COMPUTATIONAL SOLID MECHANICS

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Assignment 1

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This report contains two section of damage modelling naming 1) Rate independent models and 2) Rate dependent models using a example problem and its MATLAB coding.these hardening two types are subdivided into two i)LINEAR hardening and ii) EXPONENTIAL hardening

PART I (RATE INDEPENDENT MODELS):
Main objective of this section report is to show the understanding of the rate independent models with two type of the models non- symmetric tension-compression type and tension only type.

NON-SYMMENTRIC MODEL

LINEAR HARDENING CASE 1
In the first type of model non-Symmetric, we take the yield stress as 200 and the linear type of hardening

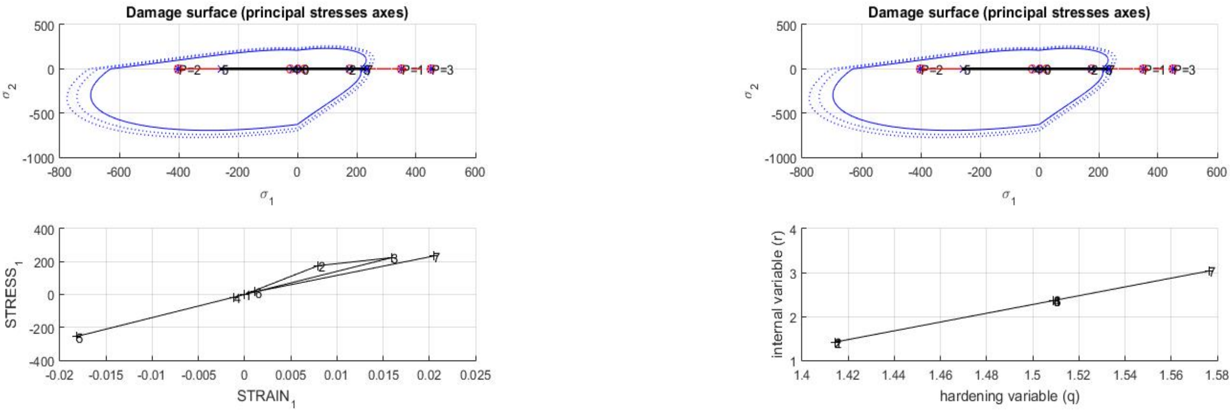
$$\Delta\bar{\sigma}_1^{(1)} = \alpha \quad ; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad \text{(uniaxial tensile loading)}$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta \quad ; \quad \Delta\bar{\sigma}_2^{(2)} = 0 \quad \text{(uniaxial tensile unloading/compressive loading)}$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma \quad ; \quad \Delta\bar{\sigma}_2^{(3)} = 0 \quad \text{(uniaxial compressive unloading/ tensile loading)}$$

Taking the value of α as 350, β as -400 & γ as 450 for the values of uni-axial tensile loading. Following graph is obtained.

Figure 1: NON-SYMMETRIC GRAPH WITH LINEAR HARDENING CASE 1



In the above graph, we can see that the during the uni-axial tensile loading the model is pulled to the positive x quadrant to a certain point which is according to the graph upto 240. At this point the uni-axial compressive loading starts during this loading the model is tending to the negative x direction and reaches the value of 250. Now we again start applying tensile load that is we are remove the compressive load at this point the model starts to gaining load which is much get greater then that of the previous tensile load so the model achieves the damage.

Now when we take below the stress-strain graph we can clearly see that the during the tensile loading the model which is at point 1 that is (0,0) move in the positive quadrant upto (0.016,250) in terms of stress. strain. Then while the model has compressive loading in other terms we can say that tensile load is removed, the strain goes to the negative 3rd quadrant upto (-0.018,-250). Finally when the load is removed and the tensile load is again add to the model, it shows the damage accounted to the model at point 7 with maximum strain of 0.0205.

Here we should note the stress in the body carries the outer part along with the stress upto a point which implies that the stress is with in outer surface of the body but acting outwards. We should also note that the graph show here goes coinciding with the theoretical words. Thus we can understand that the coding is validated.

Now at this point we are taking the comparison of the internal variable (r) can not be changed as it is the internal properties of the material of model and hence the only variable we can control is the hardening variable (q). So we plot a graph for these two so that we can see the graph show the linear change in the plot, i.e the plot has straight line which shows the linearly increasing from point 2 to point 6 and finally to point 7.

