Technical University of Catalonia

COMPUTATIONAL SOLID MECHANICS

ASSIGNMENT 2

Second Part

THREE DIMENSIONAL PLASTICITY MODELS [J2]

M.Sc. Computational Mechanics – CIMNE

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1. Introduction

This is the second part of the report for Assignment_2, the course "Computational Mechanics in Solids" which deals with ***3D Plasticity models [J2]***. The goal in this project is to implement the algorithm of constitutive model at gauss integration level in order to check the performance of 3D plastic model, so there would be no discretization of continuum model nor mesh procedure for the finite element method.

In this project, data base (Input variables) would be strains; so we have a strain driven code and along this way, backward Euler time stepping algorithm for one dimensional plasticity is implement covering both rate independent and rate dependent models. However, the code is only implemented for the rate dependent case and as a consequence of choosing zero viscosity parameter it would behave rate independent.

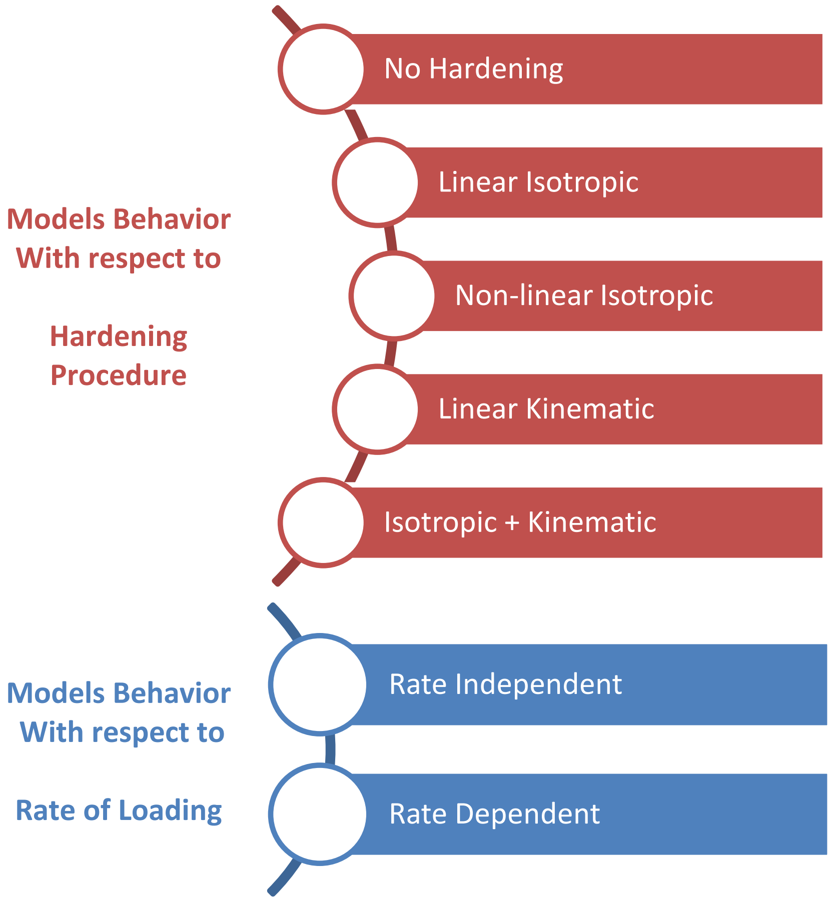

Chart1. Plasticity Models.

Different models for including hardening behaviors which are exploited in the code are introduced in ***Chart1*** and for all mentioned scenarios different numerical simulation of uniaxial cyclic plastic loading/elastic unloading examples are performed using the Matlab code which is provided and discussed in the ***Annex***.

For all cases the Young modulus is taken [***E = 200,000 MPa***] and the yield stress as [***σy = 200 MPa***]. Poisson ratio is also **0.3** for main cases. These material property values are almost in the range of *steel*.

Table1 is related to the regular three paths cyclic loading which is the main loading scenario for ***Chapter2*** to ***Chapter6*** and captures the **Tension-Compression-Tension** behavior of material.

Strain point	Loading scenario
1st point	0
2nd point	0.0025
3rd point	-0.0025
4th point	0.0005

Table1. Cyclic Loading Scenario

Based on ***Table2,*** 21 loading cases are studied In ***Chapter2*** to ***Chapter6***. In order to explain these cases, it is important to note that 5 Hardening type are considered into account and for each hardening type, 2 models of rate-independent and rate-dependent are considered. For any of which some sensitivity analysis is studied that can be checked in detail in ***Table2***.

Hardening Type	RATE INDEPENDENT			RATE DEPENDENT	
	Main case	Sensitivity analysis 1	Sensitivity analysis 2	Sensitivity analysis 1	Sensitivity analysis 2
Perfect Plasticity	***Case 1***			***Case 2*** - η	***Case 3*** - t
Linear Isotropic	***Case 4***	***Case 5*** - K		***Case 6*** - η	***Case 7*** - t
Nonlinear Isotropic	***Case 8***	***Case 9*** - δ	***Case 10*** - σu	***Case 11*** - η	***Case 12*** - t
Linear Kinematic	***Case 13***	***Case 14*** - H		***Case 15*** - η	***Case 16*** - t
Nonlinear Isotropic + Linear Kinematic	***Case 17***	***Case 18*** - δ	***Case 19*** - H	***Case 20*** - η	***Case 21*** - t

Table2. Numerical simulation cases

Table3 is related to different values of parameters in the sensitivity analysis done on different cases. In this table **Main** parameters are mentioned in the left column and 3 variants are introduced for each one, as well.

	Main	Var_1	Var_2	Var_3
K	50,000	25,000	50,000	100,000
H	50,000	25,000	50,000	100,000
δ	20,000	5,000	20,000	80,000
σu	350	250	450	550
η	5,000	1,000	5,000	10,000
t	1 sec	10 sec	1 sec	0.5 sec

Table3. Sensitivity analysis values

In each part, first of all, the step by step curve is plotted in 3 colors, each color for one path of loading. Then the sensitivity analysis results are plotted and discussed. For the rate-dependent models, the stress-strain and the stress-time curves are going to show the influence of the viscosity parameter and the loading rate. It is evident and also experienced in all cases that the rate-independent response can be recovered from the rate-dependent results using very small values for the viscosity or high values for the loading time (low loading rate). An interesting point in comparing the effect of viscosity and strain rate in rate dependent models is that by doubling the strain rate from one side and making the viscosity parameter half from the other side, the behavior of material would not undergo any change. This fact is studied in all 5 types of hardening scenarios and considering the page limit of this report, here only the results of sensitivity analysis on the viscosity are provided.

2. Perfect Plasticity

2.1. Rate Independent response

The Stress-strain curves for a rate-independent model with linear elastic and perfect plastic response is provided in ***Figure1***. Due to 3D nature of the model the material does not yield exactly when σy is reached by one of stress components. Rather it yields when the von Mises criteria is satisfied. The stress tensor σ can be interpreted as the sum of two stress tensors, namely the hydrostatic stress tensor as the cause of volume change in stressed material and the deviatoric stress tensor which tends to distort the material.

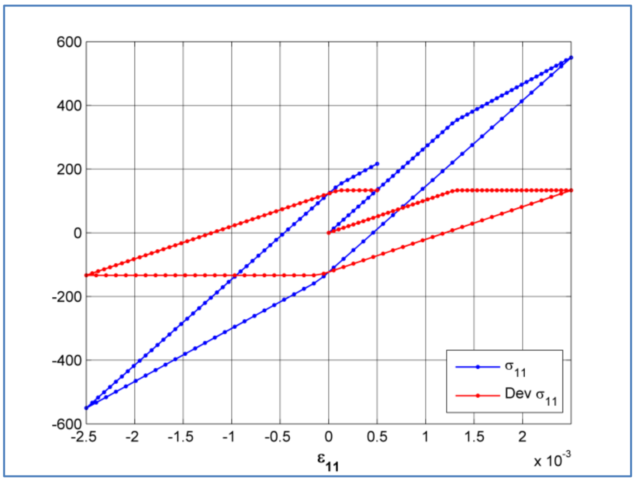

Figure1. rate-independent behavior of stress11 and deviatoric σ11 in perfect plasticity

