Continuum Damage Models

Lei Pan

Part 1-Rate independent model

1. Introduction

In part 1, I have modified the MATLAB code to implement the following situations:

i. non-symmetric tensile-compression damage model
ii. tension-only model
iii. linear and exponential hardening/softening (H<0 and H>0)

After modifying the code, I have utilized the program to get the results of Non-symmetric tensile-compression damage model and Tension-only model under different load path, which includes the strain-stress curves, damage variable curves and hardening variable curves. The result shows that the softening material is easier to be damaged. The multidirectional loading will improve the yield stress of the material.
And after the damage happens, the Elastic Modulus will decrease. Also, the damage speed of exponential model is smaller than linear model.

2. Non-symmetric tensile-compression damage model

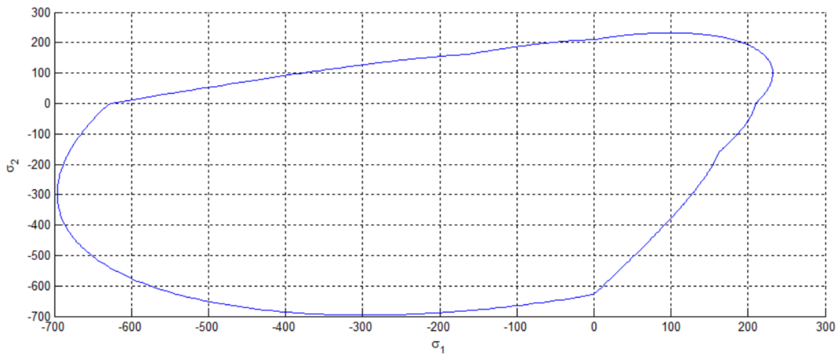

Figure 1. Non-symmetric tensile-compression damage model

A. The increments of σ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = 0$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 0$$

① The Strain-Stress and damage

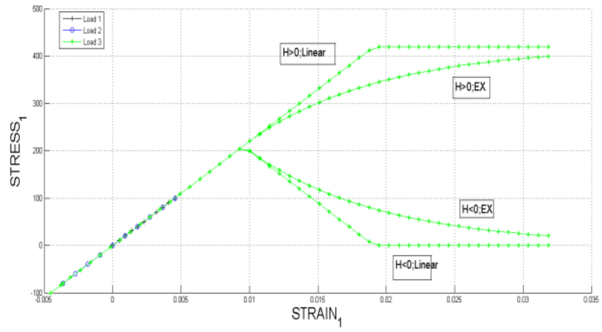

Figure 2. Strain-Stress curve of case A

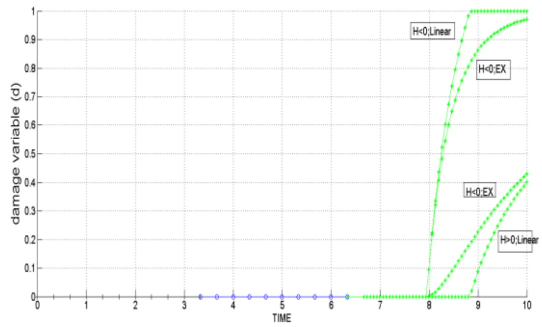

Figure 3. Damage variable respect to time of case A

The first stress increment only applies on the x direction. According to above figure, there are four situations: H>0 and Linear model, H>0 and Exponential model, H<0 and Linear model, H<0 and Exponential model.

Firstly, when H <0, we can notice that the material is easier to be damaged compared to the situation of H>0. The point when the damage happens for Linear and Exponential model is same but Linear model's damage speed is faster.

The situation for H>0 is different from H<0. We can find that when H>0, the damage level is lower than it is when H<0. For H>0 and Exponential model, the point when the damage happens is same to H<0 model and the time is about 8. But the time becomes to be about 8.9 for H>0 and Linear model. The reason is that is the same time, the hardening variable q of Linear and hardening model is higher. As a result, it makes the material more difficult to be destroyed. And the reason why hardening models' damage level is lower is that the damage surface of hardening model is expansion while the damage surface of softening model is contraction which have been shown in figure 4. To sum up, the hardening model increases the material's ability to against the extended stress.

For the Load path 1 and load path 2, because they are within the initial damage surface, so their strain-stress curves are on the same straight line. This case means that the material is in elastic stage. After this stage, I have applied the load path 3 that is beyond the damage surface so the material starts producing damage.

② The hardening variable q

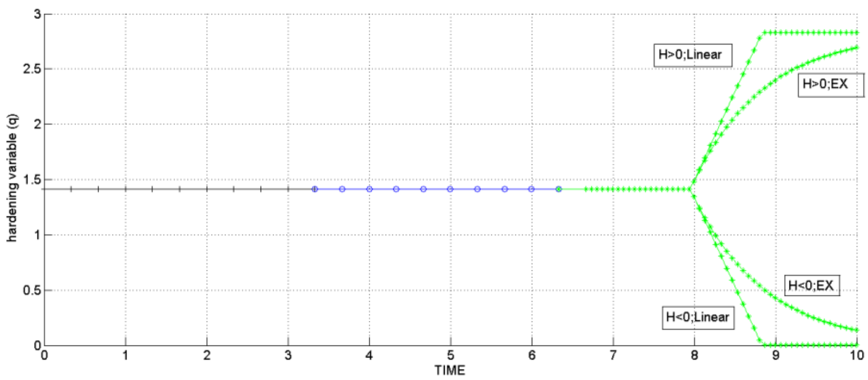

Figure 4. Hardening variable q respect to time of case A

Figure 4 shows the changing of hardening variable q with respect to time. We can see that when the load is within the initial damage surface, q doesn't change. But when the load extends the initial damage surface, which is also the time when the damage happens except for Hardening and Linear model, the q starts increasing or decreasing and the increase or decrease speed of Linear model is faster.

B. The increments of $\boldsymbol{\sigma}$ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = -200$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 800$$

In this section, the increments of σ has added $\Delta\sigma_2$.

