_g_fun = -(2*mu+2*H/3)*delta_t+sqrt(2/3)*((sigma_inf-sigma_zero)*delta_t*(-delta)...
            *sqrt(2/3)*exp(-delta*ehiso_n)*exp(-delta*gam_n1*delta_t*sqrt(2/3)))-
H*delta_t*sqrt(2/3) ;
        gam_n1_k1=gam_n1-(g_fun/der_g_fun) ;
        if abs(g_fun)<1e-6 || abs(gam_n1_k1-gam_n1)<1e-6
            flag=1;
        end
    end
    end
 sigmaiso_n1 = -(sigma_inf-sigma_zero)*(1-exp((-delta)*(ehiso_n+(sqrt(2/3))*delta_t*gam_n1)))...
     -(ehiso_n+(sqrt(2/3))*delta_t*gam_n1)*H;
else
    while flag==0
        gam_n1=gam_n1_k1;
        g_fun = fn1_trial-(2*mu+2*H/3)*gam_n1-sqrt(2/3)*((sigma_inf-sigma_zero)*(1-exp(-
delta*(ehiso_n...
            +sqrt(2/3)*gam_n1)))+H*(ehiso_n+sqrt(2/3)*gam_n1))+sigmaiso_n ;
    if abs(g_fun)<1e-6
        return
    else
        der_g_fun = -(2*mu+2*H/3)+sqrt(2/3)*((sigma_inf-sigma_zero)*(-delta)*sqrt(2/3)*exp...
(-delta*ehiso_n)*exp(-delta*gam_n1*sqrt(2/3)))-H*sqrt(2/3) ;
        gam_n1_k1=gam_n1-(g_fun/der_g_fun) ;
    if abs(g_fun)<1e-6 || abs(gam_n1_k1-gam_n1)<1e-6
        flag=1;
    end
    end
    end
     sigmaiso_n1 = -(sigma_inf-sigma_zero)*(1-exp((-delta)*(ehiso_n+(sqrt(2/3))*gam_n1)))...
        -(ehiso_n+(sqrt(2/3))*gam_n1)*H;
end
end
```

APPENDIX 5 : MATLAB Subroutines for j2 Plasticity (function to Compute deviatoric tensor) ‘dev.m’ called in ‘Plasticity_j2_main.m’

```
%This function determines the deviatoric part of A%
function devi=dev(A)

devi = A-(1/3)*trace(A)*eye(3) ;
end
```