LINEAR HARDENING CASE 2

non-Symmetric model for case 2,now we take the yield stress as 200 and the linear type of hardening . Taking

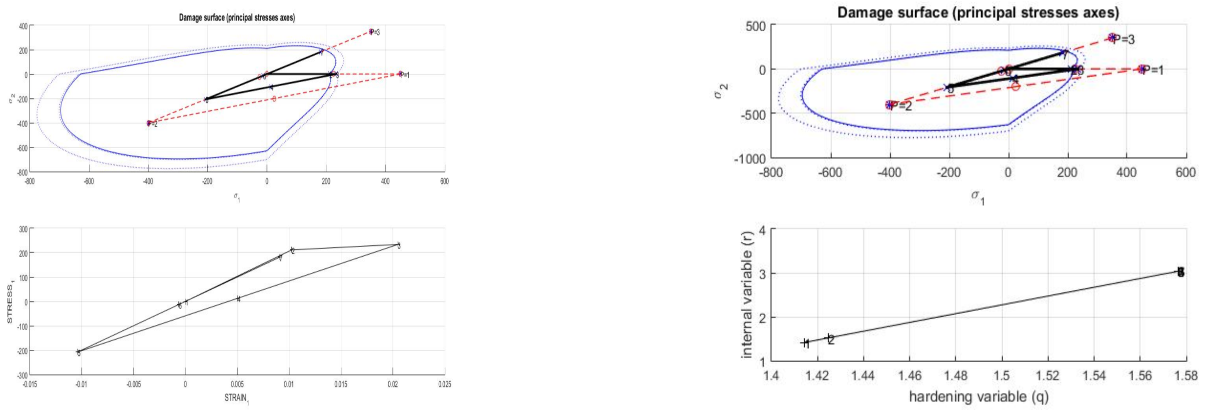
$$\Delta\bar{\sigma}_1^{(1)} = \alpha \quad ; \quad \Delta\bar{\sigma}_2^{(1)} = 0 \quad \text{(uniaxial tensile loading)}$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta \quad ; \quad \Delta\bar{\sigma}_2^{(2)} = -\beta \quad \text{(biaxial tensile unloading/compressive loading)}$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma \; ; \quad \Delta\bar{\sigma}_2^{(3)} = \gamma \quad \text{(biaxial compressive unloading/tensile loading)}$$

the value of α as 350 for the value of uni-axial tensile loading,and β as -400 & γ as 350, both β & γ are bi-axial tensile loading. Following graph is obtained.

Figure 2: NON-SYMMETRIC GRAPH WITH LINEAR HARDENING CASE 2



Here the graph of the stress-strain and the graph of the internal variable and the hardening variable are explained in the same as the above case but here we take the different values so the we can understand the hardening value much better.

LINEAR HARDENING CASE 3

Non-Symmetric model or case 3we take the yield stress as 200 and the linear type of hardening . Taking the

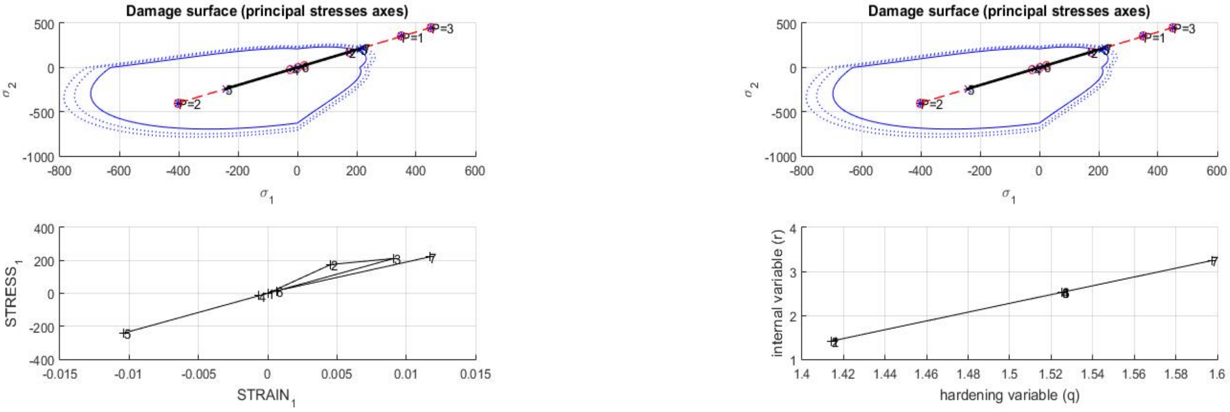
$$\Delta\bar{\sigma}_1^{(1)} = \alpha \;;\; \Delta\bar{\sigma}_2^{(1)} = \alpha \text{ (biaxial tensile loading)}$$

$$\Delta\bar{\sigma}_1^{(2)} = -\beta \;;\; \Delta\bar{\sigma}_2^{(2)} = -\beta \text{ (biaxial tensile unloading/compressive loading)}$$

$$\Delta\bar{\sigma}_1^{(3)} = \gamma \;;\; \Delta\bar{\sigma}_2^{(3)} = \gamma \text{ (biaxial compressive unloading/tensile loading)}$$

value of α as 350 β as -400 & γ as 450 for the values of bi-axial tensile loading.Following graph is obtained.