① The Strain-Stress and damage

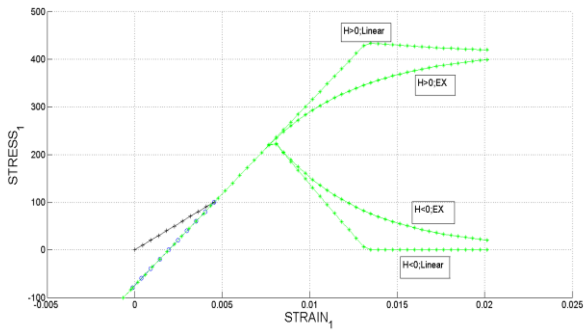

Figure 5. Strain1-Stress1 curve of case B

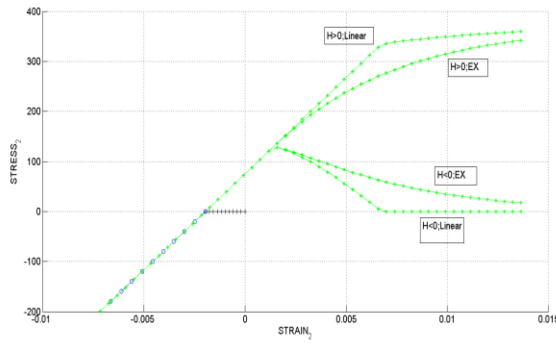

Figure 6. Strain2-Stress2 curve of case B

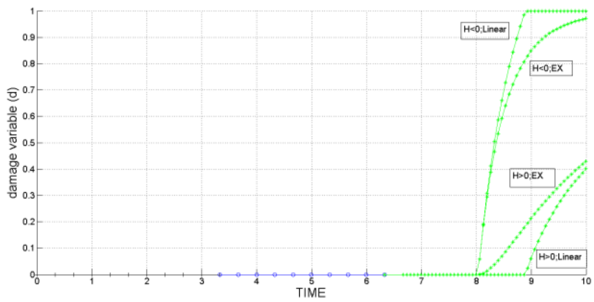

Figure 7. Damage variable respect to time of case B

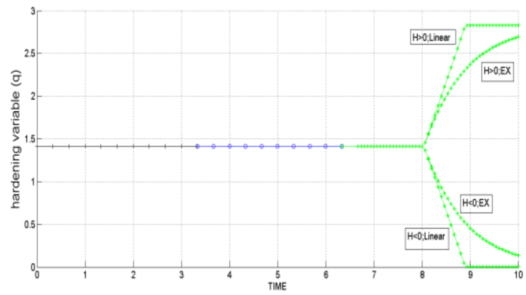

Figure 8. Hardening variable q respect to time of case B

In case B, the yield stresses on both x and y directions have increased after applying $\Delta\sigma_1$ and $\Delta\sigma_2$ compared to only applying $\Delta\sigma_1$. And we can also find this phenomenon from the damage figure and hardening variable figure because the points when damage happens and q starts changing becomes later than that in case A.

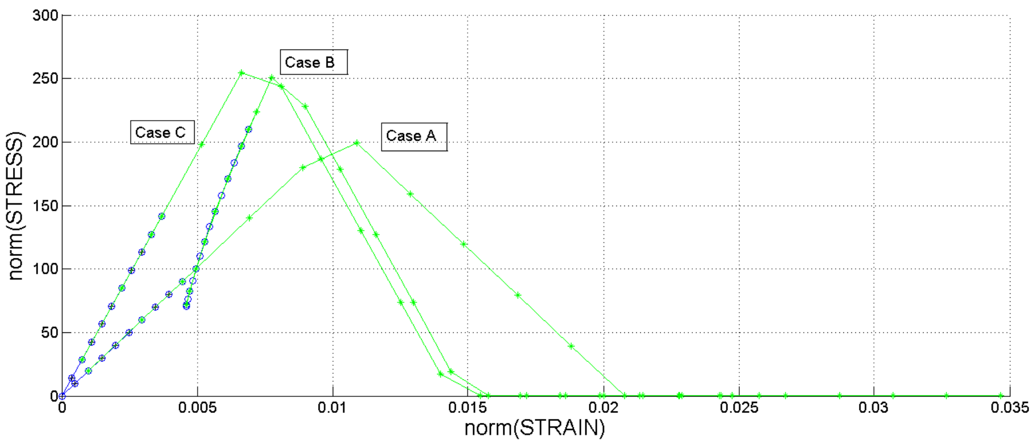

Figure 9. Comparison of norm(strain)-norm(stress) curves for different load path for Non-symmetric Model

The figure 9 has proved above phenomenon. The yield stresses sequence is C>B>A. This phenomenon is caused by the reality that the yield criterion will change for different loading combination. In this case, multidirectional loading has increased the yield stress and make the material stronger.

And we can also see that the slope of case C and case B are bigger than the slope of case A, which means that under the combined loading situation, the Elastic Modulus has increased and material becomes harder.

② The damage surface

The situation is like case A except that the time when the q starts changing in case B is later than that in case A.

C. the increments of $\boldsymbol{\sigma}$ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 100$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = -200$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 800$$

① The Strain-Stress, damage and hardening variable q

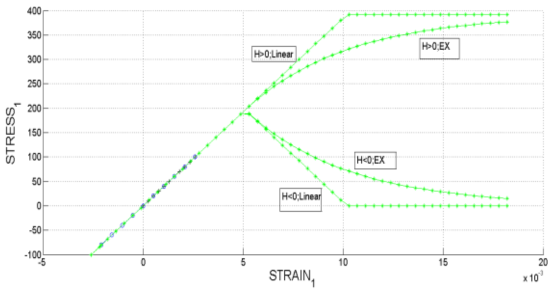

Figure 10. Strain1-Stress1 curve of case C

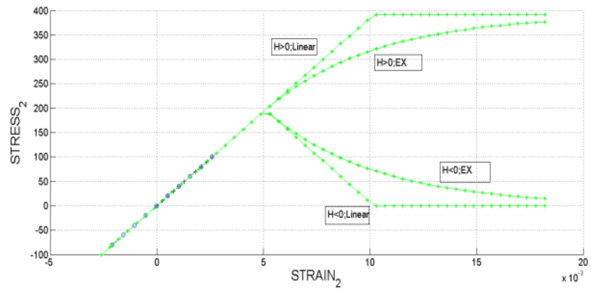

Figure 11. Strain2-Stress2 curve of case C

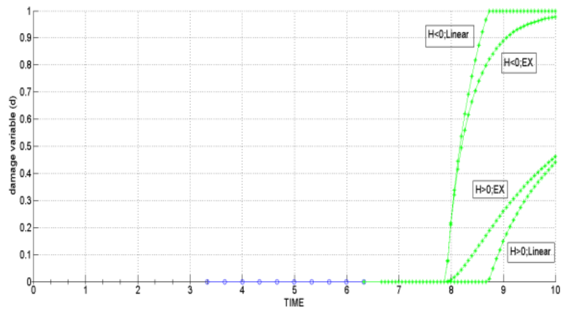

Figure 12. Damage variable respect to time of case C

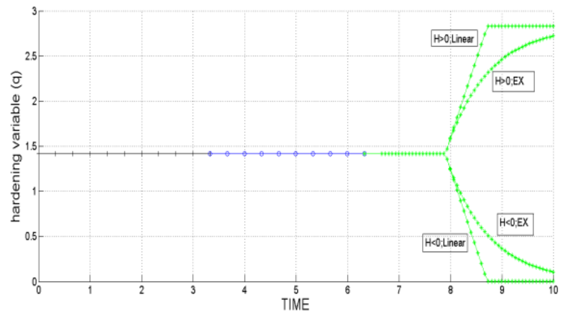

Figure 13. Hardening variable q respect to time of case C

The situation of case C is like case B, but the yield stress of case C is the highest one, meaning that the material is most difficult to be damaged in case C.