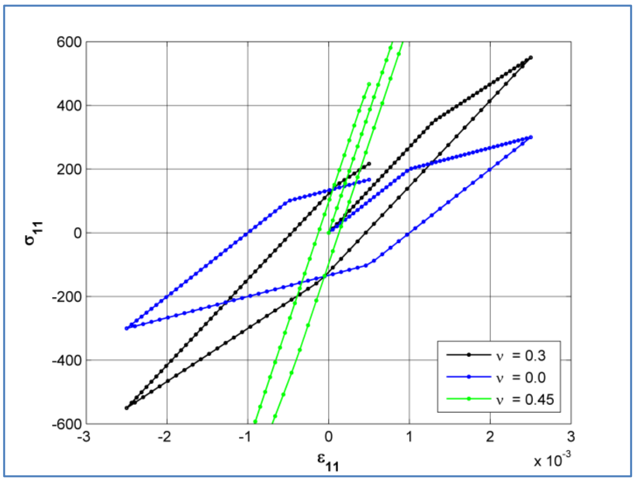

Figure2. Effect of poisson ratio on rate-independent behavior of stress11 in perfect plasticity

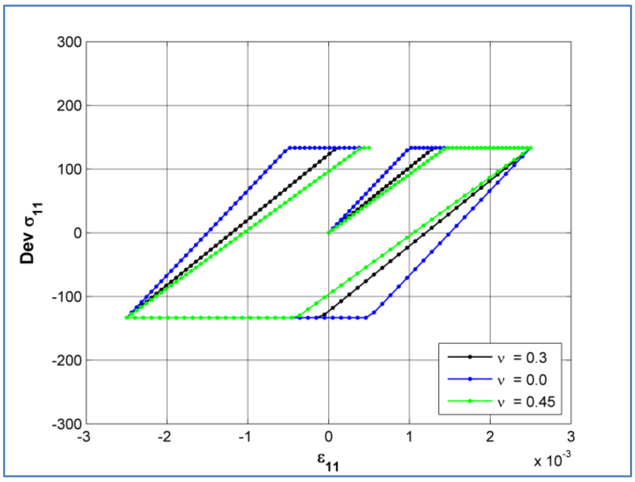

Figure3. Effect of poisson ratio on rate-independent behavior of deviatoric σ11 in perfect plasticity

The Von Mises yield condition is independent of hydrostatic stresses and predicts that the yielding of material begins exactly when the 2nd deviatoric stress invariant reaches a critical value (which is a function of the yield stress of the material in pure shear). So in this study more focus would be on the study of deviatoric part of the stress tensor which is predominant in defining the circular cylinder yield surface in the J2 model.

Figure2 and ***Figure 3*** provide the sensitivity analysis on Poisson ratio and as it was supposed by increasing the value of nu toward the limit of 0.5 since huge numbers are going to appear in the constitutive elastic tensor (since its denominators tend to zero), the stress values blow up in ***Figure2***.

2.2. Rate Dependent response

For the rate dependent models, we can observe in **Figure4** to **Figure7** that as lower the viscosity parameter would be or as higher the duration of loading would be (blue line), the closer the results would be to the rate independent case (black line) because load is being applied gradually.

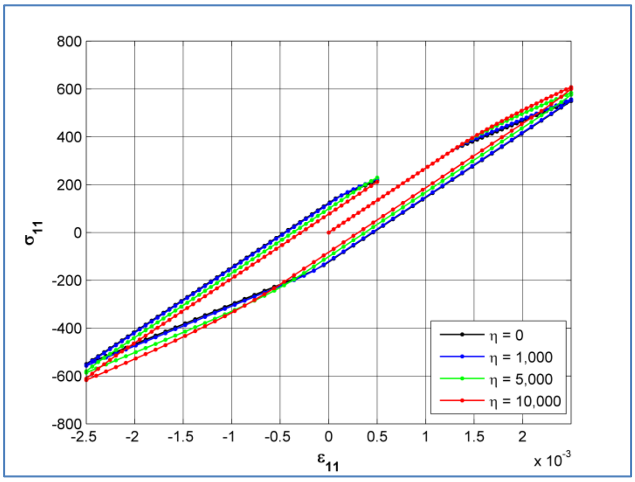

Figure4. Effect of viscosity on the rate-dependent behavior of stress11 in perfect plasticity

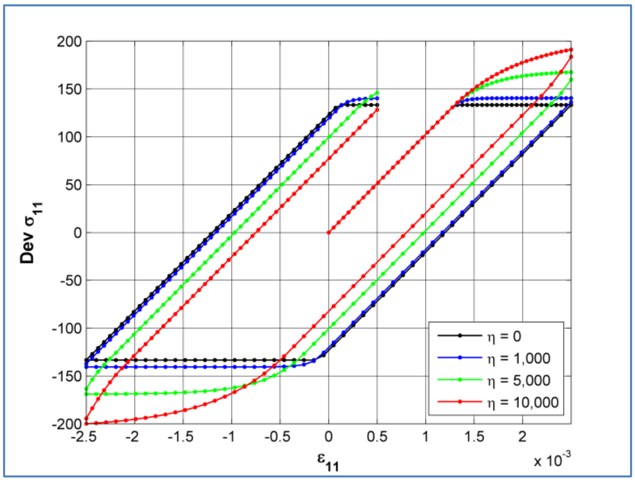

Figure5. Effect of viscosity on the rate-dependent behavior of deviatoric σ11 in perfect plasticity

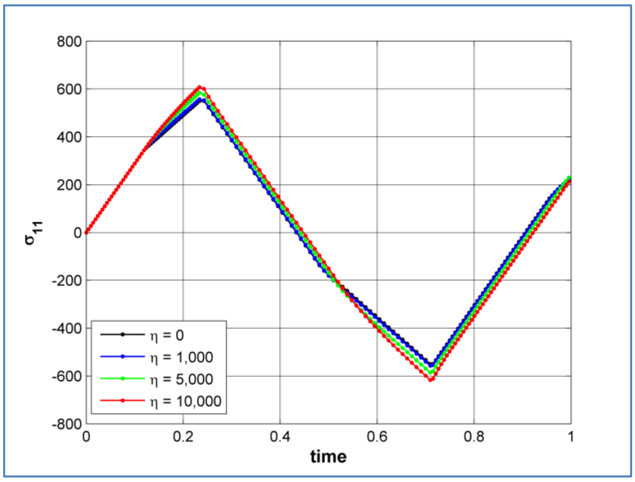

Figure6. Effect of viscosity on the rate-dependent behavior of stress11 in perfect plasticity

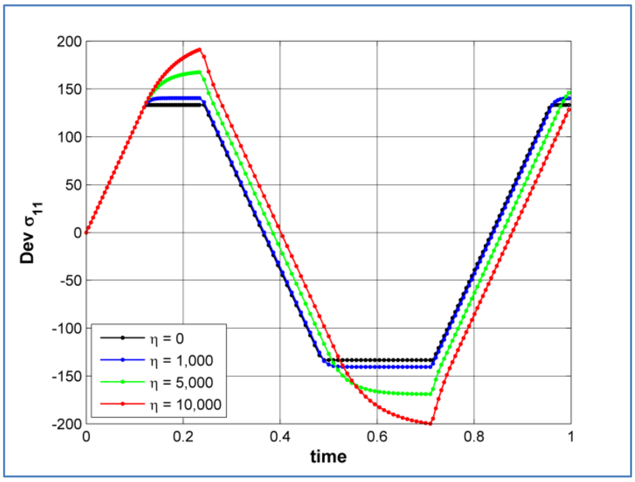

Figure7. Effect of viscosity on the rate-dependent behavior of deviatoric σ11 in perfect plasticity

3. *Linear Isotropic Hardening Plasticity*

3.1. *Rate Independent response*

The effect of expansion in yield surface due to the isotropic hardening is portrayed in ***Figure8*** and by comparing it to ***Figure1*** for the perfect plastic case we clearly observe that the max and min value of deviatoric and main stress are considerably increased.

Figure9 and ***Figure10*** also deliver a perspective related to the effect of Isotropic hardening value (K) on the behavior of this model. So, increasing the value of K would dramatically affect the expansion rate, mainly in deviatoric stress tensor.

