Assignment 2.1

On "FEM Modelling: Introduction":

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
(b) the same disk under two diametrically opposite force pairs
(c) a clamped semiannulus under a force pair oriented as shown
(d) a stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

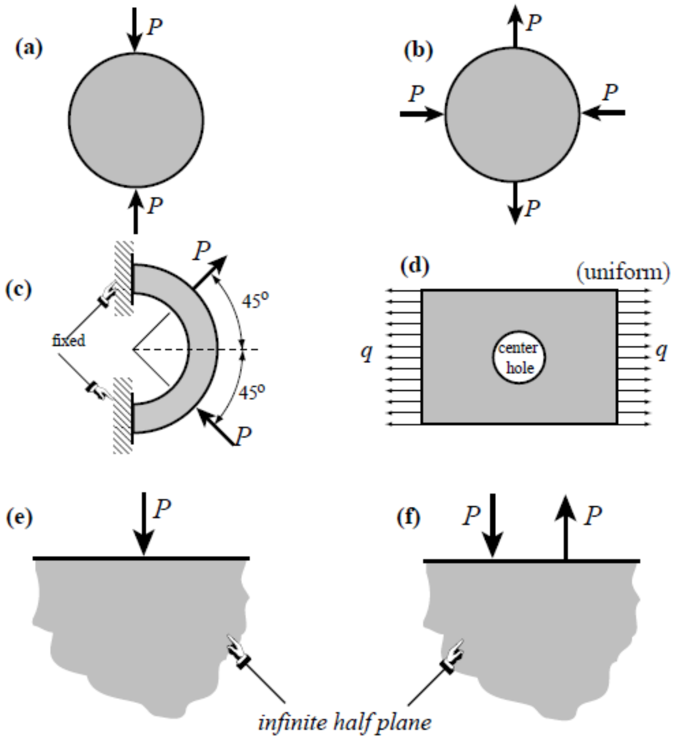

Figure 2.1.- Problems for assignment 2.1

Assignment 2.2

On “FEM Modelling: Introduction”:

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports at *I* and *N* extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

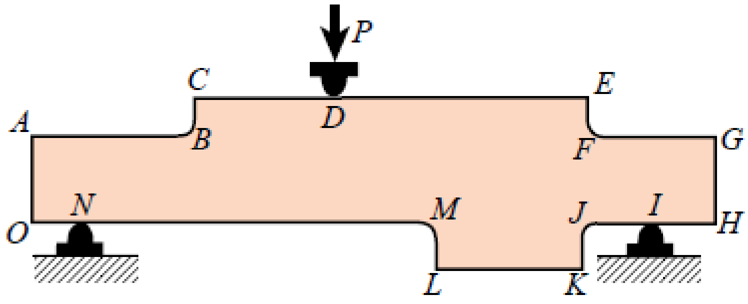

Figure 2.2.- Inplane bent plate

Assignment 2.3

On “Variational Formulation”:

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x)=\rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x=x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

Date of Assignment: **12 / 02 / 2018**
Date of Submission: **19 / 02 / 2018**

The assignment must be submitted as a pdf file named **As2-Surname.pdf** to the CIMNE virtual center.

CSMD: Assignment 2

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1 Assignment 2.1

1.1 Identification of symmetry and antisymmetry lines

The following lines can be identified in the two-dimensional problems

a) Two symmetry lines (horizontal and vertical)

b) Two symmetry lines (horizontal and vertical), and two antisymmetry lines (bisection lines)

c) One antisymmetry line (horizontal)

d) Two symmetry lines (horizontal and vertical)

e) One symmetry line (vertical)

f) One antisymmetry line (vertical)

Figure 1 shows these problems and the respective lines drawn.

1.2 Geometry reduction and FE meshes

a) can be reduced to one quarter of the geometry because it has two perpendicular symmetry lines

b) can be reduced to one eighth because it has two symmetry lines and two antisymmetry lines, that divides the problem into eight parts with symmetric geometry and antisymmetric loads

c) can be reduced to one quarter of the geometry because it has two perpendicular symmetry lines

d) can be divided into two halves (upper and lower) because of its horizontal antisymmetry line

e) can be divided in two, as it has one vertical symmetric line passing through the load

