Assignment 3

→ Assignment 3.1 (The plane stress problem)

Q I) In isotropic elastic materials..

$$\lambda = \frac{E\vartheta}{(1+\vartheta)(1-2\vartheta)} \quad \& \quad \mu = g = \frac{E}{2(1+\vartheta)}$$

ⓐ The inverse relations for E & ϑ in terms of λ & μ.

→ $$\lambda = \frac{E\vartheta}{(1+\vartheta)(1-2\vartheta)} \quad \text{---(i)} \quad \& \quad \mu = \frac{E}{2(1+\vartheta)} \quad \text{---(ii)}$$

Solving eq^n (ii),

$$E = 2\mu(1+\vartheta) \quad \text{---(iii)}$$

Substitute value of E in eq^n (i)

$$\lambda = \frac{2\mu\cancel{(1+\vartheta)}\vartheta}{\cancel{(1+\vartheta)}(1-2\vartheta)} = \frac{2\mu\vartheta}{(1-2\vartheta)}$$

ie, $\lambda(1-2\vartheta) = 2\mu\vartheta$

$$\vartheta = \frac{\lambda(1-2\vartheta)}{2\mu}$$

$$\vartheta = \frac{\lambda}{2\mu} - \frac{\vartheta\lambda}{2\mu}$$

ie, $$\vartheta + \frac{\vartheta\lambda}{2\mu} = \frac{\lambda}{2\mu}$$

$$\vartheta\left(1+\frac{\lambda}{2\mu}\right) = \frac{\lambda}{2\mu} \quad \Rightarrow \quad \vartheta = \frac{\lambda}{2\mu}\left(\frac{\mu}{\mu+\lambda}\right)$$

$$\boxed{\vartheta = \frac{\lambda}{2(\mu+\lambda)}} \quad \text{--- (iv)}$$

Substitute eq^n (iv) in eq^n (iii)

$$E = 2\mu\left(1 + \frac{\lambda}{2(\mu+\lambda)}\right)$$

$$E = 2\mu + \frac{\mu\lambda}{(\mu+\lambda)}$$

$$E = \frac{2\mu(\mu+\lambda) + \mu\lambda}{(\mu+\lambda)}$$

$$E = \frac{2\mu^2 + 2\mu\lambda + \mu\lambda}{(\mu+\lambda)}$$

$$E = \frac{2\mu^2 + 3\mu\lambda}{(\mu+\lambda)}$$

$$\boxed{E = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)}} \quad \text{— (v)}$$

(b) Elastic matrix for plane stress and plane strain -

(i) Plane Stress:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1-\vartheta^2)} \begin{bmatrix} 1 & \vartheta & 0 \\ \vartheta & 1 & 0 \\ 0 & 0 & \frac{1-\vartheta}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$$

where, E is young modulus and ϑ is poison coefficient.
To express them in λ & μ.

$$\frac{E}{(1-\vartheta^2)} = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)} \times \frac{1}{\left(1-\left(\frac{\lambda}{2(\mu+\lambda)}\right)^2\right)} \quad \text{--- from eq}^n \text{ (iv) \& (v)}$$

solving,

$$\frac{E}{(1-\vartheta^2)} = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

$$\& \quad \frac{1-\vartheta}{2} = \frac{1}{2}\left(1 - \frac{\lambda}{2(\mu+\lambda)}\right) = \frac{1}{2}\left(\frac{2(\mu+\lambda)-\lambda}{2(\mu+\lambda)}\right)$$

$$\frac{1-\vartheta}{2} = \frac{2\mu+\lambda}{4(\mu+\lambda)}$$

Substituting the values,

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)} \begin{bmatrix} 1 & \frac{\lambda}{2(\mu+\lambda)} & 0 \\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0 \\ 0 & 0 & \frac{2\mu+\lambda}{4(\mu+\lambda)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} \quad \ldots \left(\begin{matrix}\text{Plane} \\ \text{Stress}\end{matrix}\right) \ldots \text{(vi)}$$

(ii) Plane strain :

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$$

To express E & ν in term of λ & μ.