Figure 3: NON-SYMMETRIC GRAPH WITH LINEAR HARDENING CASE 3

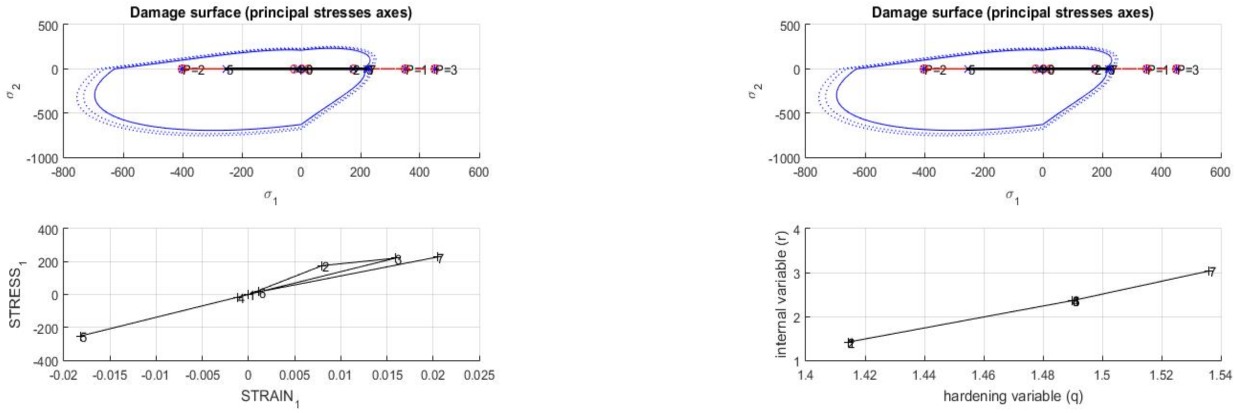


Here the graph of the stress-strain and the graph of the internal variable and the hardening variable are explained in the same as the above case but here we take the different values so the we can understand the hardening value much better. In all the above cases the graph so the agreement with the theoretical explanations.

NON-LINEAR HARDENING/EXPONENTIAL HARDENING CASES

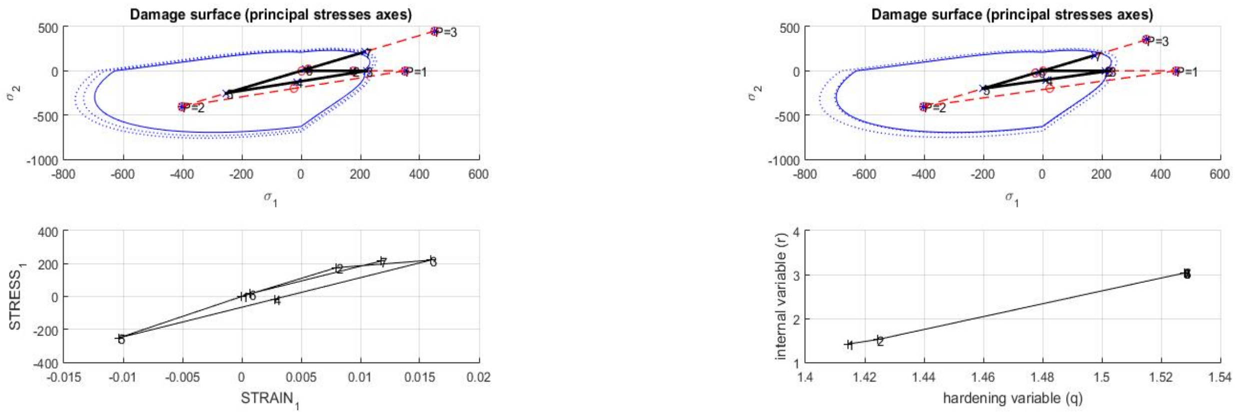
In this type of model non-Symmetric but we about to work with non-linear hardening. we take the yield stress as 200 and the non-linear type of hardening. Taking the value of α as 350, β as -400 & γ as 450 for the values of uni-axial tension loading. Following graph is obtained.

Figure 4: NON-SYMMETRIC GRAPH WITH EXPONENTIAL SOFTENING CASE 1



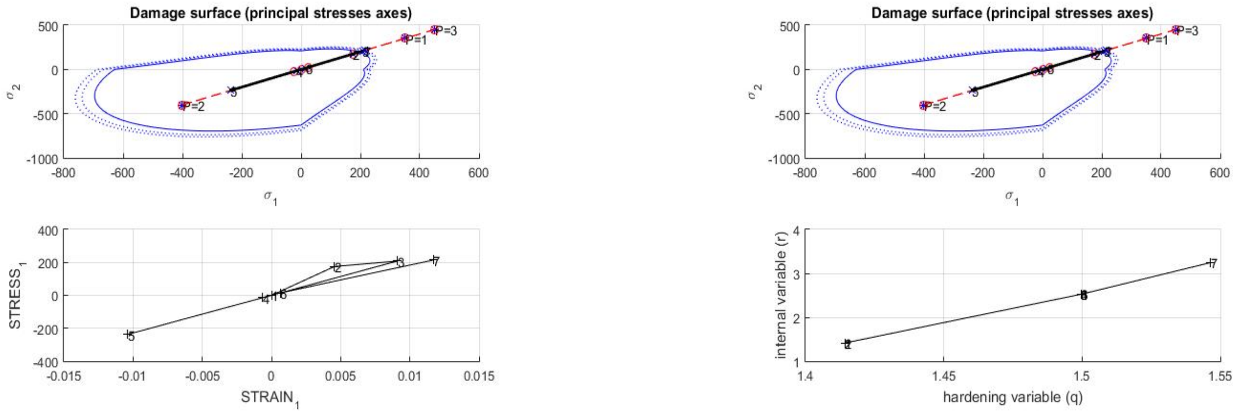
For the first case in this section an softening type is implemented and the rest with an regular hardening type is implemented.If we see the graph for the stress-strain the graph seems to have an agreement with the stress-strain graph of the linear hardening. The theoretical explanation of the damage modelling says that the two hardening two type work same way and thus looks the same way. but when we see the internal variable (r) and the hardening variable (q) have have an exponential hardening, i.e they tends to have a bend when the tensile load is removed till the load is reapplied. here since the scale is small we can not see much bend in the graph.