D. The Other load path:

$$\Delta\sigma_1^{(1)} = 400; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -400; \Delta\sigma_2^{(2)} = 0$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 0$$

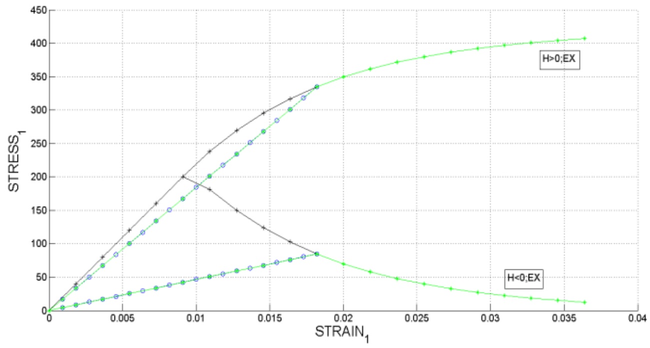

Figure 14. Strain1-Stress1 curve of case D

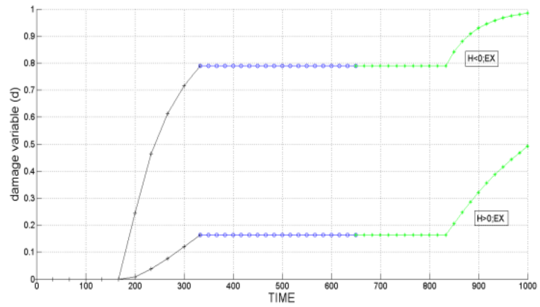

Figure 15. Damage variable respect to time of case D

As above Figures have shown, after loading path 1 which is beyond the yield stress, the material's Elastic Modulus has decreased. And the damage has divided into two stages. The loading path 2 doesn't produce any damage to the material.

3. Tension-only model

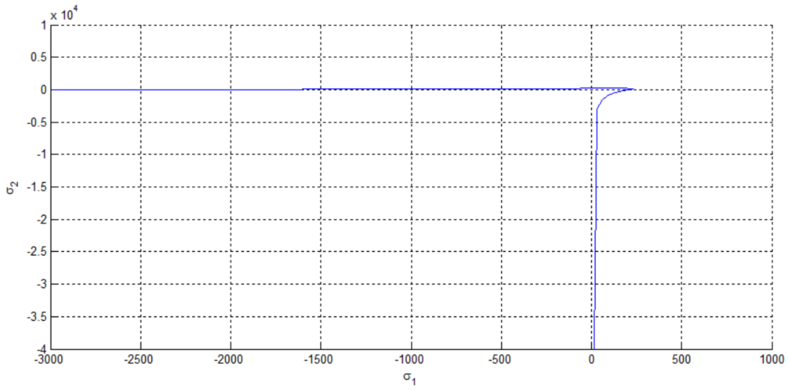

Figure 16. Tension-only model

A. the increments of σ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = 0$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 0$$

① The Strain-Stress and damage

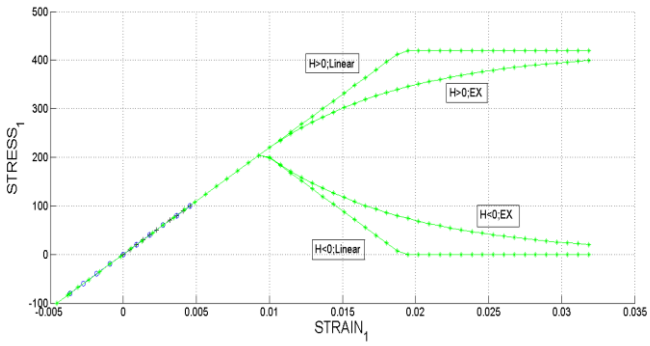

Figure 17. Strain1-Stress1 curve of case A

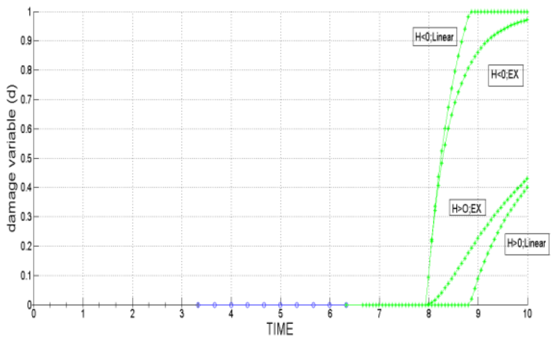

Figure 18. Damage variable respect to time of case A

② The hardening variable q

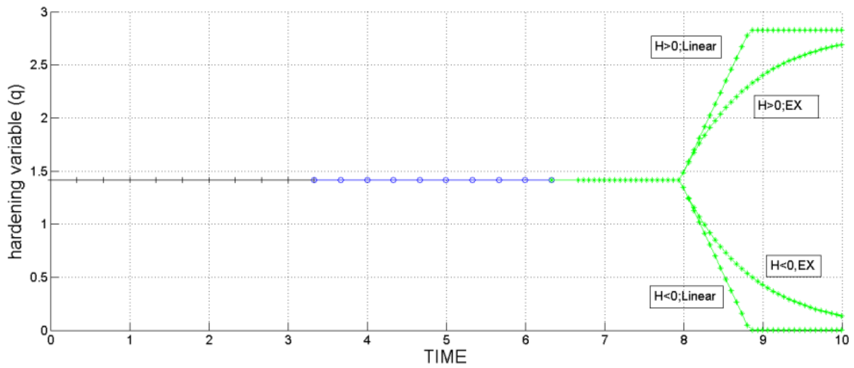

Figure 19. Hardening variable q respect to time of case A

B. the increments of σ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = -200$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 800$$