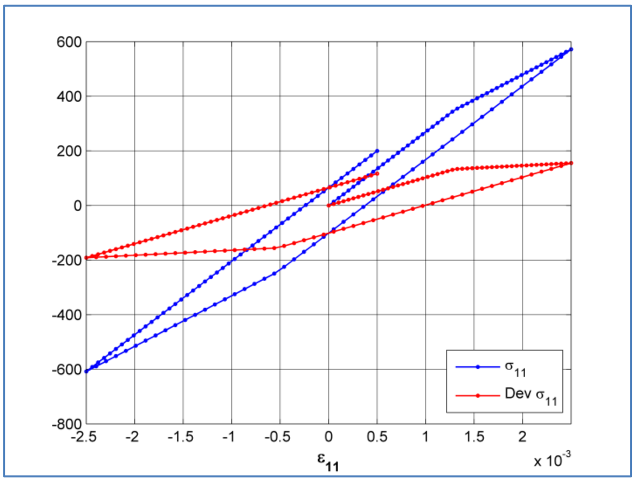

Figure8. rate-independent behavior of stress11 and deviatoric σ11 in linear isotropic hardening plasticity

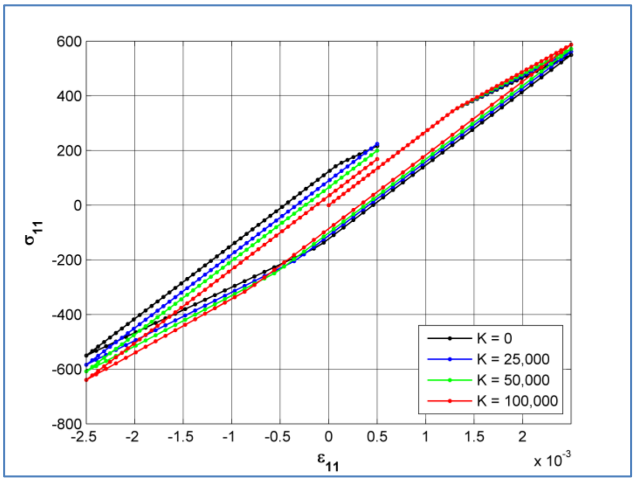

Figure9. Effect of K on the rate-independent behavior of stress11 in isotropic hardening plasticity

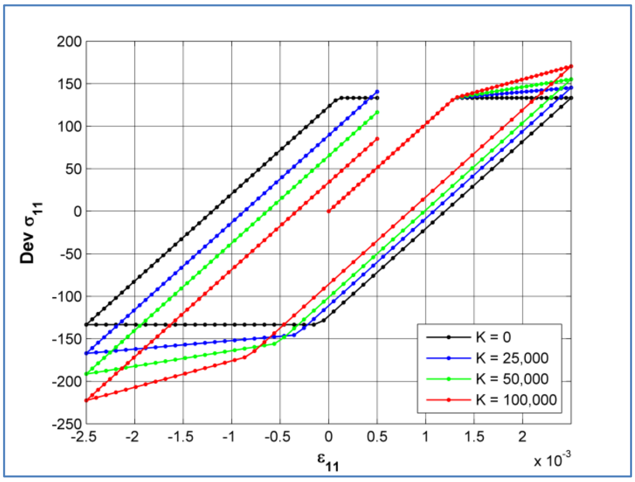

Figure10. Effect of K on the rate-dependent behavior of deviatoric σ11 in isotropic hardening plasticity

3.2. *Rate Dependent response*

Figure11 to **Figure14** prove that as lower the viscosity parameter or as higher the duration of loading would be, the closer the results would be to the rate independent case (black line).

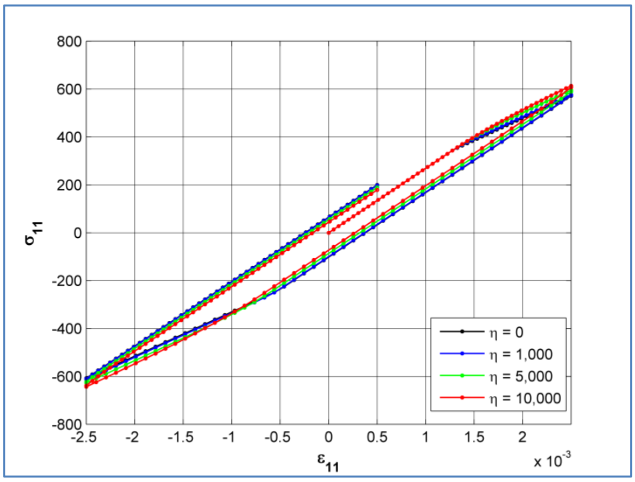

Figure11. Effect of viscosity on the rate-independent behavior of stress11 in isotropic hardening plasticity

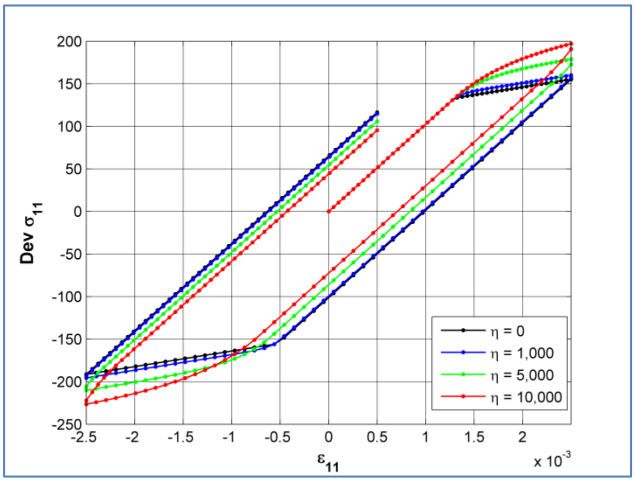

Figure12. Effect of viscosity on rate-dependent behavior of dev σ11 in isotropic hardening plasticity

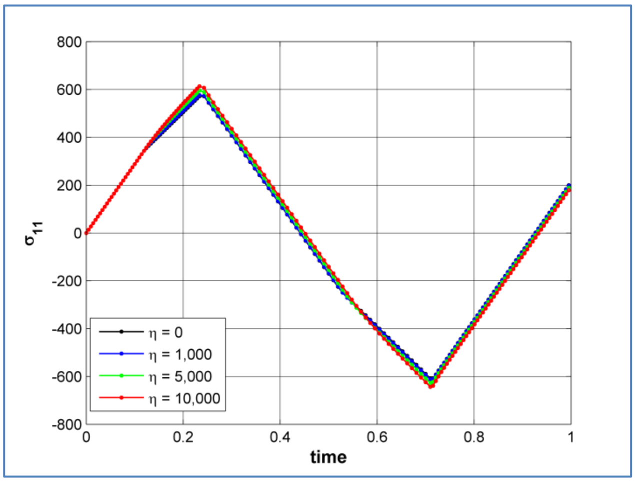

Figure13. Effect of viscosity on the rate-independent behavior of stress11 in isotropic hardening plasticity

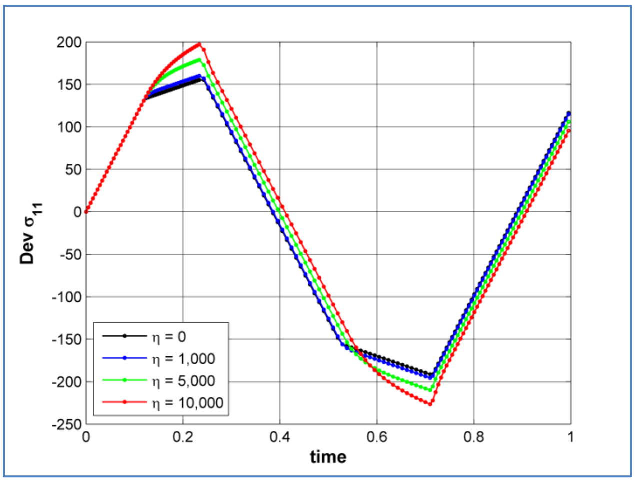

Figure14. Effect of viscosity on rate-dependent behavior of dev σ11 in isotropic hardening plasticity

4. Nonlinear Isotropic Hardening Plasticity

4.1. Rate Independent response

For the nonlinear isotropic hardening a nonlinear exponential saturation law + linear part for the isotropic hardening is available in the code and for this purpose the well-known “Newton-Raphson” iterative method is considered in order to calculate the gamma. Although for the sake of observation of exponential effect in all tests K=0 is chosen and the linear part is omitted.

Figure15 is comparable with ***Figure1*** and ***Figure8*** and demonstrates the exponential convergence to the asymptotic value of the yield stress.