Figure 5: NON-SYMMETRIC GRAPH WITH EXPONENTIAL HARDENING CASE 2



The graphs of all the cases in this section is implemented in the same method of the above thus the graphs are self explanatory.

Figure 6: NON-SYMMETRIC GRAPH WITH EXPONENTIAL HARDENING CASE 3

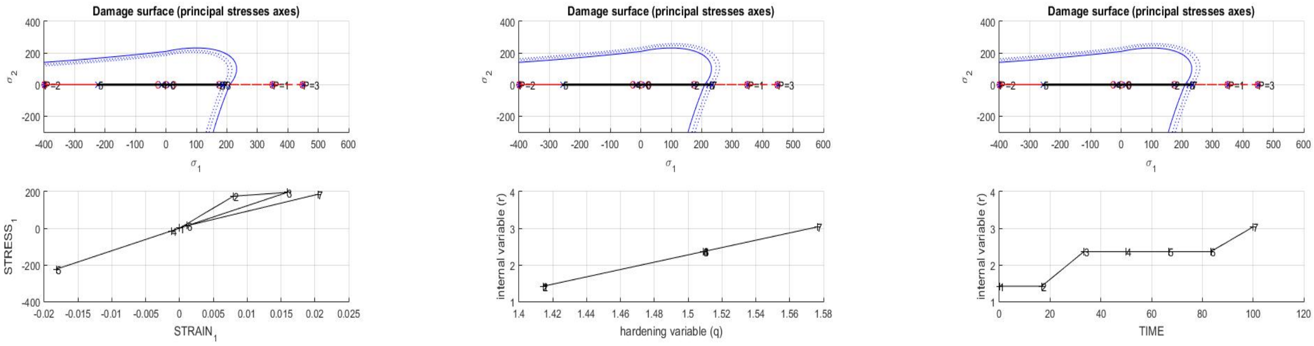


TENSION ONLY MODEL

LINEAR HARDENING CASE 1

In the this type of model tension only, we take the yield stress as 200 and the linear type of hardening. Taking the value of α as 350, β as -400 & γ as 450 for the values of uni-axial tensile loading. Following graph is obtained.

Figure 7: TENSION ONLY GRAPH WITH LINEAR HARDENING CASE 1



We can observe that the damage surface graph the softening is tending to infinity with the hardening restricted to the first quadrant, which explains us that the harden has a minimal damage effect to the model during the loading and unloading process with 240 in σ_1. while the strain is maximum of 0.02. The Q-R graph has the same effect as such of the non-symmetric model.