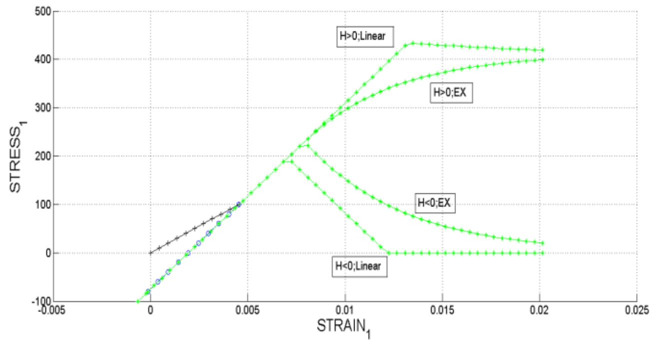

Figure 20. Strain1-Stress1 curve of case B

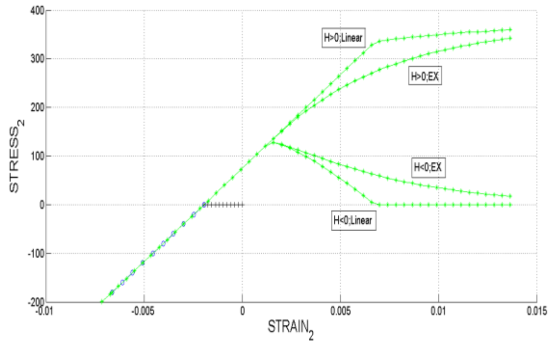

Figure 21. Strain2-Stress2 curve of case B

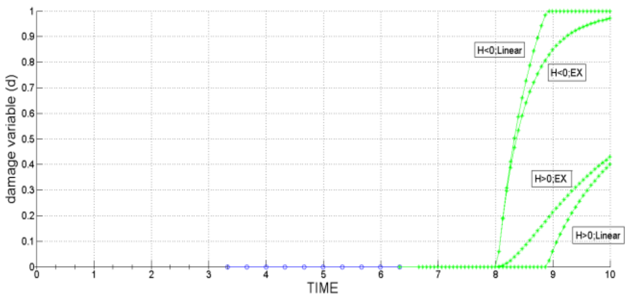

Figure 22. Damage variable respect to time of case B

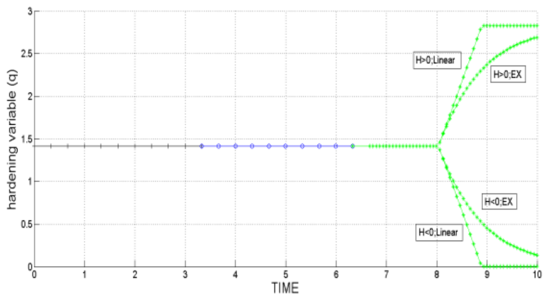

Figure 23. Hardening variable q respect to time of case B

C. the increments of σ are:

$$\Delta\sigma_1^{(1)} = 100; \Delta\sigma_2^{(1)} = 100$$
$$\Delta\sigma_1^{(2)} = -200; \Delta\sigma_2^{(2)} = -200$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 800$$

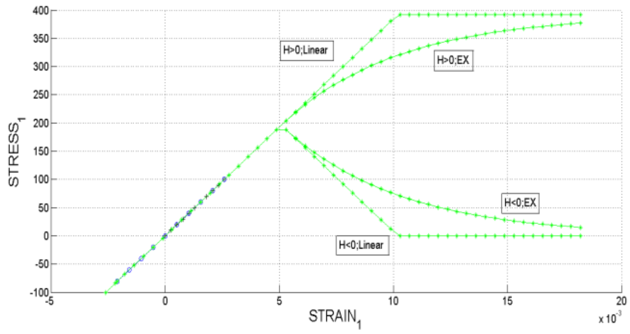

Figure 24. Strain1-Stress1 curve of case C

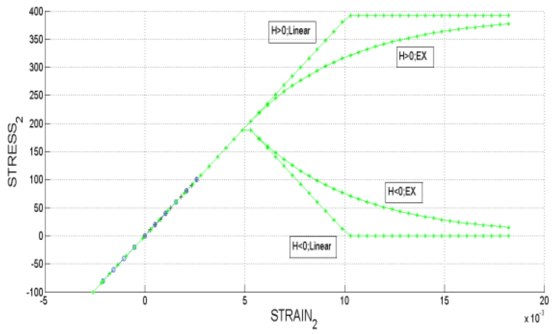

Figure 25. Strain2-Stress2 curve of case C

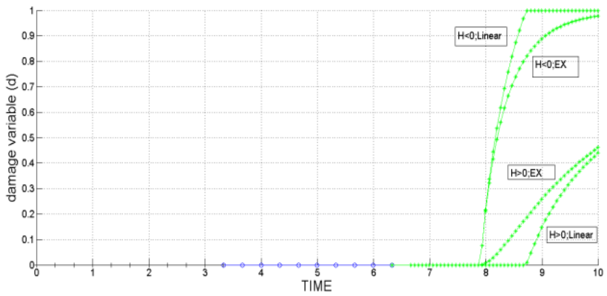

Figure 26. Damage variable respect to time of case C

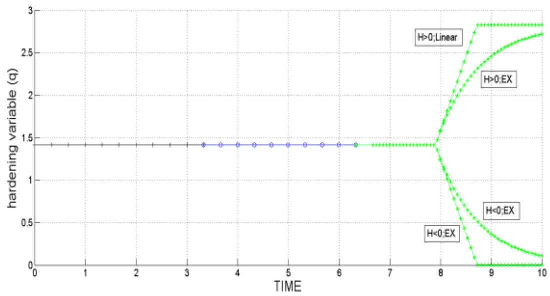

Figure 27. Hardening variable q respect to time of case C

D. Comparison

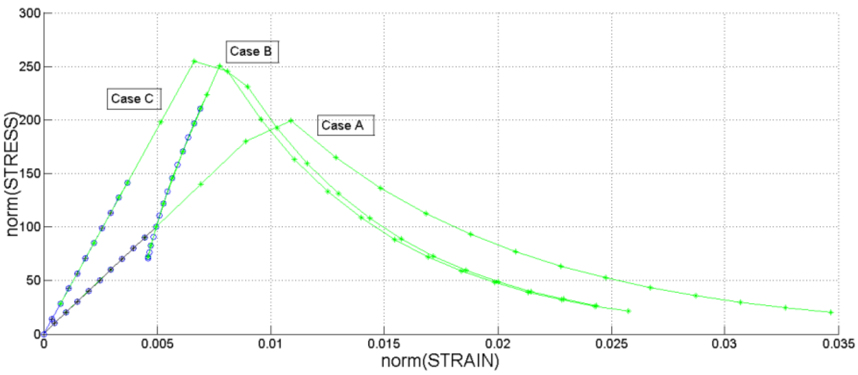

Figure 28. Comparison of norm(strain)-norm(stress) curves for different load path for Tensile-only Model

By comparing with the case of Non-symmetric tensile-compression damage model, we get that the multidirectional loading will also increase the yield stress and the Elastic Modulus in Tensile-only model.

E. The Other load path:

$$\Delta\sigma_1^{(1)} = 400; \Delta\sigma_2^{(1)} = 0$$
$$\Delta\sigma_1^{(2)} = -400; \Delta\sigma_2^{(2)} = 0$$
$$\Delta\sigma_1^{(3)} = 800; \Delta\sigma_2^{(3)} = 0$$

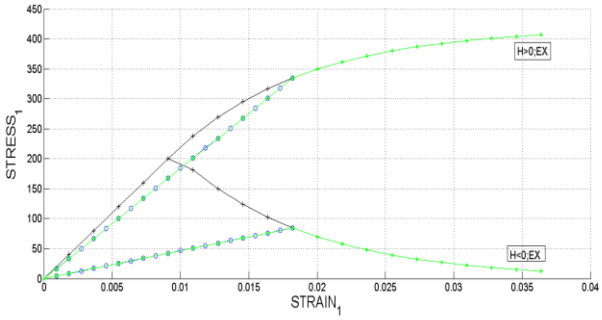

Figure 29. Strain1-Stress1 curve of case E

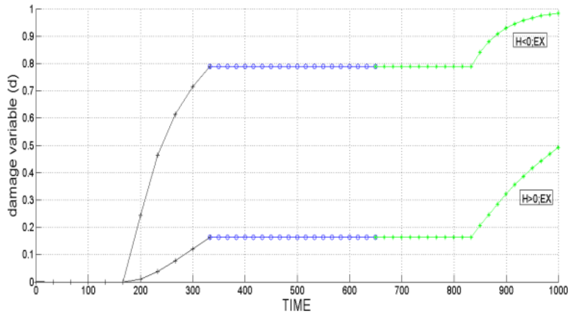

Figure 30. Damage variable respect to time of case E

Above results of Tension-only model when imposing the above loading path has also shown the same behavior of Non-symmetric model.