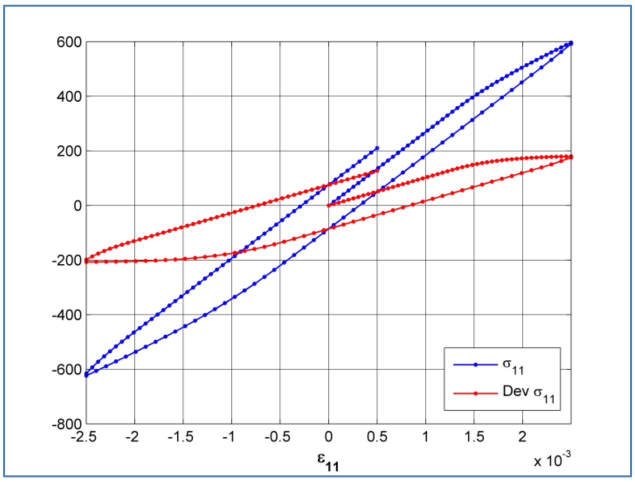

Figure15. rate-independent behavior of stress11 and deviatoric σ11 in nonlinear isotropic harden plasticity

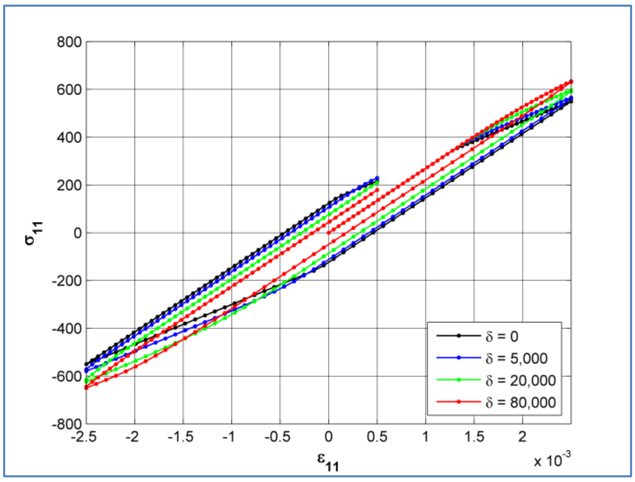

Figure16. Effect of delta on the rate-independent behavior of stress11 in nonlinear isotropic hardening

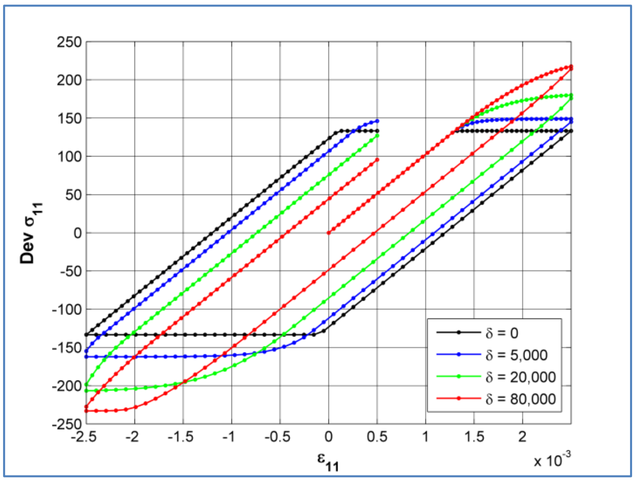

Figure17. Effect of delta on the rate-independent behavior of dev σ11 in nonlinear isotropic hardening

Figure16 and ***Figure17*** describe the effect of exponential coefficient (delta) on models behavior. This effect is more visible in deviatoric stress tensor and it is evident that by increasing the delta value the rate at which stress-strain curve attain the asymptotic value of the yield stress (sigma infinity) is increased dramatically. Delta=20000 is chosen for the rest of this part. An interesting point in ***Figure11*** is that by increasing the value of delta, material reaches to the sigma infinity so soon. So, the capacity of nonlinear behavior of material goes to end and in next cycles we would witness less and less nonlinear effect (red curve), because the asymptotic value of the yield stress is reached so early and the threshold is filled.

4.2. Rate Dependent response

Figure18 to ***Figure21*** show that the rate independent results (black line) would be achieved if a low viscosity parameter or a high duration of loading is chosen (blue line) and they also exhibit the effect of increasing viscosity on the shape of stress-time curve, in order to study the increased deviatoric stress peaks that are available by the capacity of high rate dependency.

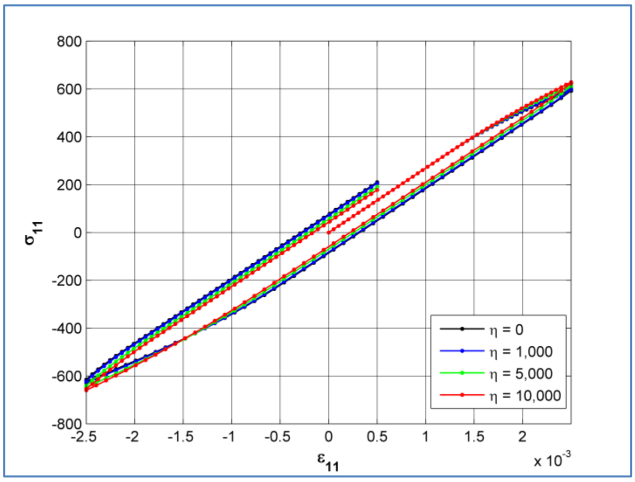

Figure18. Effect of viscosity on the rate-independent behavior of stress11 in nonlinear isotropic hardening

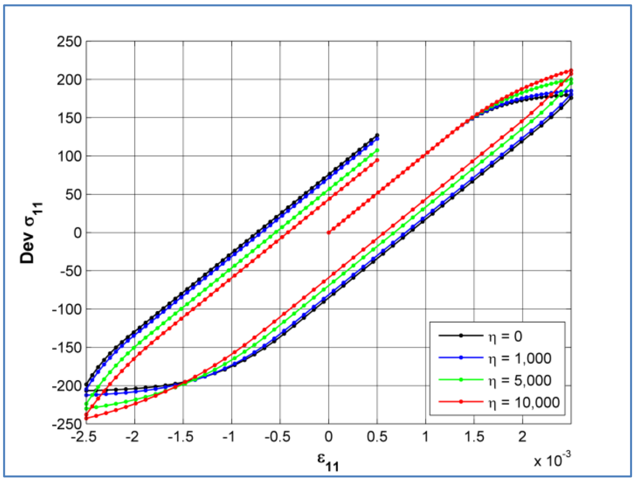

Figure19. Effect of viscosity on rate-dependent behavior of dev σ11 in nonlinear isotropic hardening

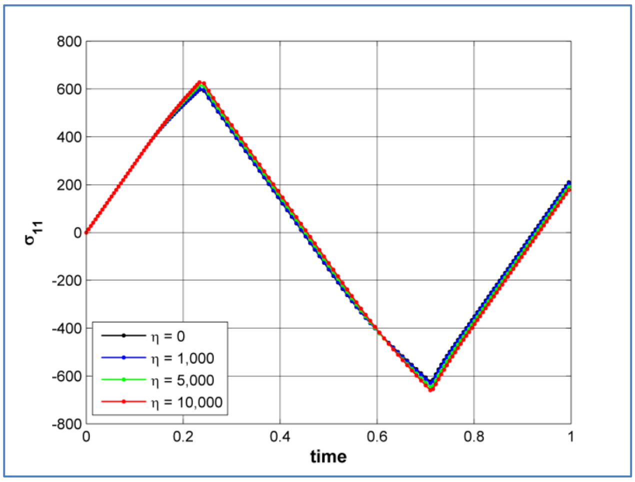

Figure20. Effect of viscosity on the rate-independent behavior of stress11 in nonlinear isotropic hardening

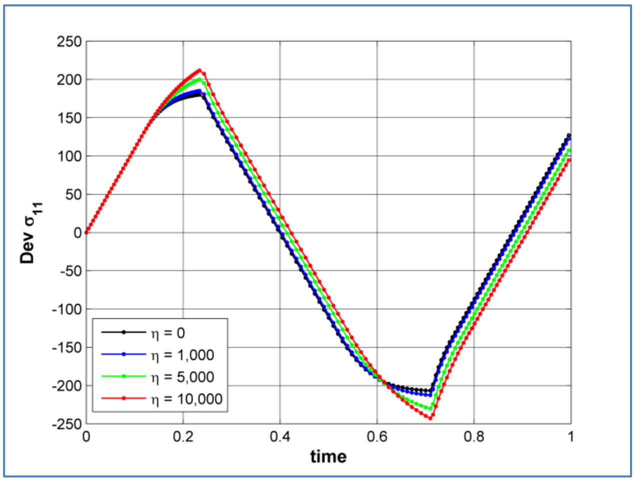

Figure21. Effect of viscosity on rate-dependent behavior of dev σ11 in nonlinear isotropic hardening

5. Linear Kinematic Hardening Plasticity

5.1. Rate Independent response

The effect of translation in yield surface due to the kinematic hardening is portrayed in ***Figure22.*** We can compare extreme tolerated deviatoric stresses by ***Figure1*** for the perfect plastic case and see that yield surface is slightly shifted upward.

Figure23 and ***Figure24*** also deliver a perspective related to the effect of kinematic hardening value (H) on the behavior of this model. So, increasing the value of H would dramatically affect the translation rate, mainly in deviatoric stress tensor, ***Figure24***.