Figure 8: TENSION ONLY GRAPH WITH LINEAR HARDENING CASE 2

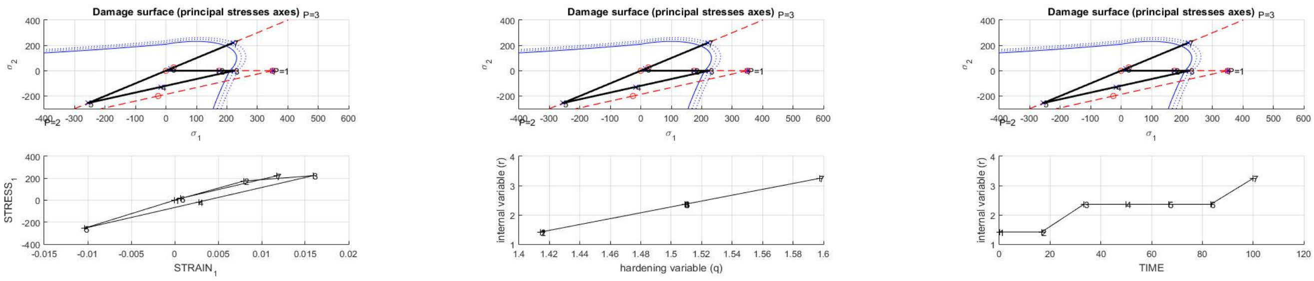
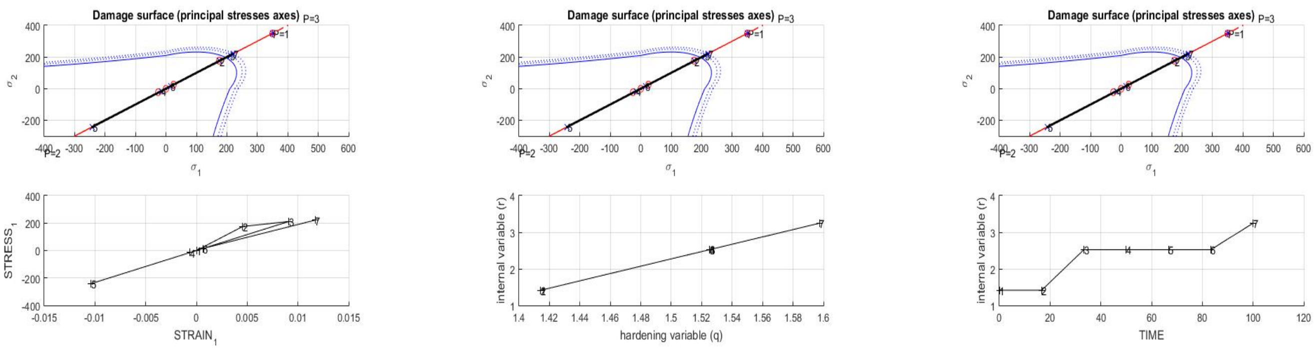


Figure 9: TENSION ONLY GRAPH WITH LINEAR HARDENING CASE 3



EXPONENTIAL HARDENING CASE 1

In the this type of model tension only, we take the yield stress as 200 and the non-linear type of hardening. Taking the value of α as 350, β as -400 & γ as 450 for the values of uni-axial tensile loading. Following graph is obtained.