Part 2-Rate dependent model

1. Introduction

In this part, I have discussed several coefficient's effects on the results of Strain-Stress curve. And I also implemented different α to discuss its effect on the C11 component of tangent and algorithmic constitutive operators. The result shows that when α is increasing, the C11 component of algorithmic constitutive operators will decrease. And higher viscosity coefficient η will cause lower slope of the strain-stress curve. The strain rate has similar effect to viscosity coefficient. Also, we should take care of choosing the appropriate value of α because when α is smaller than 0.5 and the total time is big enough, the results will show oscillation which is not reliable. In order to solve this problem, the recommended value is $\alpha \geq 0.5$.

2. Effect of viscosity coefficient η

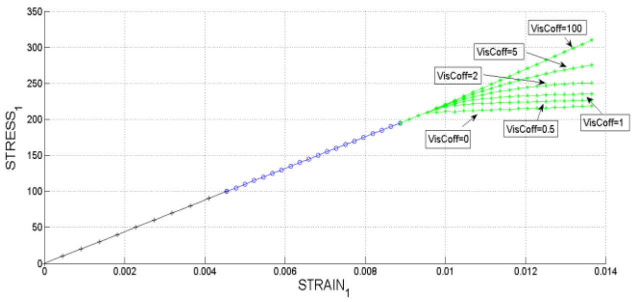

Figure 31. Strain1-Stress1 respect to viscosity coefficient

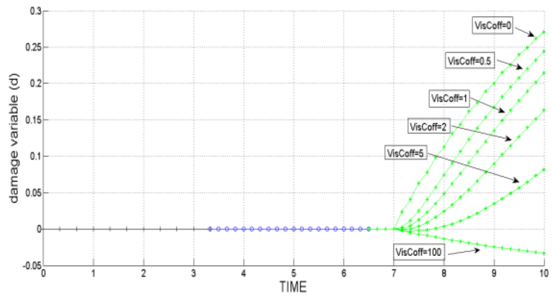

Figure 32. Damage variable respect to viscosity coefficient

When we increase the viscosity coefficient, the slope of strain-stress curve in load path 3 is approaching the Elastic Modulus while the damage level will decrease. But there is a special case when viscosity coefficient equal to 100, the damage variable becomes to be negative. Because the damage variable will never be negative, so this means that in this case, the continuum damage model is no longer applicable and credible.

3. Effect of strain rate

By increasing the total time, the strain rate will decrease accordingly.

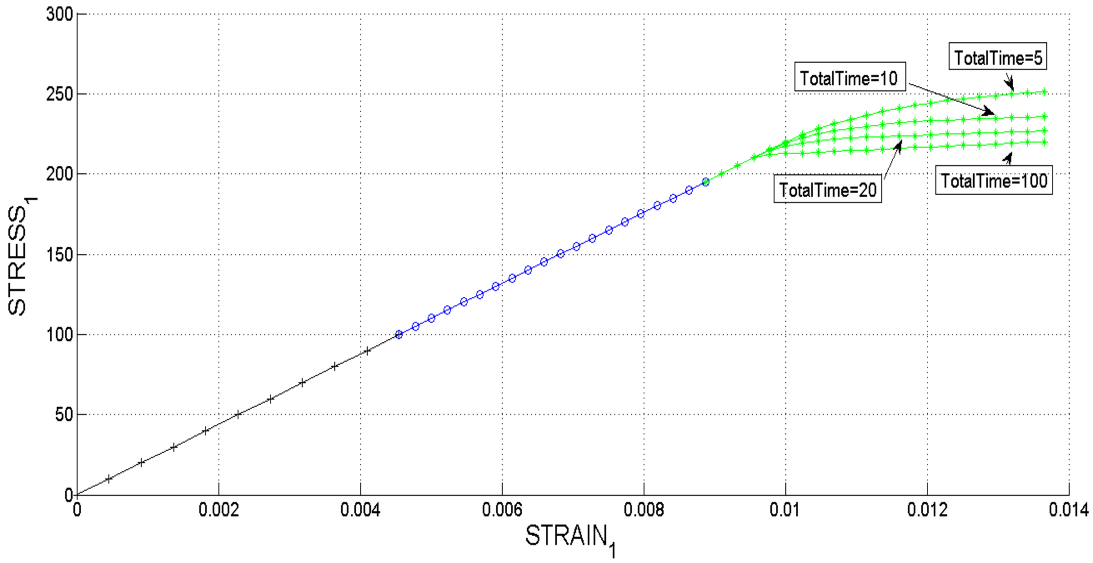

Figure 33. Strain1-Stress1 respect to strain rate

When the strain rate decreases, the slope in load path 3 will decrease, meaning that the material is getting to be harder.
The effect of strain rate is similar to the effect of viscosity coefficient $\boldsymbol{\eta}$.

4. Effect of $\boldsymbol{\alpha}$

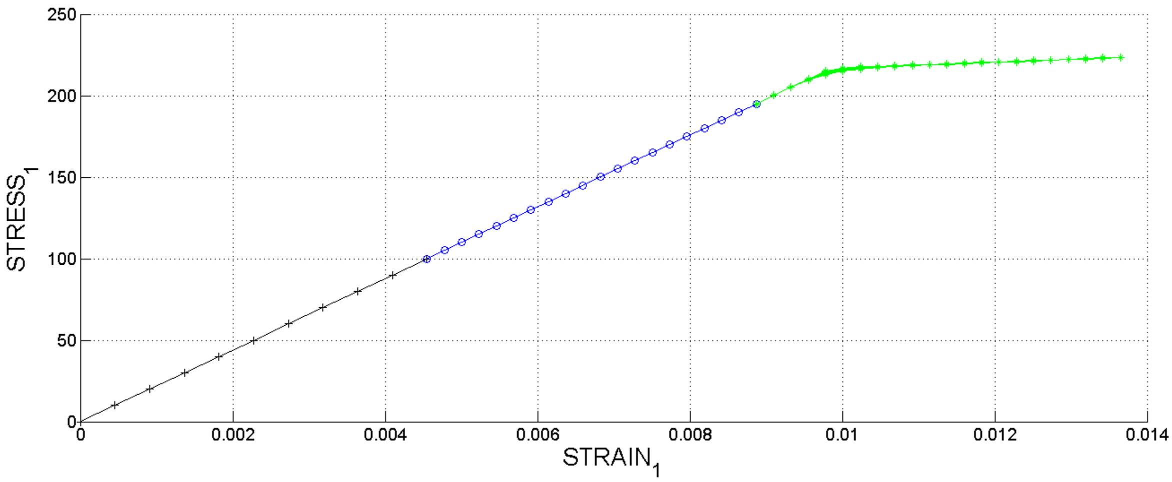

Figure 34. The Strain1-Stress1 respect to $\boldsymbol{\alpha}$ when TIME-INT=10

When total time equal to 10, the results of different values of α are close and stable.