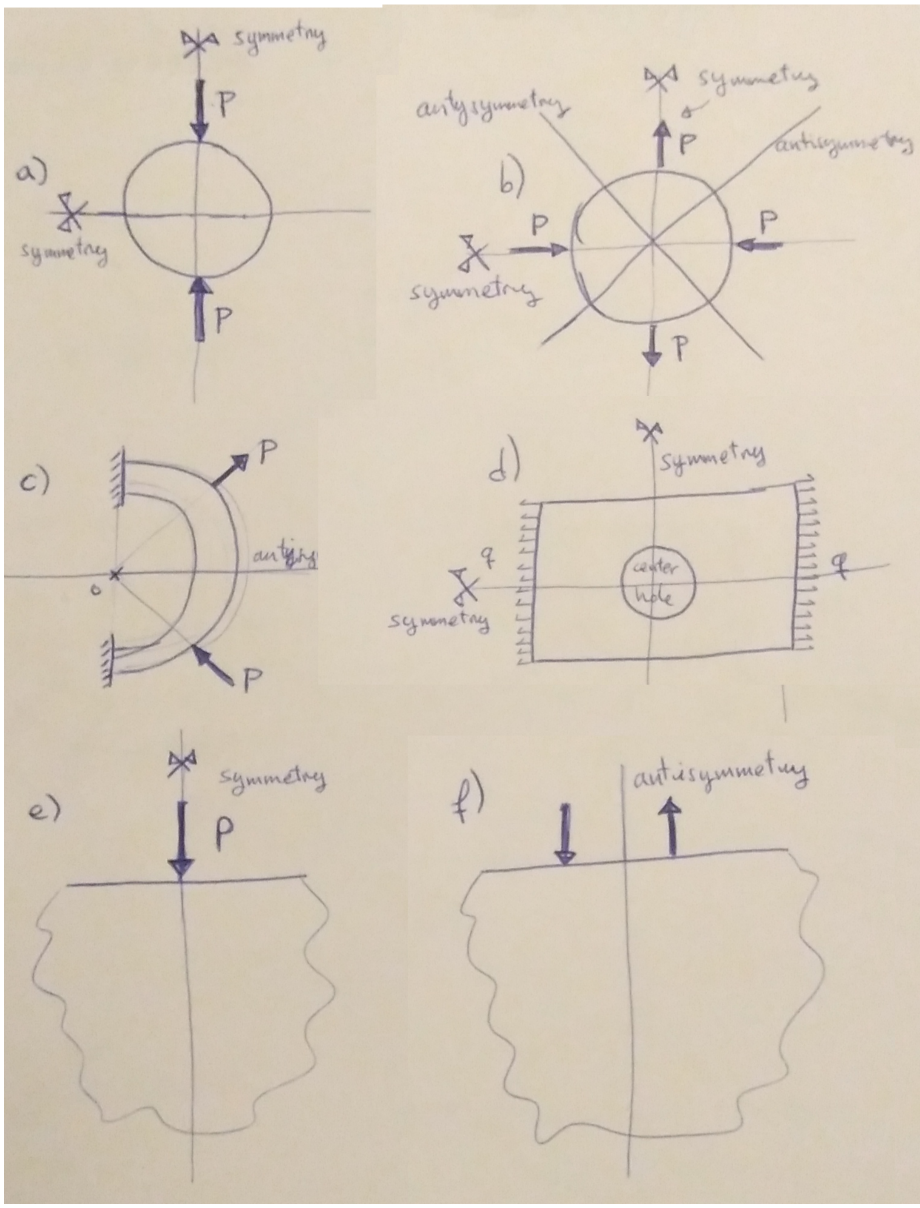

Figure 1: Symmetry and antisymmetry lines

f) can be divided in two, as it has one vertical antisymmetric line passing through the line bisection of the antisymmetric loads

Figure 2 shows a coarse FE mesh on the reduced geometries of the problems. The BC applied are shown as fixed joints, horizontal rollers and vertical rollers.

Support	**Code in figure** 2	u_x	u_y
Fixed	F	0	0
Horizontal roller	HR	-	0
Vertical roller	VR	0	-
Inclined roller	IR	u	u

2 Assignment 2.2

The trouble spots that require finer meshing can be found in the following table:

Spot ID	**Refinement due to**
B, M	Entrant corners
D	Vicinity of concentrated load and sharp contact area
N, H	Vicinity of sharp contact areas
F, J	Abrupt thickness change and entrant corner

3 Assignment 2.3

Recalling the element stiffness equations, we can define the 1-D problem as $\mathbf{K}\ \mathbf{u} = \mathbf{f}$, where the elemental external force vector $\mathbf{f}$ is

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \int_{-1}^{1} q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi \tag{1}$$

In this case, the variable ξ is the parametric expression of x, so that $\xi = \frac{x-x_i}{l}$. And then, we can introduce the expression of the force $q(x) = \rho A \omega^2 x$ and the area $A(x) = A_i(1-\frac{x}{l}) + A_j\frac{x}{l}$ into $\mathbf{f}$

$$\begin{aligned} \mathbf{f} &= \int_0^l \rho\omega^2 x (A_i(1-\frac{x}{l}) + A_j\frac{x}{l}) \begin{bmatrix} 1-\frac{x}{l} \\ \frac{x}{l} \end{bmatrix} dx = \\ &= \int_0^1 \rho\omega^2 \xi (A_i(1-\xi) + A_j\xi) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l^2 d\xi \end{aligned} \tag{2}$$