Figure 10: TENSION ONLY GRAPH WITH EXPONENTIAL HARDENING CASE 1

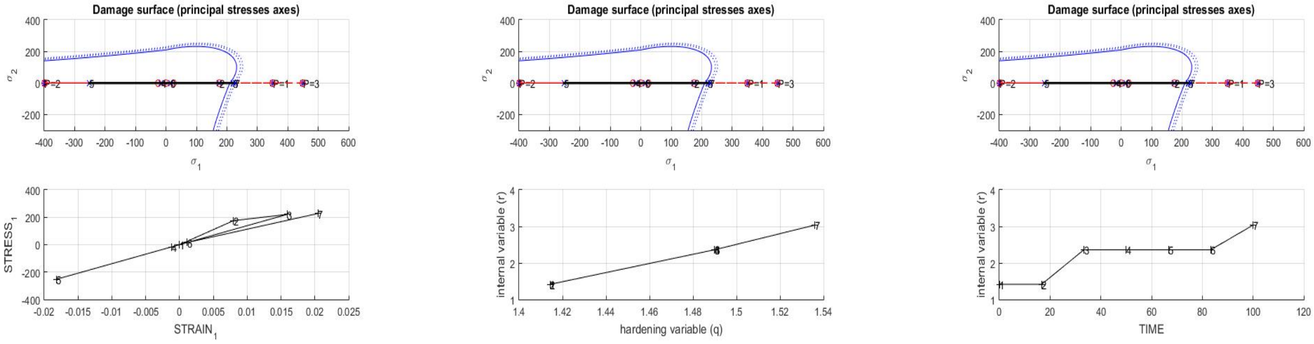


Figure 11: TENSION ONLY GRAPH WITH EXPONENTIAL HARDENING CASE 2

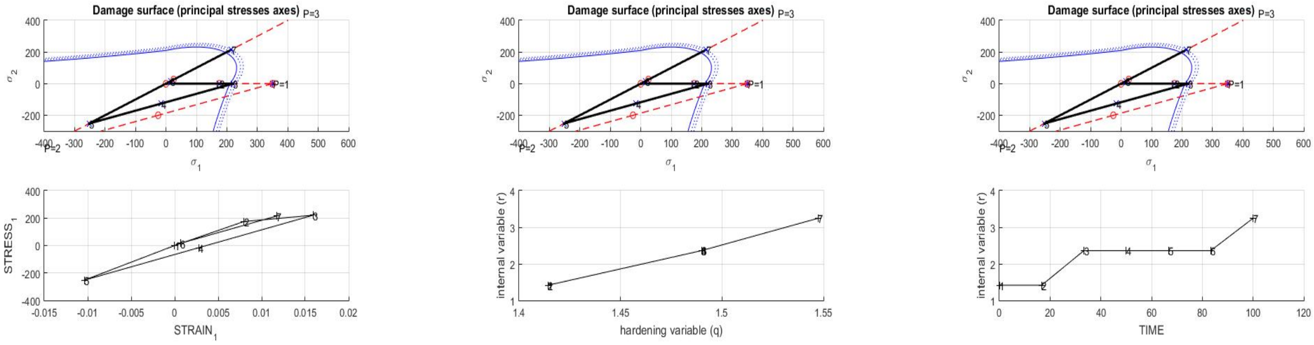
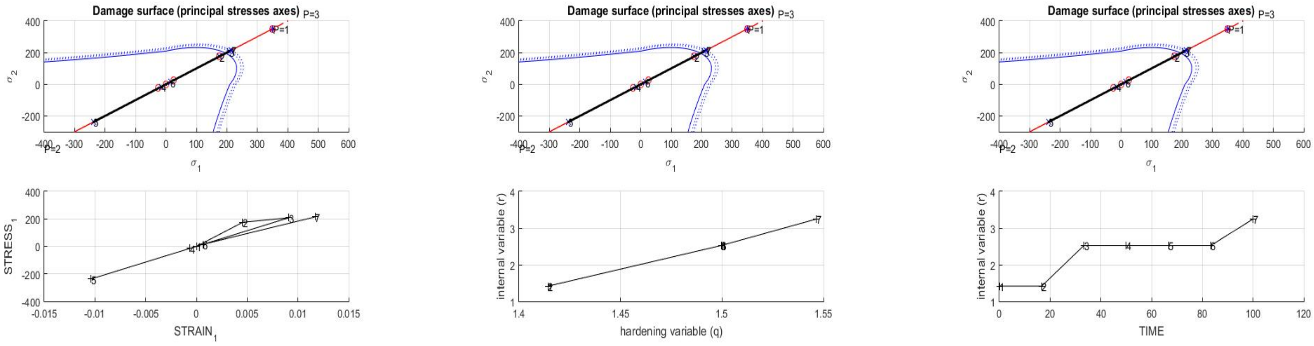


Figure 12: TENSION ONLY GRAPH WITH EXPONENTIAL HARDENING CASE 3



thus with all above we can conclude that the code works fine and the case of the problem have been worked out well and the correctness of the program is validates well.

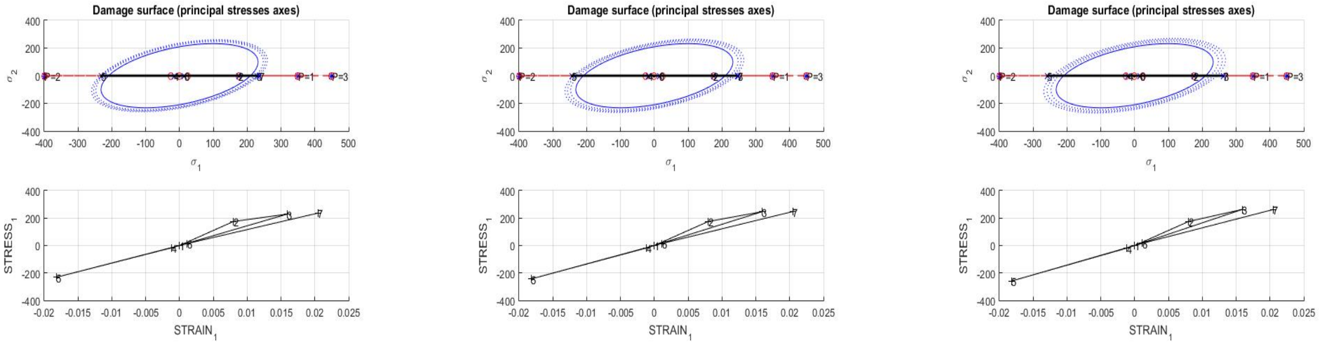
PART II (RATE DEPENDENT MODEL):

In this section we are going to work with rate dependent model the method we use method visco-damage in symmetric tension-compression model. So to being with we add the viscosity to model.

Different viscosity parameters η:

Now we have changed the value of the viscosity with changing the values of ηas 1,5 & 10 and giving the values of yield stress as 200 and the linear type of hardening. Taking the value of σ_1 as 350, σ_2 as -400 & σ_3 as 450 for the values of uni-axial tensile loading. Taking α as 1.We obtain the following stress-strain graphs:

Figure 13: Visco-Damage in Symmetric Tension-Compression Model with viscosity parameters η as 1,5 & 10