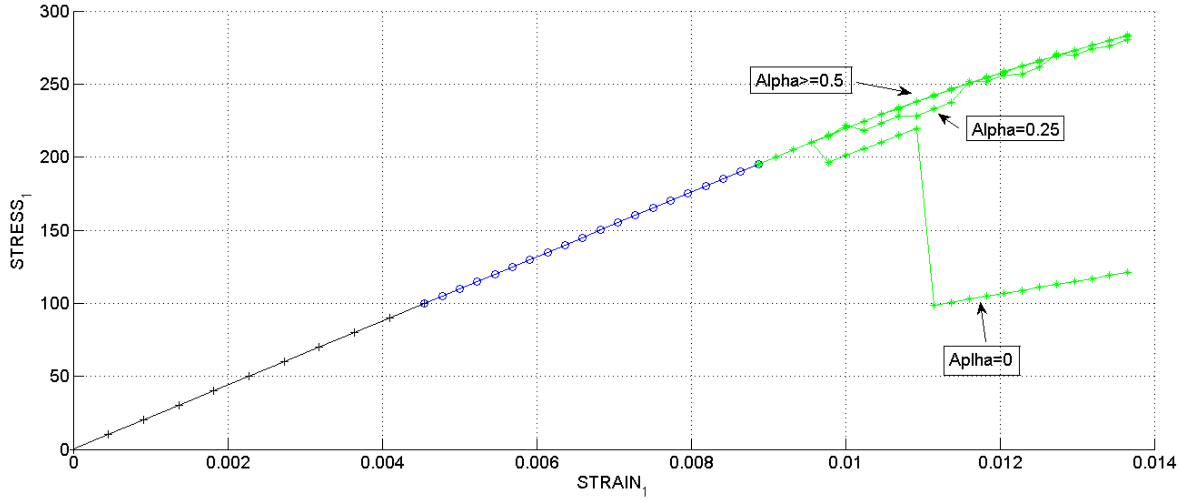

Figure 35. The Strain1-Stress1 respect to α when TIME-INT=1000

In figure 35, I have applied total time that is equal to 1000. When $\alpha = 0$ and $\alpha = 0.25$, the results are not stable. On the contrary, the results become to be sable when $\alpha >= 0.5$.

5. Effect of α on C11 component of tangent and algorithmic constitutive operators

① C11 component of algorithmic constitutive operators

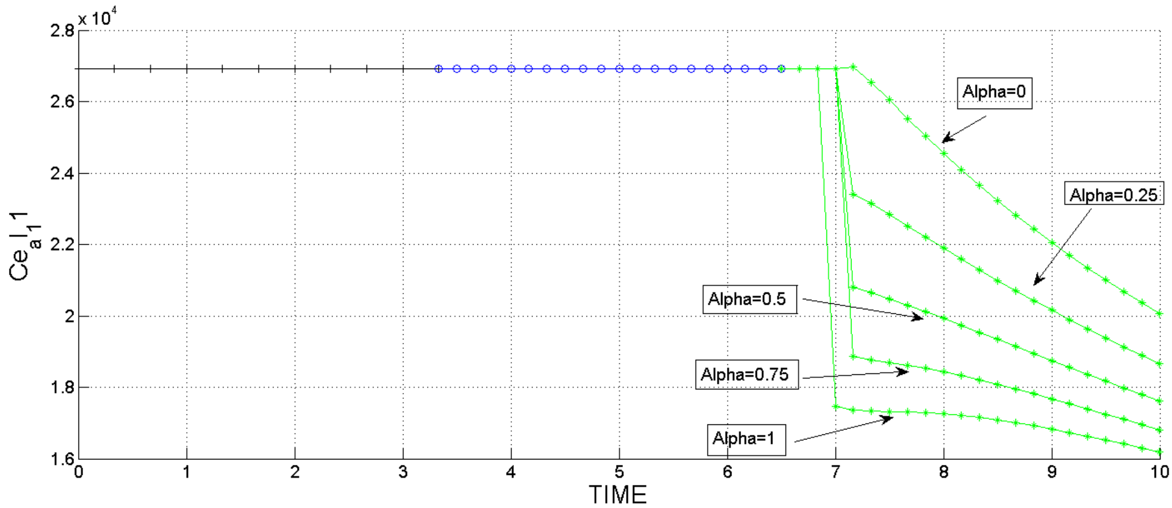

Figure 37. C11 component of algorithmic constitutive operator respect to α when TIME-INT=10

When α increases, the C11 component of algorithmic constitutive operators will decrease accordingly. And the results show the stability, but this only happens when total time is small. When we apply huge total time like 1000, the results with $\alpha < 0.5$ will become unstable. Like figure and figure have shown.

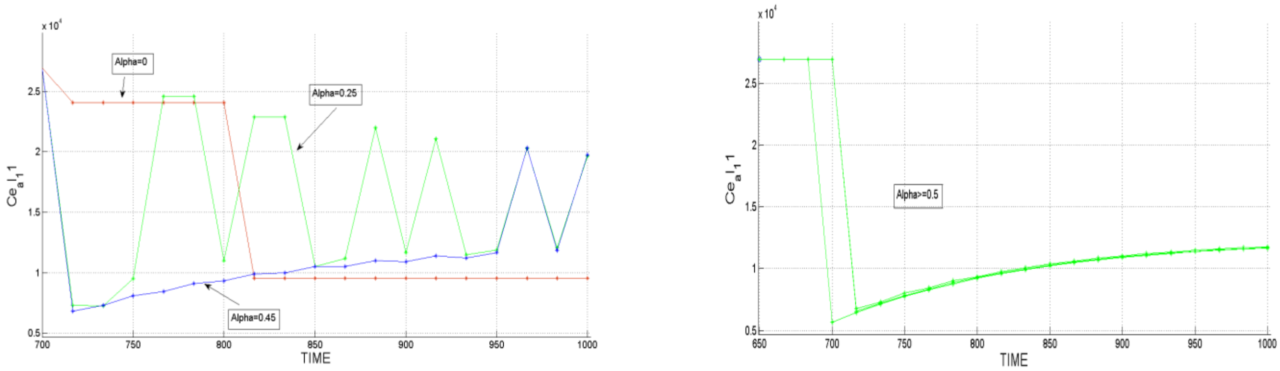

Figure 37. C11 component of algorithmic constitutive operator respect to $\boldsymbol{\alpha}$ when TIME-INT=1000

② C11 component of tangent operators

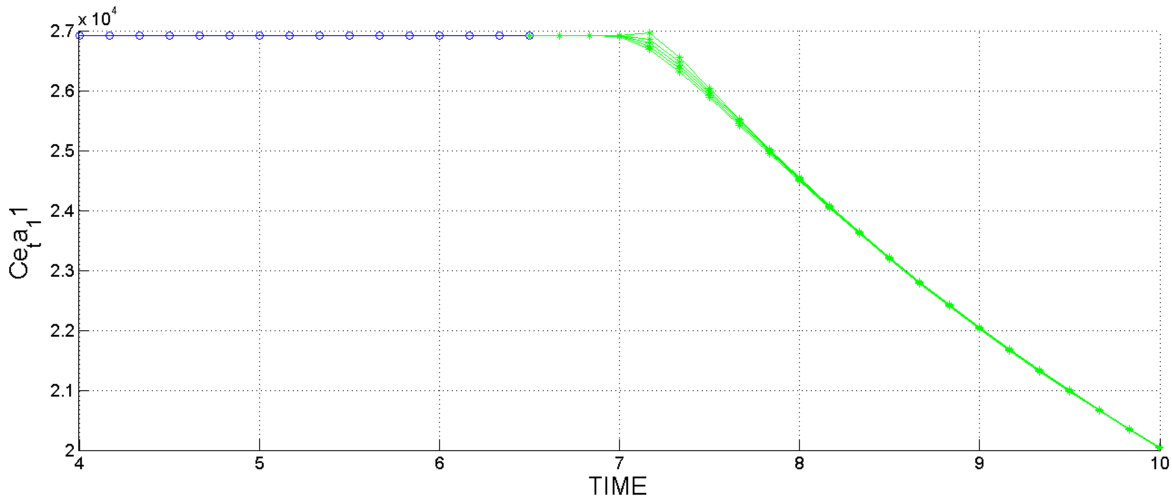

Figure 38. C11 component of tangent operator respect to $\boldsymbol{\alpha}$ when TIME-INT=10

When we apply small total time value, the different α values will have little influence on the results of C11 component of tangent operator. But when applying the huge total time value, the oscillation will also happen.

