Assignment 3

Q. I)

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = G = \frac{E}{2(1+\nu)}$$

The inverse relation E & ν in terms λ & μ

$$\mu = \frac{E}{2(1+\nu)}$$

$$\boxed{E = 2\mu(1+\nu)}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$= \frac{2\mu\cancel{(1+\nu)}\,\nu}{\cancel{(1+\nu)}(1-2\nu)}$$

$$\lambda(1-2\nu) = 2\mu\nu$$

$$\boxed{\nu = \frac{\lambda}{2(\mu+\lambda)}}$$

substituting

$$E = 2\mu\left(1 + \frac{\lambda}{2(\mu+\lambda)}\right)$$

$$\boxed{E = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}$$

(b) Elastic matrix for plane stress & plane strain

For plane stress

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix}$$

Substituting the values of E & ν

$$= \frac{\dfrac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{1-\left(\dfrac{\lambda}{2(\mu+\lambda)}\right)^2}\begin{bmatrix}1 & \frac{\lambda}{2(\mu+\lambda)} & 0\\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0\\ 0 & 0 & \frac{1-\lambda}{2(\mu+\lambda)}\end{bmatrix}[e.]$$

$$= \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}\begin{bmatrix}1 & \frac{\lambda}{2(\mu+\lambda)} & 0\\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0\\ 0 & 0 & \frac{\lambda+2\mu}{4(\mu+\lambda)}\end{bmatrix}[e]$$

ii) <u>Plain strain</u>

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}(1-\nu) & \nu & 0\\ \nu & (1-\nu) & 0\\ 0 & 0 & \frac{1-2\mu}{4(\lambda-\mu)}\end{bmatrix}[e]$$

substituting the values of E & ν.

$$= \frac{\frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}}{\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)\left(1-\cancel{2}\frac{\lambda}{\cancel{2}(\mu+\lambda)}\right)}\begin{bmatrix}1 & \frac{\frac{\lambda}{2(\mu+\lambda)}}{1-\frac{\lambda}{2(\mu+\lambda)}} & 0\\ \frac{\frac{\lambda}{2(\mu+\lambda)}}{1-\frac{\lambda}{2(\mu+\lambda)}} & 1 & 0\\ 0 & 0 & \frac{1-2\frac{\lambda}{2(\mu+\lambda)}}{2\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}(\lambda+2\mu) & \lambda & 0\\ \lambda & (\lambda+2\mu)' & 0\\ 0 & 0 & \mu\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix}$$

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3)

$$E = E_\lambda + E_\mu.$$

$$\begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

$$= \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad E_\lambda = \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

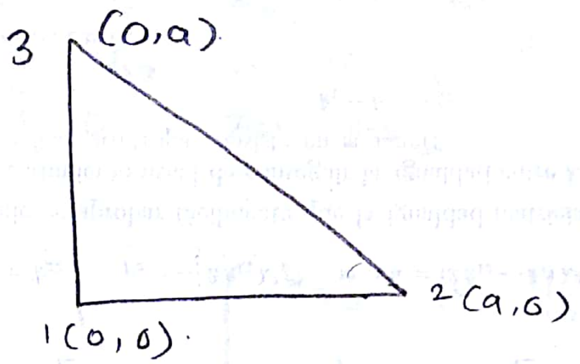
$$= \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_\mu = \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.2

Plane triangular domain of thickness h with horizontal & vertical length $a = 1$ & thickness $h = 1$.



$x_1 = 0 \qquad y_1 = 0$

$x_2 = a \qquad y_2 = 0$

$x_3 = 0 \qquad y_3 = a$

Area $\quad A = \det \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$

$$= \det \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$A = \frac{a}{2} = \frac{1}{2}$$

Plain - stress

$$E_{stress} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

initially $\nu = 0$

$$E_{stress} = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

Stiffness matrix

$$k^e_{tri} = \frac{h}{4A} \begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & y_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix}$$

$$\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$= \frac{E}{2} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$k^e_{tri} = \frac{E}{2} \begin{bmatrix} 1.5 & 0.5 & -1 & -0.5 & -0.5 & 0 \\ 0.5 & 1.5 & 0 & -0.5 & -0.5 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -0.5 & -0.5 & 0 & 0.5 & 0.5 & 0 \\ -0.5 & -0.5 & 0 & 0.5 & 0.5 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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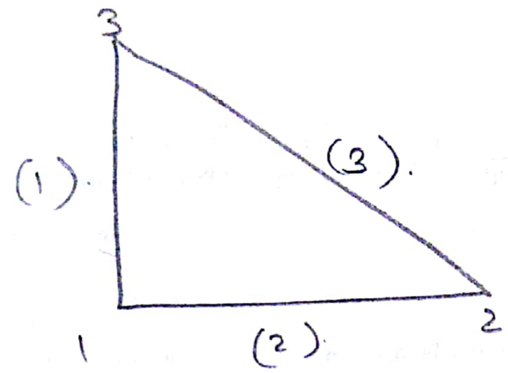
For bar Element.



$$L_1 = L_2 = a = 1$$

$$L_3 = \sqrt{2}\,a = \sqrt{2}.$$

$$k^e = \frac{(EA)^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & c^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -c^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\phi \qquad c = \cos\phi$

<u>Element 1</u> $\Rightarrow$.

$s = 1 \quad c = 0.$

$$k^{(1)} = \frac{EA}{1} \begin{array}{c} \begin{array}{cccc} 1 & 1 & 3 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 1 \\ 3 \\ 3 \end{array}$$

Element (3).

$s = \frac{\sqrt{2}}{2} \qquad c = \frac{-\sqrt{2}}{2}$

$$k^{(3)} = \frac{EA}{\sqrt{2}} \begin{array}{c} \begin{array}{cccc} 2 & 2 & 3 & 3 \end{array} \\ \begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix} \end{array} \begin{array}{c} \\ 2 \\ 2 \\ 3 \\ 3 \end{array}$$

For Element 2

$$S=0 \quad C=1$$

$$k^{(2)} = \frac{EA}{1} \begin{array}{c} \begin{array}{cccc} 1 & 1 & 2 & 2 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 1 \\ 2 \\ 2 \end{array}$$

Global stiffness Matix

$$k = E \begin{bmatrix} A_2 & 0 & -A_2 & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 & 0 & -A_1 \\ -A_2 & 0 & \left(A_2+\frac{A_3}{2\sqrt{2}}\right) & \frac{-A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} \\ 0 & 0 & \frac{-A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} \\ 0 & 0 & \frac{-A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} & \frac{A_3}{2\sqrt{2}} \\ 0 & -A_1 & \frac{A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} & \frac{-A_3}{2\sqrt{2}} & A_1+\frac{A_3}{2\sqrt{2}} \end{bmatrix}$$

- kbar & k_{tri} are not same totally different.
- If we take $A_1 = A_2 = 1/2$ & $A_3 = \sqrt{2}$ they will be equivalent but not exact.

3) In case of bar element, axial displacement is considered on the other hand in case of K_{tri}, stresses & Strains within the triangular domain are calculated.

4) ν calculates lateral strain to the longitudinal strain. If we do not consider that K_{tri} & k_{bar} will be totaly different.

Which may conclude that the efficiency of the K_{tri} will reduce than K_{bar} which is actually more efficient than K_{bar}.