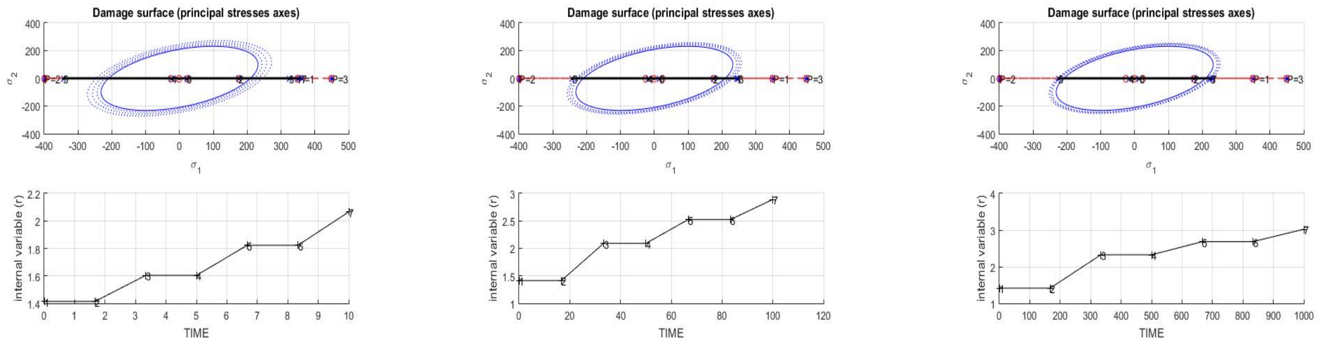


We when see the graph as comparison for different values of the η the damage in model move outside. So we can say in other words the stress goes outside and thus damage is less. As the viscosity increase the damage reduces and stress moves outside the model.

Different strain rate ϵ values

Now we have add the value of the viscosity η with the values as 5. Giving the values of yield stress as 200 and the linear type of hardening. Taking the value of σ_1 as 350, σ_2 as -400 & σ_3 as 450 for the values of uni-axial tensile loading. Taking α as 0.75.We obtain the following Time-Internal variable (r) graphs:

Figure 14: Visco-Damage in Symmetric Tension-Compression Model with viscosity parameters Time with 10,100 & 1000



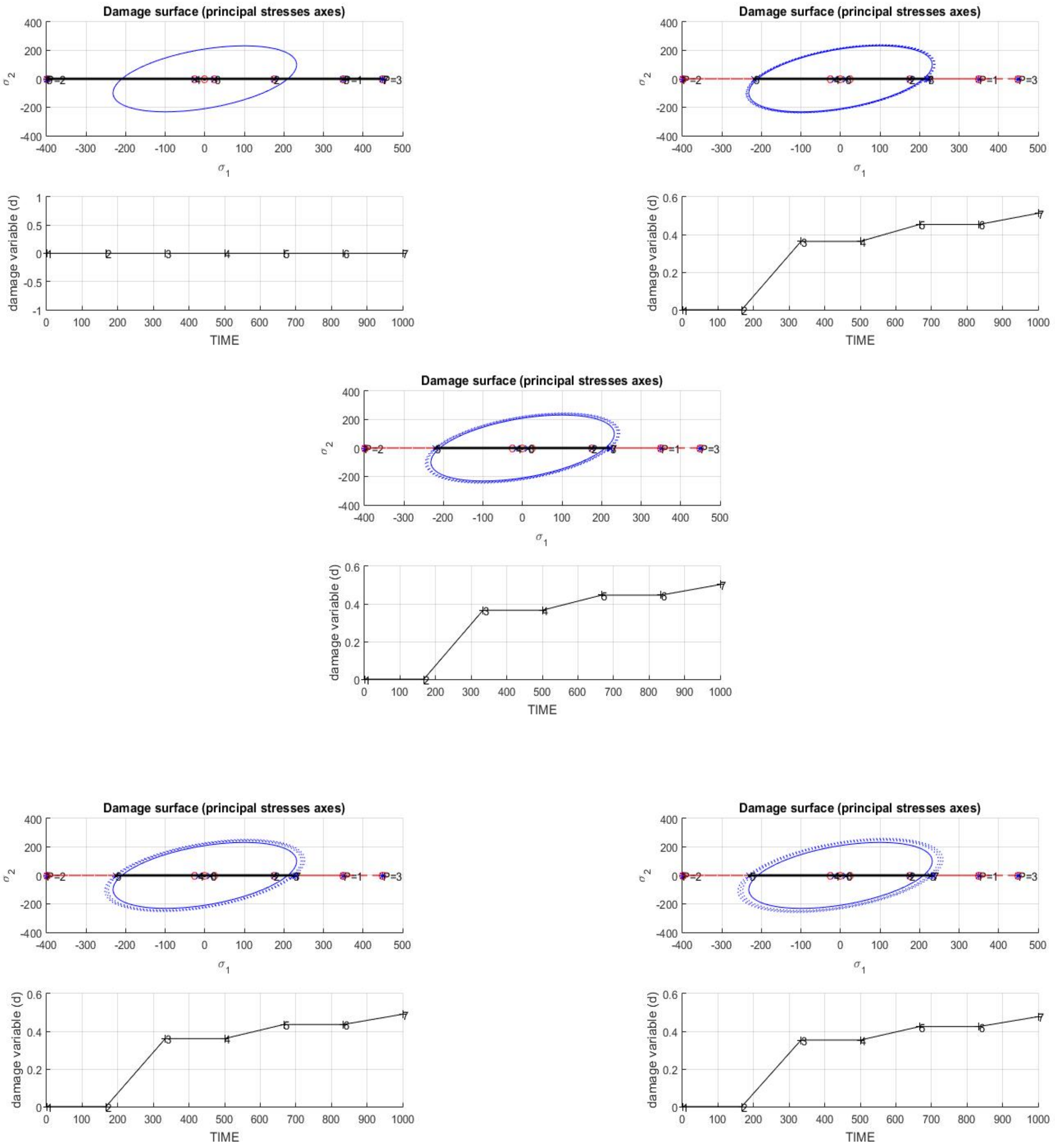
From the above graphs comparison,we can see graph look similar, but damage went up from 2 to 3 even though loading are same.this is because of the slower strain time outside damage surface is more.