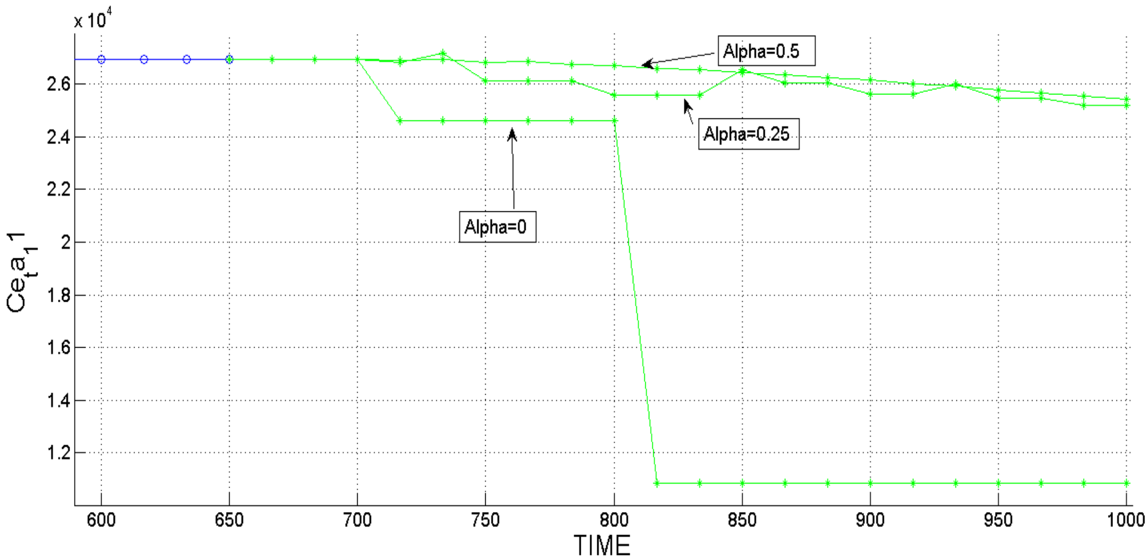

Figure 39. C11 component of tangent operator respect to $\boldsymbol{\alpha}$ when TIME-INT=1000 ($\boldsymbol{\alpha} \leq \mathbf{0.5}$)

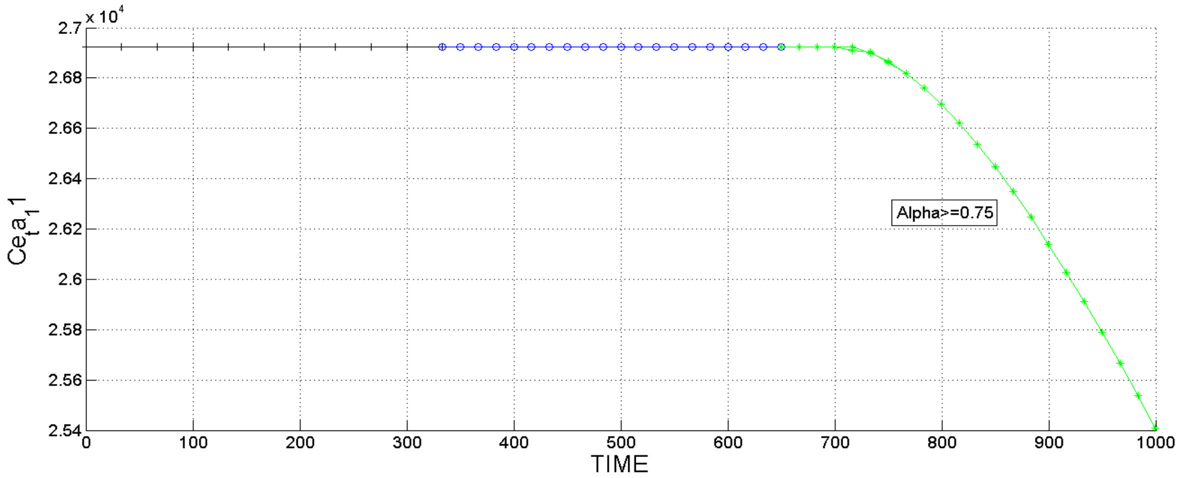

Figure 40. C11 component of tangent operator respect to $\boldsymbol{\alpha}$ when TIME-INT=1000 ($\boldsymbol{\alpha} \geq \mathbf{0.75}$)

In this case, the results become unstable when $\alpha = 0$ and $\alpha = 0.25$. When $\alpha \geq 0.5$, the oscillation will disappear.

6. Conclusion for α

From above discussion, we realize that when we apply huge total time values, the results given by using $\alpha < 0.5$ will become unstable. So the recommended α value will be $\alpha \geq 0.5$.