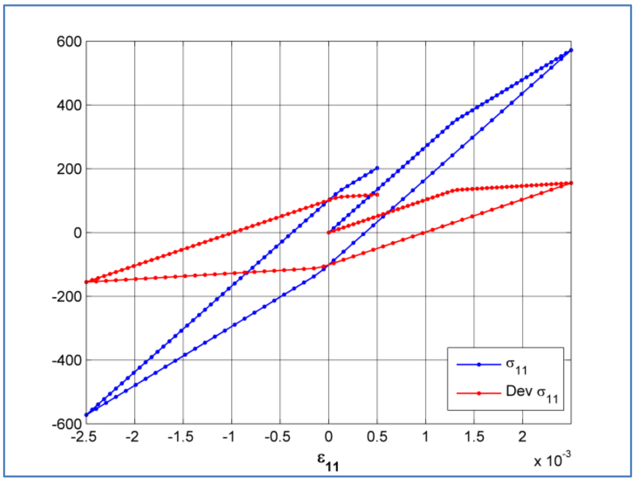

Figure22. rate-independent behavior of stress11 and deviatoric σ11 in liniear kinematic harden plasticity

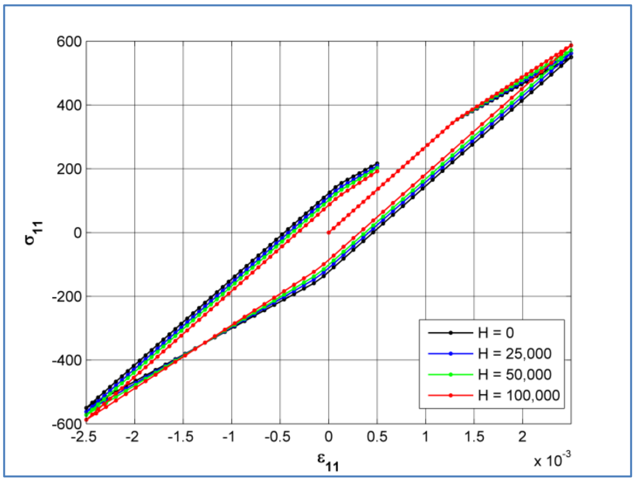

Figure23. Effect of H on the rate-independent behavior of stress11 in kinematic hardening plasticity

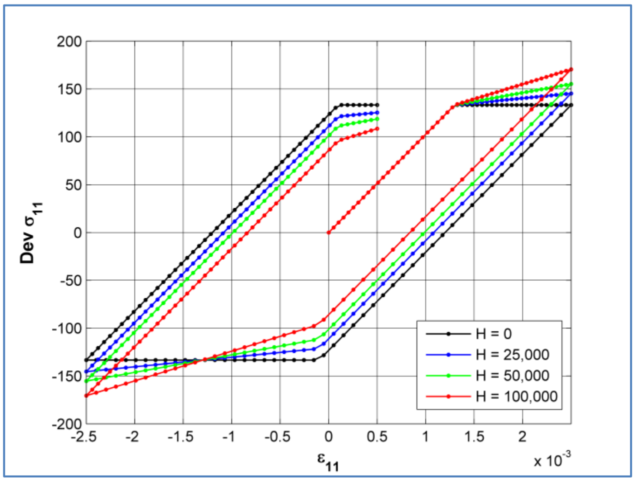

Figure24. Effect of H on the rate-independent behavior of dev σ11 in kinematic hardening plasticity

5.2. *Rate Dependent response*

For the rate dependent model, we can observe in ***Figure25*** and ***Figure26*** that like previous cases as lower the viscosity parameter would be or as higher the duration of loading would be, closer the results would be to the rate independent case (black line).

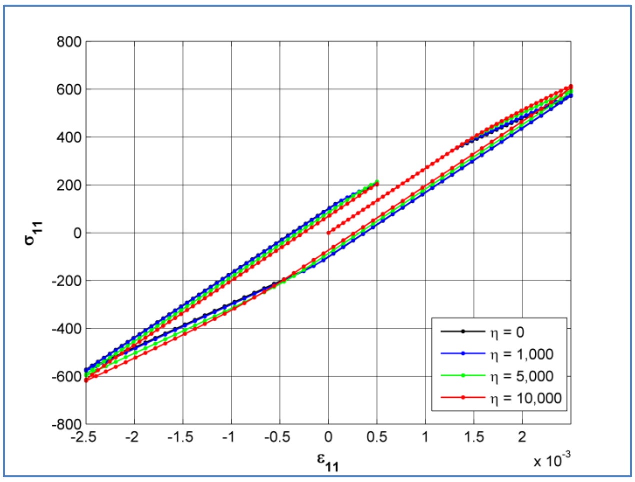

Figure25. Effect of viscosity on the rate-independent behavior of stress11 in kinematic hardening plasticity

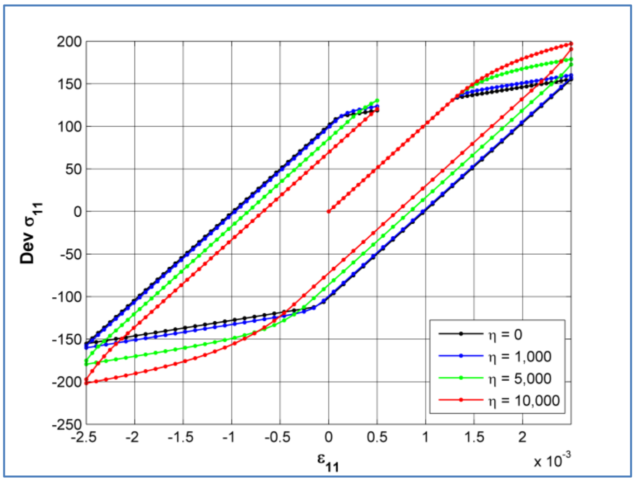

Figure26. Effect of viscosity on the rate-independent behavior of dev σ11 in kinematic hardening plasticity

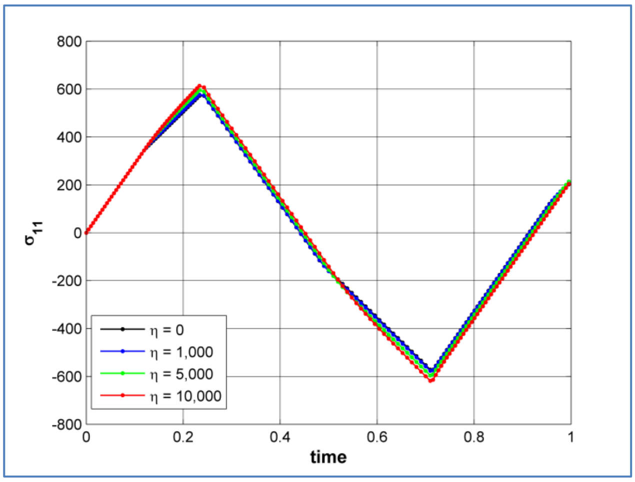

Figure27. Effect of viscosity on the rate-independent behavior of stress11 in kinematic hardening plasticity

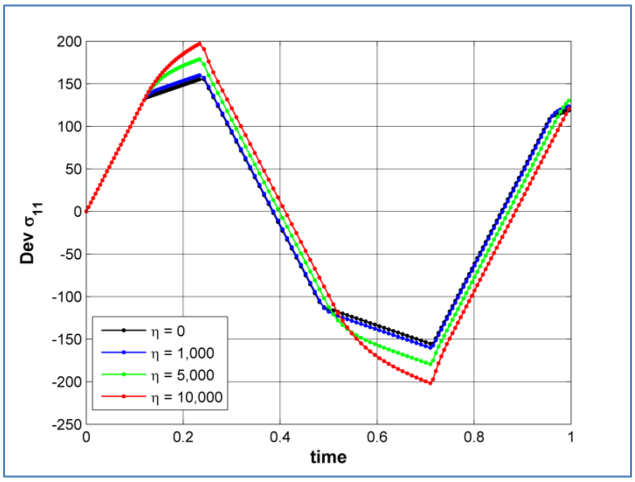

Figure28. Effect of viscosity on the rate-independent behavior of dev σ11 in kinematic hardening plasticity

Figure27 and ***Figure28*** plot the stress-time curve for linear kinematic hardening and shows that the increased viscosity would case higher peaks in stress capacity in tension and compression and as lower the viscosity would be, closer the results would be to the rate independent case (black line).

6. Nonlinear Isotropic and Linear Kinematic Hardening Plasticity

6.1. Rate Independent response

This part includes both isotropic (nonlinear) hardening and kinematic (linear) hardening. ***Figure29*** has both effects and is comparable with ***Figure15*** for the nonlinear isotropic and ***Figure22*** for the linear kinematic cases.