$$\frac{E}{(1+\nu)(1-2\nu)} = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)} \times \frac{1}{\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)\left(1-2\left(\frac{\lambda}{2(\mu+\lambda)}\right)\right)} \quad \ldots \text{from eq}^n \text{ (iv) \& (v)}$$

$$= \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)} \times \frac{2(\mu+\lambda)^2}{(2\mu+3\lambda)\mu}$$

$$\frac{E}{(1+\nu)(1-2\nu)} = 2(\mu+\lambda)$$

$$\cancel{(1+\nu)} = \cancel{1+}\frac{\lambda}{2(\mu+\lambda)} = \frac{2(\mu+\lambda)\cancel{+\lambda}}{\cancel{2(\mu+\lambda)}} = \cancel{\frac{2\mu+3\lambda}{2(\mu+\lambda)}}$$

$$\cancel{(1-\nu)} = \cancel{\frac{2\mu+3\lambda}{2(\mu+\lambda)}}$$

$$(1-2\nu) = 1-\frac{2\lambda}{2(\mu+\lambda)} = \frac{\mu+\lambda-\lambda}{(\mu+\lambda)} = \frac{\mu}{\mu+\lambda}$$

$$(1-\nu) = 1-\frac{\lambda}{2(\mu+\lambda)} = \frac{2\mu+2\lambda-\lambda}{2(\mu+\lambda)} = \frac{2\mu+\lambda}{2(\mu+\lambda)}$$

Substituting the values,

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = 2(\mu+\lambda) \begin{bmatrix} \frac{2\mu+\lambda}{2(\mu+\lambda)} & \frac{\lambda}{2(\mu+\lambda)} & 0 \\ \frac{\lambda}{2(\mu+\lambda)} & \frac{2\mu+\lambda}{2(\mu+\lambda)} & 0 \\ 0 & 0 & \frac{\mu}{2(\mu+\lambda)} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} \quad \text{(Plane strain)} \quad \text{-- (VII)}$$

(c) Split the stress-strain matrix E for plane strain

$$E = E_\mu + E_\lambda \quad \text{— (VIII)}$$

$$E = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} \quad \text{-- } \cancel{\text{(VIII)}}$$

We know that, E_μ contains only μ &
E_λ contains only λ.

$\therefore$,

$$E_\mu = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \quad \text{— (IX)}$$

$$E_\lambda = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{— (X)}$$

Substituting the eq^n (ix) & (x) in eq^n (viii)

$$E = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} + \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This is termed as B-bar formulation of near incompressible finite elements.

(d) E_λ & E_μ in terms of E & ϑ.

$$E_\lambda = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{E\vartheta}{(1+\vartheta)(1-2\vartheta)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_\mu = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} = \begin{bmatrix} \frac{E}{(1+\vartheta)} & 0 & 0 \\ 0 & \frac{E}{(1+\vartheta)} & 0 \\ 0 & 0 & \frac{E}{2(1-\vartheta)} \end{bmatrix}$$

$$E_\mu = \frac{E}{(1+\vartheta)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

→ Assignment 3.2 (The 3noded plane stress triangle)

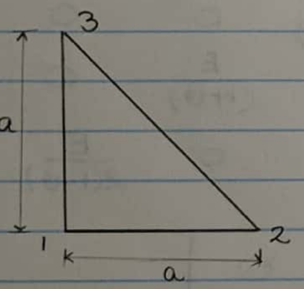
Q II) Consider a plane triangular domain.....

(A) A plane linear turner triangle with the same dimensions.

(B) A set of three bar elements placed over the edges of the triangular domain. The cross sections for the bars are $A_1 = A_2$ and A_3.

(a) Stiffness matrices K_{tri} & K_{bar} for both discrete models.

(A) Linear Turner Triangle -



Pt 1 (0, 0)
Pt 2 (a, 0)
Pt 3 (0, a)

(i) Element Stiffness Matrix -

$$K^e = \int_\Omega h B^T E B \, d\Omega$$

$$\boxed{K^e = \frac{h}{4A} B^T E B}$$

(ii) Area (A):

$$2A = \det\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} = \det\begin{bmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$2A = a^2$$

$$\boxed{A = \frac{a^2}{2}}$$

(iii) Strain Displacement Matrix (B)

$$\epsilon = DNU^e = BU^e$$

ie, $B = DN$.

ie,
$$B = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

where, $y_{jk} = y_j - y_k$ & $x_{jk} = x_j - x_k$

$$B = \frac{1}{a^2}\begin{bmatrix} -a & 0 & a & 0 & 0 & 0 \\ 0 & -a & 0 & 0 & 0 & a \\ -a & -a & 0 & a & a & 0 \end{bmatrix}$$

(iv) Constitutive Matrix (E) (Elastic Matrix)