Part 3-Appendix

1. Modification in function `rmap_dano1`

```
if viscpr== 1 % Viscosity model

    rtrial_alp=(1-alpha)*rtrial_n+alpha*rtrial;

    if(rtrial_alp> r_n)
    %*   Loading

    fload=1;
    delta_r=rtrial-r_n;
    r_n1= ((eta-delta_t*(1-alpha))*r_n+rtrial_alp*delta_t)/(eta+alpha*delta_t)  ;

    if hard_type == 0
        %  Linear

      if HARDSOFT_MOD>0
           q_n1= q_n+ H*delta_r;
          q_inf=2*r0-zero_q;
          if q_n1>q_inf          %q_n1<=q_inf when HARDSOFT_MOD>0
              q_n1=q_inf;
          end
      else
           q_n1= q_n+ H*delta_r;
           q_inf=zero_q;
           if q_n1<q_inf
               q_n1=q_inf;      %q_n1>=q_inf when HARDSOFT_MOD<0
           end
      end
      H_n1=H;

    else
        %exponential
            if HARDSOFT_MOD>0

        q_inf=2*r0-zero_q;
        q_n1=q_inf-(q_inf-r0)*exp(H*(1-rtrial/r0));
        H_n1=H*(q_inf-r0)*exp(H*(1-rtrial/r0))/r0;

    if q_n1>q_inf
        q_n1=q_inf;
    end

        else if HARDSOFT_MOD<0

         q_inf=zero_q;
         q_n1=q_inf-(q_inf-r0)*exp(H*(rtrial/r0-1));% because A should alway be positive
          H_n1=H*(q_inf-r0)*exp(H*(rtrial/r0-1))/r0;% so here take (-H) to make A be positive
          if q_n1<q_inf
              q_n1=q_inf;
          end
            end
          end
    end

else

    %*    Elastic loading/unloading
    fload=0;
    r_n1= r_n  ;
    q_n1= q_n  ;
    H_n1=0;
   end
```

```
else % Non viscosity model
  if(rtrial > r_n)
    %*   Loading

    fload=1;
    delta_r=rtrial-r_n;
    r_n1= rtrial   ;
      if hard_type == 0
        %  Linear
            if HARDSOFT_MOD>0
           q_n1= q_n+ H*delta_r;
          q_inf=2*r0-zero_q;
          if q_n1>q_inf
              q_n1=q_inf;
          end
      else
           q_n1= q_n+ H*delta_r;
           q_inf=zero_q;
           if q_n1<q_inf
               q_n1=q_inf;
           end
           end

      else
        %Exponential
         if HARDSOFT_MOD>0
         q_inf=2*r0-zero_q;
         q_n1=q_inf-(q_inf-r0)*exp(H*(1-rtrial/r0));
          H_n1=H*(q_inf-r0)*exp(H*(1-rtrial/r0))/r0;
         if q_n1>q_inf
               q_n1=q_inf;
            end
         else if HARDSOFT_MOD<0
          q_inf=zero_q;
          q_n1=q_inf-(q_inf-r0)*exp(H*(rtrial/r0-1));
           H_n1=H*(q_inf-r0)*exp(H*(rtrial/r0-1))/r0;
             if q_n1<q_inf
                q_n1=q_inf;
             end
             end
         end
      end



  else

    %*     Elastic load/unload
    fload=0;
    r_n1= r_n   ;
    q_n1= q_n   ;

  end
end
```

```
% compute C11 component of tangent and algorithmic constitutive operators
%************
if viscpr== 1
     rtrial_alp=(1-alpha)*rtrial_n+alpha*rtrial;
    if(rtrial_alp> r_n)% loading
     Ce_al=(1-dano_n1)*ce-((alpha*delta_t)/(eta+alpha*delta_t))*((q_n1-H_n1*r_n1)/(r_n1^2))*(sigma_n1'*sigma_n1)/rtrial;
     Ce_ta=(1-dano_n1)*ce;
     Ce_al_11=Ce_al(1,1);
     Ce_ta_11=Ce_ta(1,1);
    else
        Ce_ta=(1-dano_n1)*ce;
        Ce_al=Ce_ta;
        Ce_ta_11=Ce_ta(1,1);
     Ce_al_11=Ce_al(1,1);

    end
end
```

```
if viscpr==1
hvar_n1(7)=Ce_ta_11;
hvar_n1(8)=Ce_al_11;
end
```

2. Modification in function `damage_main`

```
LABELPLOT = {'hardening variable (q)','internal variable','damage variable (d)','Ce_ta_11','Ce_al_11'};
```

```
% INITIALIZING  (i = 1) !!!!
% ***********i*
i = 1 ;
r0 = sigma_u/sqrt(E);
hvar_n(5) = r0; % r_n
hvar_n(6) = r0; % q_n
hvar_n(7)=ce(1,1);
hvar_n(8)=ce(1,1);
eps_n1 = strain(i,:) ;
sigma_n1 =ce*eps_n1'; % Elastic
sigma_v{i} = [sigma_n1(1)  sigma_n1(3) 0;sigma_n1(3) sigma_n1(2) 0 ; 0 0  sigma_n1(4)];

nplot = 5 ;
vartoplot = cell(1,totalstep+1) ;
vartoplot{i}(1) = hvar_n(6) ; % Hardening variable (q)
vartoplot{i}(2) = hvar_n(5) ; % Internal variable (r)
vartoplot{i}(3) = 1-hvar_n(6)/hvar_n(5)  ; %  Damage variable (d)
vartoplot{i}(4)=hvar_n(7);
vartoplot{i}(5)=hvar_n(8);
```

```
% VARIABLES TO PLOT (set label on cell array LABELPLOT)
% ----------------
vartoplot{i}(1) = hvar_n(6) ; % Hardening variable (q)
vartoplot{i}(2) = hvar_n(5) ; % Internal variable (r)
vartoplot{i}(3) = 1-hvar_n(6)/hvar_n(5)  ; %  Damage variable (d)
vartoplot{i}(4)=hvar_n(7);% C11 component of tangent operators
vartoplot{i}(5)=hvar_n(8);% C11 component of algorithmic constitutive operators
```

3. Modification in function `dibujar_criterio_dano1`

```
elseif MDtype==2
   tetha=[-0.5*pi*0.99999:0.01:pi*0.99999]; % the range is between -pi/2 to pi for only-tensile;
    D=size(tetha);                          %*  Range
    m1=cos(tetha);
    n1=m1;
    n1(n1<0)=0;% when the sigma is minus, it should be 0;
    m2=sin(tetha);
    n2=m2;
    n2(n2<0)=0;% when the sigma is minus, it should be 0;

    Contador=D(1,2);                        %
    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;

        for i=1:Contador
         radio(i)= q/sqrt([n1(i) n2(i) 0 nu*(n1(i)+n2(i))]*ce_inv*[m1(i) m2(i) 0 ...
             nu*(m1(i)+m2(i))]');

         s1(i)=radio(i)*m1(i);
         s2(i)=radio(i)*m2(i);

        end
    hplot =plot(s1,s2,tipo_linea);
```

```
elseif MDtype==3
    tetha=[0:0.01:2*pi];
    D=size(tetha);                          %*  Range
    m1=cos(tetha);
    n1=m1;
    n1(n1<0)=0;
    m2=sin(tetha);                          %*
    n2=m2;
    n2(n2<0)=0;
    Contador=D(1,2);                        %*
    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;



    for i=1:Contador
          sigma=[n1(i) n2(i) 0 nu*(n1(i)+n2(i))];
          sigma_abs=[m1(i) m2(i) 0 nu*(m1(i)+m2(i))];
          sita=sum(sigma,2)/sum(abs(sigma_abs),2);
        radio(i)=q/((sita+(1-sita)/n)*sqrt([m1(i) m2(i) 0 nu*(m1(i)+m2(i))]*ce_inv*[m1(i) m2(i) 0 ...
            nu*(m1(i)+m2(i))]'));
        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);
end
```

4. Modification in function `Modelos_de_dano1`

```
elseif (MDtype==2)  %* Only tension
    sigma = ce*eps_n1';
    sigma_only_tensile=sigma;
    sigma_only_tensile(sigma_only_tensile<0)=0;
  rtrial=sqrt(eps_n1*sigma_only_tensile);


elseif (MDtype==3)  %*Non-symmetric
     sigma = eps_n1*ce;
     sigma_abs=abs(sigma);
     sigma_non_symmetric=sigma;
     sigma_non_symmetric(sigma_non_symmetric<0)=0;
     sita=sum(sigma_non_symmetric,2)/sum(sigma_abs,2);
     C=sita+((1-sita)/n);
     rtrial= C*sqrt(eps_n1*ce*eps_n1');

end
```