Figure30 and ***Figure31*** describe the effect of exponential coefficient (delta). As ***chapter 4.1*** it is again evident that increase of delta would cause increase of the rate at which stress-strain curve reaches the asymptotic value of the sigma infinity, in both main and deviatoric stress parts.

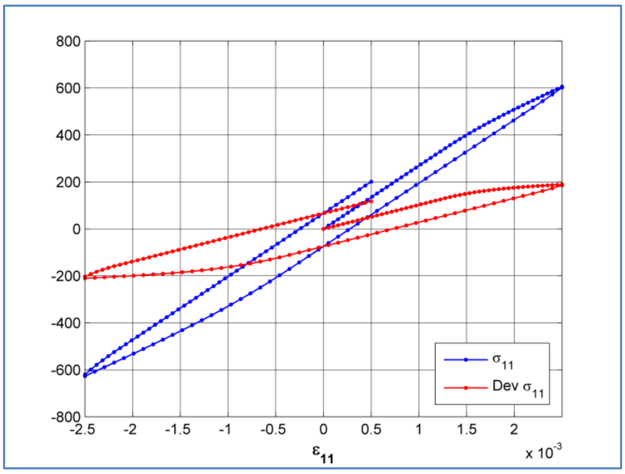

Figure29. rate-independent behavior of stress11 and deviatoric σ11 in mixed hardening plasticity

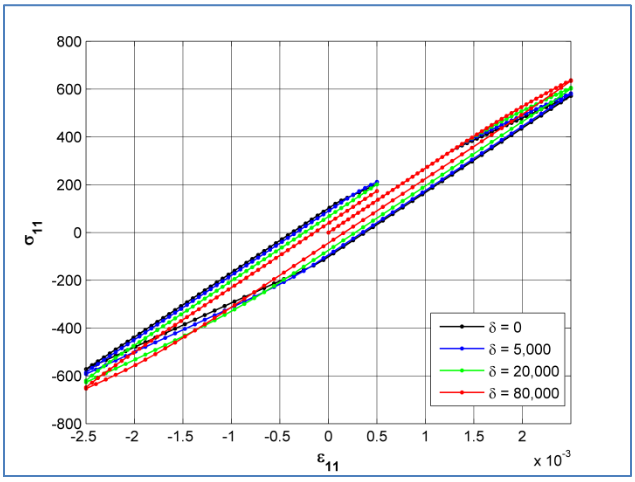

Figure30. Effect of delta on the rate-independent behavior of stress11 in mixed hardening plasticity

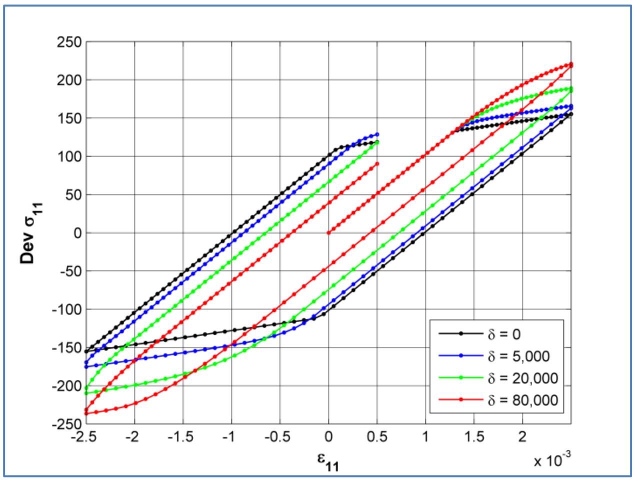

Figure31. Effect of delta on the rate-independent behavior of dev σ11 in mixed hardening plasticity

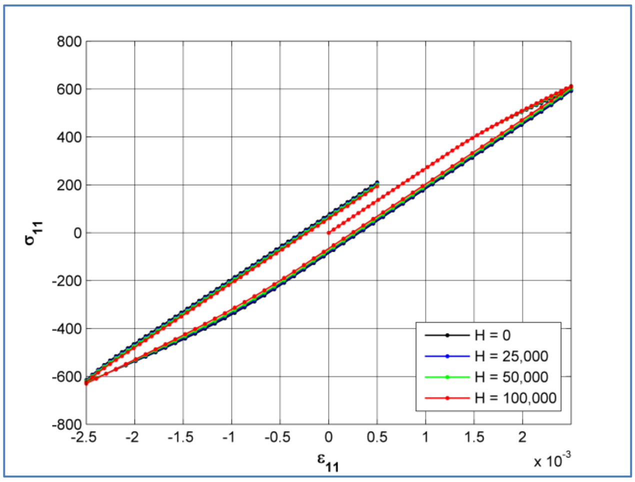

Figure32. Effect of H on the rate-independent behavior of stress11 in mixed hardening plasticity

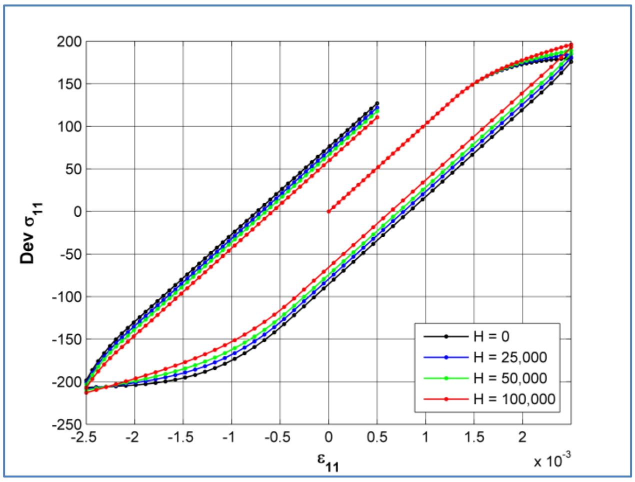

Figure33. Effect of H on the rate-independent behavior of dev σ11 in mixed hardening plasticity

Moreover, ***Figure32*** and ***Figure33*** clearly show the impact of adding the linear kinematic hardening term (H) beside nonlinear isotropic hardening model.

6.2. Rate Dependent response

The rate dependent behavior is somehow predictable from chapter4 and 5. ***Figure34*** to ***Figure35*** show again that as lower the viscosity parameter would be or as higher the duration of loading would be (blue line), the closer the results would be to the rate independent case (black line) because load is being applied gradually.

Figure36 and ***Figure37*** also plot the stress-time curve for nonlinear isotropic + linear kinematic hardening and show that the increased viscosity would case higher peaks in stress capacity in tension and compression, mainly in deviatoric part of stress and as lower the viscosity would be, closer the results would be to the rate independent case (black line).

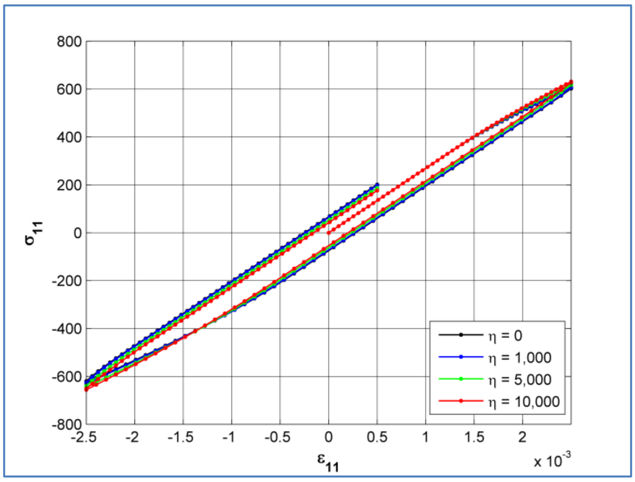

Figure34. Effect of viscosity on the rate-independent behavior of stress11 in mixed hardening plasticity

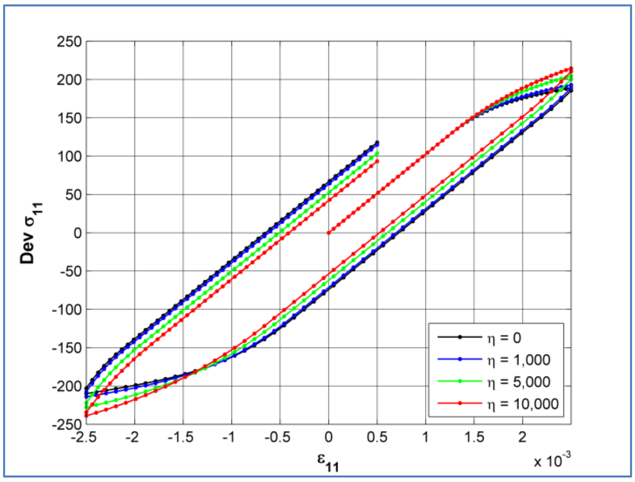

Figure35. Effect of viscosity on the rate-independent behavior of dev σ11 in mixed hardening plasticity