Plane stress –
$$\underset{\sim}{E} = \frac{E}{(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Now, solving for stiffness matrix.

$$K_{tri} = \frac{Eh}{2a^2(1-\nu^2)}\begin{bmatrix} -a & 0 & -a \\ 0 & -a & -a \\ a & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{bmatrix} -a & 0 & a & 0 & 0 & 0 \\ 0 & -a & 0 & 0 & 0 & a \\ -a & -a & 0 & a & a & 0 \end{bmatrix}$$

$$K_{tri} = \frac{Eh}{2a^2(1-\nu^2)}\begin{bmatrix} a^2+\frac{a^2(1-\nu)}{2} & a^2\nu+\frac{a^2(1-\nu)}{2} & -a^2 & -\frac{a^2(1-\nu)}{2} & -\frac{a^2(1-\nu)}{2} & -a^2\nu \\ a^2\nu+\frac{a^2(1-\nu)}{2} & a^2+\frac{a^2(1-\nu)}{2} & -a^2\nu & -\frac{a^2(1-\nu)}{2} & -\frac{a^2(1-\nu)}{2} & -a^2 \\ -a^2 & -a^2\nu & a^2 & 0 & 0 & a^2\nu \\ -\frac{a^2(1-\nu)}{2} & -\frac{a^2(1-\nu)}{2} & 0 & \frac{a^2(1-\nu)}{2} & \frac{a^2(1-\nu)}{2} & 0 \\ -\frac{a^2(1-\nu)}{2} & -\frac{a^2(1-\nu)}{2} & 0 & \frac{a^2(1-\nu)}{2} & \frac{a^2(1-\nu)}{2} & 0 \\ -a^2\nu & -a^2 & a^2\nu & 0 & 0 & a^2 \end{bmatrix}$$

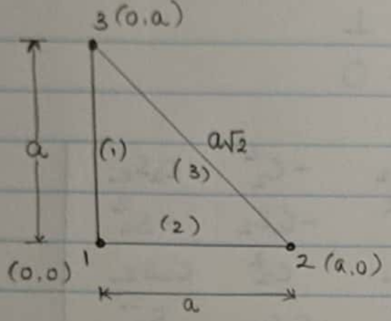
Taking a^2 common.

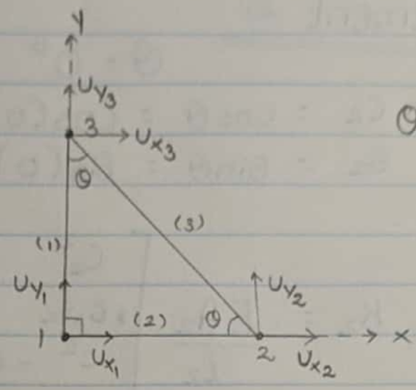
$$K_{tri} = \frac{Eh}{2(1-\nu^2)} \begin{bmatrix} 1+\frac{(1-\nu)}{2} & \nu+\frac{(1-\nu)}{2} & -1 & -\frac{(1-\nu)}{2} & -\frac{(1-\nu)}{2} & -\nu \\ \nu+\frac{(1-\nu)}{2} & 1+\frac{(1-\nu)}{2} & -\nu & -\frac{(1-\nu)}{2} & -\frac{(1-\nu)}{2} & -1 \\ -1 & -\nu & 1 & 0 & 0 & \nu \\ -\frac{(1-\nu)}{2} & -\frac{(1-\nu)}{2} & 0 & \frac{(1-\nu)}{2} & \frac{(1-\nu)}{2} & 0 \\ -\frac{(1-\nu)}{2} & -\frac{(1-\nu)}{2} & 0 & \frac{(1-\nu)}{2} & \frac{(1-\nu)}{2} & 0 \\ -\nu & -1 & \nu & 0 & 0 & 1 \end{bmatrix}$$

Substituting $h = 1$, $a = 1$ & $\nu = 0$

$$K_{tri} = \frac{E}{2} \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

B 3 Bar Element





$\theta = 45^\circ$

(i) Element Stiffness Matrix.

