Computational Structural Mechanics and Dynamics

Analysis of Rev. shells

Aditya Mangaonkar

a) Describe in extension, how can be applied a non symmetric load on this formulation.

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For n-noded strip with n nodes we have.

$$u' = \sum_{l=0}^{m} \sum_{i=1}^{n} N_i \left(\bar{S}^{l} \bar{a}_i^{l} + \bar{\bar{S}}^{l} \bar{\bar{a}}_i^{l} \right)$$

u' is displacement & ($\bar{\ }$) & ($\bar{\bar{\ }}$) denotes Symmetric & antyi symetric components of displacement

The loads. expanded in Fouries series using Same harmonic function as for the displacement.

i.e.

$$t = \sum_{l=0}^{m} \left(\bar{S}^{l} \bar{t}^{l} + \bar{\bar{S}}^{l} \bar{\bar{t}}^{l} \right)$$

The analysis can be simplified by computing independently the symetric & antisyme. solu^n.

The finite strip formulation for the symmetric & anti-symmetric cases can be treated in a uniform manner. The local generalized strain matrix is idenfical for both cases.

The troncoconical strip simply ~~chan~~ interchanging

m by $-l$ for symmetric case

m by l for anti-symmetric case.

b) Using thin beam formulation, describ the shape of $B^{(e)}$ matrix & comment on the integration rule

→ In kirchhoff & Reissner-Mindlin difference is of assumption made for the rotation of normal.

In mathematical form we can write,

$$\theta = \left.\frac{\partial w'}{\partial s}\right|_{z'=0}$$

$$\theta = \frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}$$

$$\gamma_{x'z'} = \frac{1}{C_s}\left(\frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s} - \left(\frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}\right)\right) = 0$$

local displacement vector is defined as,

$$u' = \left[u'_0, w'_0, \frac{\partial w'_0}{\partial s}\right]^T$$

Expressions for axial & circumferential strains are deduced

$$\varepsilon_{x'} = \frac{1}{C_s}\left[\frac{\partial u'_0}{\partial s} - \frac{w'_0}{R_s} - z'\left(\frac{\partial^2 w'_0}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{u'_0}{R_s}\right)\right)\right]$$

$$\varepsilon_{y'} = \frac{1}{C_\alpha}\left[\frac{u'_0\cos\phi - w'_0\sin\phi}{r} - \frac{z'\cos\phi}{r}\left(\frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}\right)\right]$$

$$\varepsilon' = S_2\begin{Bmatrix}\hat{\varepsilon}'_m\\ \hat{\varepsilon}'_b\end{Bmatrix} \; ; \; S_2 = [S, -z'S]$$

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Membrane & bending generalized strains,

$$\varepsilon'_m = \begin{Bmatrix} \frac{\partial u'_0}{\partial s} - \frac{w'_0}{R_s} \\ \frac{u'_0 \cos\phi - w'_0 \sin\phi}{r} \end{Bmatrix} ; \quad \hat{\varepsilon}'_b = \begin{Bmatrix} \frac{\partial^2 w'_0}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{u'_0}{R_s}\right) \\ \frac{\cos\phi}{r}\left\{\frac{\partial w'_0}{\partial s} + \frac{u'_0}{R_s}\right\} \end{Bmatrix}$$

Troncoconical shell elements based on kirchoff theory

$$\varepsilon'_m = \begin{Bmatrix} \frac{\partial u'_0}{\partial s} \\ \frac{u'_0 \cos\phi - w'_0 \sin\phi}{r} \end{Bmatrix} \qquad \varepsilon'_b = \begin{Bmatrix} \frac{\partial^2 w'_0}{\partial s^2} \\ \frac{\cos\phi}{r}\frac{\partial w'_0}{\partial s} \end{Bmatrix}$$

Two noded kirchhoff troncoconical element.

$$u'_0 = \sum_{i=1}^{2} N_i^u u'_{0i} \quad \text{with } N_i^u = \frac{1+\xi\xi_i}{2}$$

$$a_i^{(e)} = \begin{Bmatrix} u'_{0i} \\ w'_{0i} \\ \left(\frac{\partial w'_0}{\partial s}\right) \end{Bmatrix} \qquad B' = \begin{Bmatrix} B'_m \\ \vdots \\ B'_f \end{Bmatrix} =$$

$$\begin{bmatrix} -1/l_m^{(e)} & 0 & 0 & 1/l^{(e)} & 0 & 0 \\ \frac{1-\xi'}{2r}c^{(e)} & (-N_1^w)\frac{s^{(e)}}{4r} & (-\bar{N}_1^w)\frac{s^{(e)}}{4r} & \frac{1+\xi'}{2r}c^{(e)} & (-N_2^w)\frac{s^{(e)}}{4r} & (-\bar{N}_2^w)\frac{s^{(e)}}{4r} \\ 0 & \frac{6\xi}{(l^{(e)})^2} & \frac{2(-1+3\xi)}{(l^{(e)})^2} & 0 & \frac{-6\xi}{(l^{(e)})^2} & \frac{-2(1+3\xi)}{(l^{(e)})^2} \\ 0 & (\xi^2-1)\frac{3c^{(e)}}{2rl^{(e)}} & H_1\frac{c^{(e)}}{2rl^{(e)}} & 0 & (1-\xi^2)\frac{3c^{(e)}}{2rl^{(e)}} & H_2\frac{c^{(e)}}{2rl^{(e)}} \end{bmatrix}$$

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$$C^{(e)} = \cos\phi^{(e)} \qquad S^{(e)} = \sin\phi^{(e)}.$$

$$N_i^{W} = \frac{1}{4}\left(2 + 3\xi\xi_i - \xi^3\xi_i\right)$$

$$\bar{N}_i^{W} = \frac{1}{4}\left(\xi^3 + \xi^2\xi_i - \xi - \xi_i\right)$$

$$H_i = 3\xi^2 + 2\xi\xi_i - 1).$$

$$w_0^{\prime} = \sum_{i=1}^{2}\left[N_i^{W} w_{0_i}^{\prime} + \bar{N}_i^{W}\left(\frac{\partial w_0^{\prime}}{\partial s}\right)_i\right]$$

$$\varepsilon^{1} = \left[B_1^{\prime}, B_2^{\prime}\right] a^{\prime(e)} = B^{\prime} a^{\prime(e)}$$

$$B_i^{1} = \begin{Bmatrix} B_{m_i}^{1} \\ B_{b_i}^{1} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i^{u}}{\partial s} & 0 & 0 \\ \frac{N_i^{u}\cos\phi}{r} & -\frac{N_i^{W}\sin\phi}{r} & -\frac{\bar{N}_i^{w}\sin\phi}{r} \\ 0 & \frac{\partial^2 N_i^{W}}{\partial s^2} & \frac{\partial^2 \bar{N}_i^{w}}{\partial s^2} \\ 0 & \frac{\cos\phi}{r}\frac{\partial N_i^{W}}{\partial s} & \frac{\cos\phi}{r}\frac{\partial \bar{N}_i^{W}}{\partial s} \end{bmatrix}$$