We can see that the nodal forces are

$$\begin{aligned} f_1 &= \rho\omega^2 \int_0^1 \xi(A_i(1-\xi) + A_j\xi)(1-\xi) d\xi \\ f_2 &= \rho\omega^2 \int_0^1 \xi^2(A_i(1-\xi) + A_j\xi) d\xi \end{aligned} \tag{3}$$

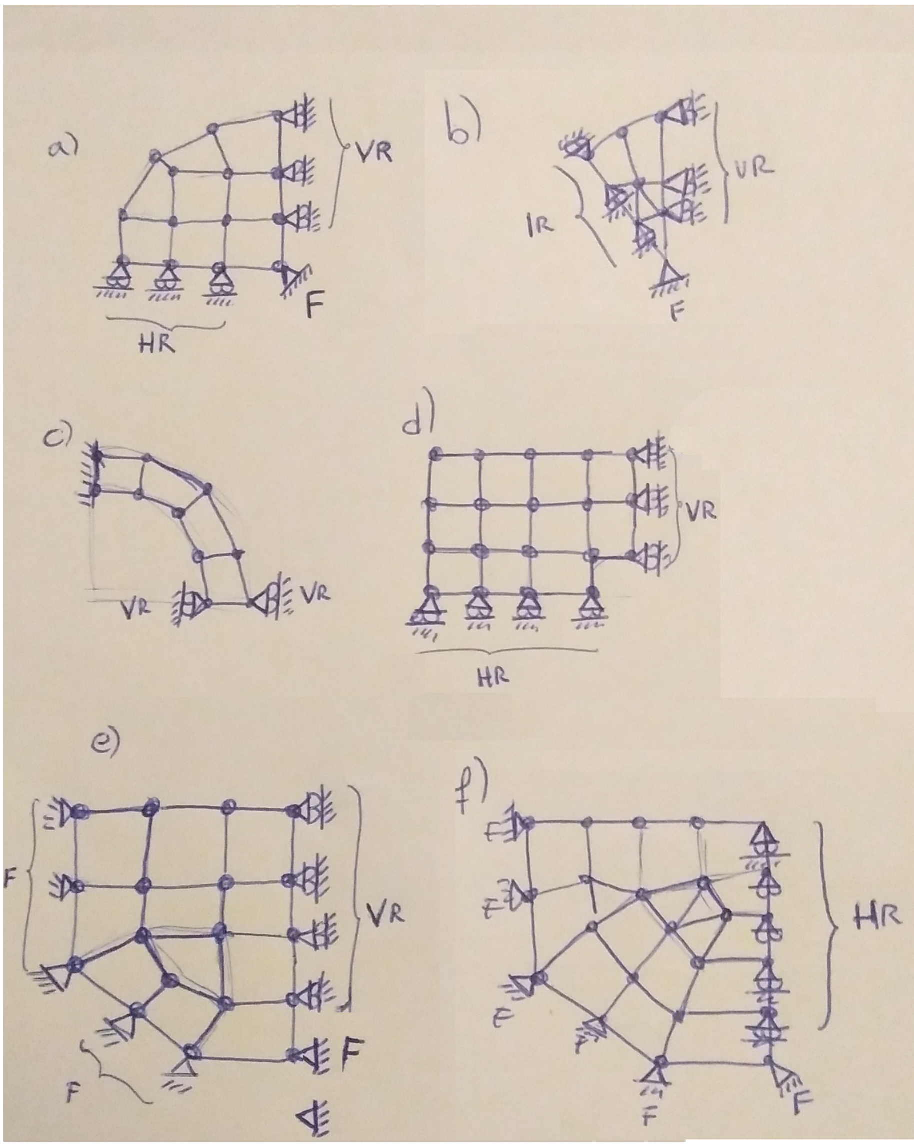

Figure 2: Example of coarse mesh taking into account only the geometry, not the loads

This is the integral of a polynomial, so:

$$f_1 = \rho\omega^2 l^2 \int_0^1 A_i(\xi) + (A_j - 2A_i)\xi^2 + (A_i - A_j)\xi^3 d\xi =$$
$$= \rho\omega^2 l^2 [A_i(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}) + A_j(\frac{1}{3} - \frac{1}{4})] = \frac{1}{12}\rho\omega^2 l^2 (A_i + A_j)$$
$$f_2 = \rho\omega^2 l^2 \int_0^1 A_i\xi^2 - A_i\xi^3 + A_j\xi^3 d\xi$$
$$= \rho\omega^2 l^2 [A_i(\frac{1}{3} - \frac{1}{4}) + A_j\frac{1}{4}] = \rho\omega^2 l^2 \frac{1}{12}(A_i + 3A_j)$$

For a prismatic bar, where $A_i = A_j = A$, nodal forces are

$$f_1 = \rho\omega^2 l^2 \frac{1}{6} A$$
$$f_2 = \rho\omega^2 l^2 \frac{1}{3} A$$

And we can see that this way we recover the external force:

$$f_1 + f_2 = \frac{1}{2}\rho\omega^2 l^2 A = \int_0^1 \rho\omega^2 \xi l A l d\xi$$