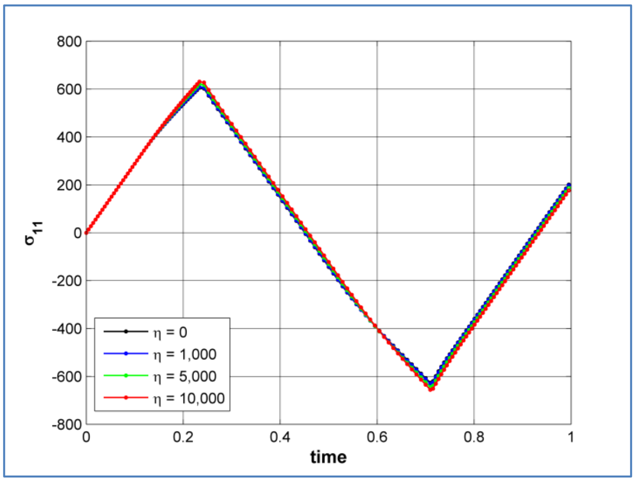

Figure36. Effect of viscosity on the rate-independent behavior of stress11 in mixed hardening plasticity

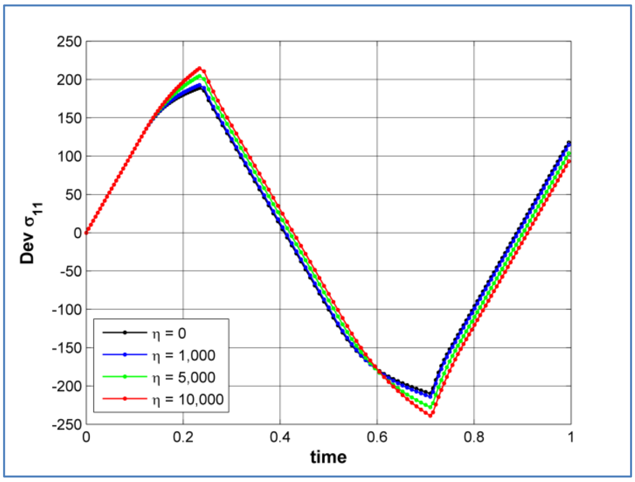

Figure37. Effect of viscosity on the rate-independent behavior of dev σ11 in mixed hardening plasticity

❖ *Appendix: Code*

```
%% 3D Plasticity Model [J2]

%==============================================================%
%            Linear isotropic hardening plasticity             %
%         Nonlinear isotropic hardening plasticity             %
%            Linear kinematic hardening plasticity             %
%==============================================================%
%                                                              %
%--INPUTS------------------------------------------------------%
%                                                              %
%   SIGy         : Yield Stress                                %
%   SIGu         : Ultimate Stress                             %
%   E            : Young's Modulus                             %
%   K            : Isotropic Hardening Modulus                 %
%   H            : Kinematic Hardening Modulus                 %
%   Delta        : Exponential Parameter                       %
%   Eta          : Viscosity Coefficient                       %
%   nPath        : Number of Loading path                      %
%   nStep        : Number of time steps in each path           %
%   Strain_rate  : Number of time steps in each path           %
%   nu           : Poisson ratio                               %
%                                                              %
%--OUTPUTS-----------------------------------------------------%
%                                                              %
%   EPS_input    : Strain Cyclic Loading Points [INPUT DATA]   %
%   EPS          : Strain Evolution                            %
%   EPS_p        : Plastic Strain Evolution                    %
%   SIG          : Stress Evolution                            %
%   EPS_p_tr     : Trial Plastic Strain                        %
%   EPS_e_tr     : Trial elastic Strain                        %
%   SIG_tr       : Trial Stress                                %
%   EPS_Hist     : Strain History                              %
%   SIG_Hist     : Stress History                              %
%                                                              %
%--------------------------------------------------------------%
clc; clear all; %close all;

colors = input('colors = 1:black - 2:blue - 3:green - 4:red ');
axislimit     = 4;

nu            = 0.3;
SIGy          = 200;
SIGu          = 350;
E             = 200000;

K             = 50000*0;
H             = 50000*1;
Delta         = 5000*4;

Eta           = 1000*10;

nPath         = 3;
nStep         = 50;
% Strain_rate  = 0.00105;     % for t = 10 sec
Strain_rate  = 0.01050;     % for t = 1 sec
% Strain_rate  = 0.02100;     % for t = 0.5 sec

EPS_input(1)  =  0.0000;     %Strain Cyclic Loading Points [INPUT DATA]
EPS_input(2)  =  0.0025;     %max number = nPath+1
EPS_input(3)  = -0.0025;
EPS_input(4)  =  0.0005;
```

Input data by user

Cyclic loading Strain points

Building required arrays for strain and stress evolution

```
%% DEFINING STRAIN & STRESS ARRAY

TotStep  = nPath*nStep;
StepTime = zeros(nPath,1);
dt       = zeros(TotStep,1);
Timing   = zeros(TotStep,1);
EPS_Path = zeros(nPath,nStep);

for i=1:nPath
    StepTime(i) = abs(EPS_input(i+1) - EPS_input(i)) / Strain_rate;
    dt( (i-1)*nStep+1 : i*nStep ) = StepTime(i) / nStep;
    EPS_Path(i,:) = linspace( EPS_input(i) , EPS_input(i+1) , nStep );
end
TotTime = sum(StepTime(:));

for i=2:TotStep
    Timing(i) = Timing(i-1) + dt(i);
end

SIG_Hist = zeros(TotStep,1);
EPS_Hist = EPS_Path(1,:)';
for i=2:nPath
    EPS_Hist = [EPS_Hist ; EPS_Path(i,:)'];
end

%---------------------------------------------------------------------------
EPS{1}   = zeros(TotStep,6);    % { epsilon }
EPS{2}   = zeros(TotStep,1);    % { 0 }
EPS{3}   = zeros(TotStep,6);    % { 0 }

EPS_p{1} = zeros(TotStep,6);    % { epsilon plastic }
EPS_p{2} = zeros(TotStep,1);    % { exi }
EPS_p{3} = zeros(TotStep,6);    % { exi_bar }

SIG{1}   = zeros(TotStep,6);    % { sigma }
SIG{2}   = zeros(TotStep,1);    % { q }
SIG{3}   = zeros(TotStep,6);    % { q_bar }

%---------------------------------------------------------------------------
E11 = E *  (1-nu) /  ( (1+nu) * (1-2*nu) );
E12 = E *   nu    /  ( (1+nu) * (1-2*nu) );
E44 = E           /  ( (1+nu) * 2        );

EE = [  E11 E12 E12 0   0   0;
        E12 E11 E12 0   0   0;
        E12 E12 E11 0   0   0;
        0   0   0   E44 0   0;
        0   0   0   0   E44 0;
        0   0   0   0   0   E44];
KK = K;
HH =(2/3) * H * eye(6);
mu = E44;
```