Different α values:

Now we have add the value of the viscosity η with the values as 5. Giving the values of yield stress as

200 and the linear type of hardening. Taking the value of σ_1 as 350, σ_2 as -400 & σ_3 as 450 for the values of uni-axial tensile loading. Taking given α value as 0,0.5,0.75 & 1.We obtain the following Time-Damage variable (d) graphs: From the results given above, we can say that when alpha ¿ 0.5 gives almost same result as they

Figure 15: Visco-Damage in Symmetric Tension-Compression Model with α values as 0, 1/4 , 1/2 , 3/4 & 1



satisfy convergence criteria as mentioned in the theoretical part.But other alphas which are less then 0,seriously underestimate damage parameter.This is due to instability of scheme below 0.5

ANNEX

rmap_dano1.m

```
if(rtrial > r_n)          % LINEAR

     fload=1;
     delta_r=rtrial-r_n;
     r_n1= rtrial   ;
     if hard_type == 0
         %  Linear
         q_n1= q_n+ H*delta_r;
     else

%   EXPONENTIAL

       q_infi=r0*1.3;
          A = (H*r0)/(q_infi-r0);
          H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
          q_n1= q_n + H_new*delta_r;
     end
```

```
%FOR VISCOUS ('viscrp == 1')
    rtrial=(1-alpha)*r_n+alpha*rtrial;

    if(rtrial > r_n)
    %*    Loading

    fload=1;
    delta_r=rtrial-r_n;
    r_n1= (((eta-delta_t*(1-alpha))/(eta+alpha*delta_t))*r_n) + ....
        ((delta_t/(eta+alpha*delta_t))*rtrial);
    if hard_type == 0
        %  Linear
        q_n1= q_n+ H*delta_r;
    else

% *******************************************************
%         Hardening/Softening exponential law
          q_infi=r0*1.3;
          A = (H*r0)/(q_infi-r0);
          H_new= (A*(q_infi-r0)*exp(A*(1-rtrial/r0)))/r0;
          q_n1= q_n + H_new*delta_r;
    end

    if(q_n1<zero_q)
        q_n1=zero_q;
    end
```

Modelos_de_dano1.m

```
elseif (MDtype==2)  %* Only tension
stress=ce*eps_n1';
stress(stress<0)=0;
rtrial=sqrt(eps_n1*stress);

elseif (MDtype==3)  %*Non-symmetric
    stress=ce*eps_n1';
    stress_plus=stress;
    stress_plus(stress_plus<0)=0;
    num = sum((stress_plus));
    den = sum((abs(stress)));
    theta = num/den;
    rtrial = (theta + (1-theta)/n)* sqrt(eps_n1 * ce*eps_n1');

end
```

dibujar_criterio_dano1.m

```
elseif MDtype==2
    tetha=[0:0.01:2*pi];
%*********************************************************************
**********
    %* RADIUS
    D=size(tetha);                          %*   Range
    m1=cos(tetha);                          %*
    n1=m1;
    n1(n1<0)=0;
    m2=sin(tetha);                          %*
    n2=m2;
    n2(n2<0)=0;
    Contador=D(1,2);                        %*

    radio = zeros(1,Contador) ;
    s1    = zeros(1,Contador) ;
    s2    = zeros(1,Contador) ;

    for i=1:Contador
        radio(i)= q/sqrt([n1(i) n2(i) 0 nu*(n1(i)+n2(i))]*ce_inv*[m1(i) m2(i)
0 ...
            nu*(m1(i)+m2(i))]');

        s1(i)=radio(i)*m1(i);
        s2(i)=radio(i)*m2(i);

    end
    hplot =plot(s1,s2,tipo_linea);
     axis([-400 600 -300 400])




elseif MDtype==3
 tetha=[0:0.01:2*pi];
 %* RADIUS
     D=size(tetha);                          %*   Range
     m1=cos(tetha);
     m2=sin(tetha);
     Contador=D(1,2);
     radio = zeros(1,Contador) ;
     s1    = zeros(1,Contador) ;
     s2    = zeros(1,Contador) ;

    for i = 1:Contador
        den = abs(m1(i))+abs(m2(i));
        n1 = m1(i);
        n2 = m2(i);

        if n1<0
             n1 = 0;
        end
        if n2<0
             n2 = 0;
        end

        num = n1+n2;

        radio(i)= q/(((num/den)+(1-(num/den))/n)*sqrt([m1(i) m2(i) 0
nu*(m1(i)+m2(i))]*ce_inv*[m1(i) m2(i) 0 ...
        nu*(m1(i)+m2(i))]'));

         s1(i)=radio(i)*m1(i);
         s2(i)=radio(i)*m2(i);
    end
```