Constitutive Elastic Tensor

Loop for each loading step

Trial State Stresses

Linear hardening

Nonlinear hardening

Newton-Raphson algorithm

```
%% The Loading Cycle Loop

EPS{1}(:,1) = EPS_Hist;

for i=2:TotStep
%---------------------------------------------------------------Trial state
    EPS_p_tr{1} = EPS_p{1}(i-1,:);
    EPS_p_tr{2} = EPS_p{2}(i-1,:);
    EPS_p_tr{3} = EPS_p{3}(i-1,:);

    EPS_e_tr{1} = EPS{1}(i,:) - EPS_p_tr{1};
    EPS_e_tr{2} = EPS{2}(i,:) - EPS_p_tr{2};
    EPS_e_tr{3} = EPS{3}(i,:) - EPS_p_tr{3};

    SIG_tr{1}   = (EE * EPS_e_tr{1}')';
    SIG_tr{2}   =  KK * EPS_e_tr{2};
    SIG_tr{3}   = (HH * EPS_e_tr{3}')';

    SIG_mean_tr = sum(SIG_tr{1}(1:3))/3;
    SIG_HYD_tr  = SIG_mean_tr * [1,1,1,0,0,0];
    SIG_DEV_tr  = SIG_tr{1} - SIG_HYD_tr;

    n_tr = ( SIG_DEV_tr - SIG_tr{3} ) / norm( SIG_DEV_tr - SIG_tr{3} );

    F_tr = norm( SIG_DEV_tr - SIG_tr{3} ) - sqrt(2/3)*(SIGy - SIG_tr{2});
    if  F_tr <= 0
    %-----------------------------------------------------------Elastic Part
        EPS_p{1}(i,:) = EPS_p_tr{1};
        EPS_p{2}(i,:) = EPS_p_tr{2};
        EPS_p{3}(i,:) = EPS_p_tr{3};

        SIG{1}(i,:) = SIG_tr{1};
        SIG{2}(i,:) = SIG_tr{2};
        SIG{3}(i,:) = SIG_tr{3};

        EE_ep = EE;
        Gam  = 0;
    else
    %-----------------------------------------------------------Plastic Part
        if Delta==0         %====== Linear  Hardening =====%
            Gam = F_tr / ( 2*mu + 2/3*K + 2/3*H + Eta/dt(i) ) / dt(i);
        else                %==== Nonlinear Hardening ====%
            tol = 1e-6;
            g   = 0.01;
            Gam = 0;
            while g>=tol
                X1   = EPS_p{2}(i,:);
                X2   = EPS_p{2}(i,:) + Gam * dt(i) * sqrt(2/3);
                Pi1  = ( SIGu - SIGy ) * ( 1 - exp(-Delta * X1) ) + K * X1;
                Pi2  = ( SIGu - SIGy ) * ( 1 - exp(-Delta * X2) ) + K * X2;
                Pii2 = ( SIGu - SIGy ) * Delta * exp(-Delta * X2) + K ;

                g    = F_tr - Gam * dt(i) * ( 2*mu + 2/3*H + Eta/dt(i) ) -
sqrt(2/3)*(Pi2 - Pi1);
                Dg   = -dt(i) * ( 2*mu + 2/3*Pii2 + 2/3*H + Eta/dt(i) );
                DGam = -g / Dg;
                Gam  = Gam + DGam;
            end
        end

        X1  = EPS_p{2}(i,:);
        X2  = EPS_p{2}(i,:) + Gam * dt(i) * sqrt(2/3);
        Pi1 = ( SIGu - SIGy ) * ( 1 - exp(-Delta * X1) ) + K * X1;
```

```
        Pi2 = ( SIGu - SIGy ) * ( 1 - exp(-Delta * X2) ) + K * X2;

        SIG{1}(i,:)    = SIG_tr{1} - Gam * dt(i) * 2 * mu * n_tr;
        SIG{2}(i,:)    = SIG_tr{2} - (Pi2-Pi1);
        SIG{3}(i,:)    = SIG_tr{3} + Gam * dt(i) * 2/3 * H * n_tr;

        EPS_p{1}(i,:) = EPS_p{1}(i-1,:) + Gam * dt(i) * n_tr;
        EPS_p{2}(i,:) = EPS_p{2}(i-1,:) + Gam * dt(i) * sqrt(2/3);
        EPS_p{3}(i,:) = EPS_p{3} (i-1,:) - Gam * dt(i) * n_tr;

    end

    SIG_mean = sum(SIG{1}(i,1:3))/3;
    SIG_HYD  = SIG_mean * [1,1,1,0,0,0];
    SIG_DEV  = SIG{1}(i,:) - SIG_HYD;

    SIG_Hist(i,1) = SIG{1}(i,1);
    SIG_Hist(i,2) = SIG_DEV(1);
end

%% Plot
plotcurves_3D(EPS_Path,EPS_Hist,SIG_Hist,Timing,nStep,colors,axislimit);
```

Final stress and strain value at step

Plotting

Plot function

```
function plotcurves_3D(EPS_Path,EPS_Hist,SIG_Hist,Timing,nStep,colors,axislimit)

%--------------------------------------------------------------------------
figure(1);
plot(EPS_Path(1,:),SIG_Hist(1          :  nStep,1)
,'g','LineWidth',1,'Marker','o','MarkerSize',5); grid on; hold on;
plot(EPS_Path(2,:),SIG_Hist(1+  nStep :2*nStep,1)
,'m','LineWidth',1,'Marker','^','MarkerSize',5); grid on; hold on;
plot(EPS_Path(3,:),SIG_Hist(1+2*nStep :3*nStep,1)
,'r','LineWidth',1,'Marker','d','MarkerSize',5); grid on; hold on;
switch axislimit
    case 1; axis([-0.003,0.003,-300,300])
    case 2; axis([-0.003,0.003,-400,400])
    case 3; axis([-0.003,0.003,-500,500])
    case 4;
end
xlabel('\epsilon_{11}','FontSize',12,'FontWeight','bold')
ylabel('\sigma_{11}','FontSize',12,'FontWeight','bold')
set(gca,'GridLineStyle','-');
```

Stress11-strain

```
%--------------------------------------------------------------------------
figure(2);
plot(EPS_Path(1,:),SIG_Hist(1          :  nStep,2)
,'g','LineWidth',1,'Marker','o','MarkerSize',5); grid on; hold on;
plot(EPS_Path(2,:),SIG_Hist(1+  nStep :2*nStep,2)
,'m','LineWidth',1,'Marker','^','MarkerSize',5); grid on; hold on;
plot(EPS_Path(3,:),SIG_Hist(1+2*nStep :3*nStep,2)
,'r','LineWidth',1,'Marker','d','MarkerSize',5); grid on; hold on;
switch axislimit
    case 1; axis([-0.003,0.003,-300,300])
    case 2; axis([-0.003,0.003,-400,400])
    case 3; axis([-0.003,0.003,-500,500])
    case 4;
end
xlabel('\epsilon_{11}','FontSize',12,'FontWeight','bold')
ylabel('Dev \sigma_{11}','FontSize',12,'FontWeight','bold')
set(gca,'GridLineStyle','-');
```

Dev (Stress11)-strain

```
%--------------------------------------------------------------------------
switch colors
    case 1; cc='k'; mm='o';
    case 2; cc='b'; mm='d';
    case 3; cc='g'; mm='^';
    case 4; cc='r'; mm='.';
end
```

Color selection

```
%--------------------------------------------------------------------------
figure(3);
plot(EPS_Hist,SIG_Hist(:,1) ,cc,'LineWidth',1,'Marker','o','MarkerSize',2); grid
on; hold on;
switch axislimit
    case 1; axis([-0.003,0.003,-300,300])
    case 2; axis([-0.003,0.003,-400,400])
    case 3; axis([-0.003,0.003,-500,500])
    case 4;
end
xlabel('\epsilon_{11}','FontSize',12,'FontWeight','bold')
ylabel('\sigma_{11}','FontSize',12,'FontWeight','bold')
legend('bbb');
set(gca,'GridLineStyle','-');

%--------------------------------------------------------------------------
```

Stress11-strain

Dev (Stress11)-strain

```
figure(4);
plot(EPS_Hist,SIG_Hist(:,2) ,cc,'LineWidth',1,'Marker','o','MarkerSize',2); grid
on; hold on;
switch axislimit
    case 1; axis([-0.003,0.003,-300,300])
    case 2; axis([-0.003,0.003,-400,400])
    case 3; axis([-0.003,0.003,-500,500])
    case 4;
end
xlabel('\epsilon_{11}','FontSize',12,'FontWeight','bold')
ylabel('Dev \sigma_{11}','FontSize',12,'FontWeight','bold')
legend('bbb');
set(gca,'GridLineStyle','-');


%-----------------------------------------------------------------------------
figure(5);
plot(Timing,SIG_Hist(:,1) ,cc,'LineWidth',1,'Marker','o','MarkerSize',2); grid on;
hold on;
xlabel('time','FontSize',12,'FontWeight','bold')
ylabel('\sigma_{11}','FontSize',12,'FontWeight','bold')
legend('bbb');
set(gca,'GridLineStyle','-');
```

Stress11-time

```
%-----------------------------------------------------------------------------
figure(6);
plot(Timing,SIG_Hist(:,2) ,cc,'LineWidth',1,'Marker','o','MarkerSize',2); grid on;
hold on;
xlabel('time','FontSize',12,'FontWeight','bold')
ylabel('Dev \sigma_{11}','FontSize',12,'FontWeight','bold')
legend('bbb');
set(gca,'GridLineStyle','-');
```

Dev (Stress11)-time

```
%-----------------------------------------------------------------------------
figure(7);
plot(EPS_Hist,SIG_Hist(:,1),'b','LineWidth',1,'Marker','o','MarkerSize',2); grid
on; hold on;
plot(EPS_Hist,SIG_Hist(:,2),'r','LineWidth',1,'Marker','o','MarkerSize',2); grid
on; hold on;
switch axislimit
    case 1; axis([-0.003,0.003,-300,300])
    case 2; axis([-0.003,0.003,-400,400])
    case 3; axis([-0.003,0.003,-500,500])
    case 4;
end
xlabel('\epsilon_{11}','FontSize',12,'FontWeight','bold')
legend('bbb');
set(gca,'GridLineStyle','-');

end
```

Stress11 and Dev (Stress11)-time