$$K^e = (T^e)^T \bar{k}^e (T^e)$$

$$K^e = \frac{E^e A^e}{l^e}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \quad \text{--- (I)}$$

where, 'C' is cosθ & 'S' is sinθ

(ii) Element wise stiffness matrix -

Element 1 $\quad \theta = 90^\circ$, $A_1^e = A_1$ & $l_1 = a$

$C_1 = \cos\theta = \cos(90) = 0$

$S_1 = \sin\theta = \sin(90) = 1$

$$K_1 = \frac{EA_1}{l_1}\begin{bmatrix} C_1^2 & C_1S_1 & -C_1^2 & -C_1S_1 \\ C_1S_1 & S_1^2 & -C_1S_1 & -S_1^2 \\ -C_1^2 & -C_1S_1 & C_1^2 & C_1S_1^2 \\ -C_1S_1 & -S_1^2 & C_1S_1 & S_1^2 \end{bmatrix}$$

Substituting the values,

$$K_1 = \frac{EA_1}{a}\begin{array}{c} \begin{array}{cccc} x_1 & y_1 & x_3 & y_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}\begin{array}{c} x_1 \\ y_1 \\ x_3 \\ y_3 \end{array} \quad \text{---- (II)}$$

Element 2

$\theta = 0^\circ$, $A = A_2$, $l_2 = a$

$C_2 = \cos\theta = \cos(0) = 1$

$S_2 = \sin\theta = \sin(0) = 0$

$$K_2 = \frac{EA_2}{l_2}\begin{bmatrix} C_2^2 & C_2S_2 & -C_2^2 & -C_2S_2 \\ C_2S_2 & S_2^2 & -C_2S_2 & -S_2^2 \\ -C^2 & -C_2S_2 & C_2^2 & C_2S_2 \\ -C_2S_2 & -S_2^2 & C_2S_2 & S_2^2 \end{bmatrix}$$

Substituting the values,

$$K_2 = \frac{EA_2}{a}\begin{array}{c c} \begin{array}{cccc} x_1 & y_1 & x_2 & y_2 \end{array} & \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} x_1 \\ y_1 \\ x_2 \\ y_2 \end{array} \end{array} \quad \text{— III}$$

Element 3

$\theta = -45^\circ \ (135^\circ)$, $A = A_3$, $l_3 = a\sqrt{2}$

$C_3 = \cancel{\cos(-45)} \ \cancel{= \frac{1}{\sqrt{2}}} \quad \cos(135) = -\frac{1}{\sqrt{2}}$

$S_3 = \cancel{\sin(-45)} \ \cancel{= -\frac{1}{\sqrt{2}}} \quad \sin(135) = \frac{1}{\sqrt{2}}$

$$K_3 = \frac{EA_3}{l_3}\begin{bmatrix} C_3^2 & C_3S_3 & -C_3^2 & -C_3S_3 \\ C_3S_3 & S_3^2 & -C_3S_3 & -S_3^2 \\ -C_3^2 & -C_3S_3 & C_3^2 & C_3S_3 \\ -C_3S_3 & -S_3^2 & C_3S_3 & S_3^2 \end{bmatrix}$$

$$K_3 = \frac{EA_3}{a\sqrt{2}}\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K_3 = \frac{EA_3}{a2\sqrt{2}} \begin{array}{c} \begin{array}{cccc} x_2 & y_2 & x_3 & y_3 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ x_2 \\ y_2 \\ x_3 \\ y_3 \end{array} \longrightarrow \text{(IV)}$$

(ii) The globalized stiffness matrix -

$$K_{bar} = K_1 + K_2 + K_3.$$

$$K_{bar} = \begin{bmatrix} \frac{EA_2}{a} & 0 & -\frac{EA_2}{a} & 0 & 0 & 0 \\ 0 & \frac{EA_1}{a} & 0 & 0 & 0 & -\frac{EA_1}{a} \\ -\frac{EA_2}{a} & 0 & \frac{EA_2}{a} + \frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} & \frac{EA_3}{2\sqrt{2}a} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}a} & \frac{EA_3}{2\sqrt{2}a} & \frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}a} & \frac{EA_3}{2\sqrt{2}a} & \frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} \\ 0 & -\frac{EA_1}{a} & \frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} & -\frac{EA_3}{2\sqrt{2}a} & \frac{EA_1}{a} + \frac{EA_3}{2\sqrt{2}a} \end{bmatrix}$$

— Substituting $a = 1$

$$K_{bar} = \begin{bmatrix} EA_2 & 0 & -EA_2 & 0 & 0 & 0 \\ 0 & EA_1 & 0 & 0 & 0 & -EA_1 \\ -EA_2 & 0 & EA_2 + \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & 0 & -\frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} \\ 0 & -EA_1 & \frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & -\frac{EA_3}{2\sqrt{2}} & EA_1 + \frac{EA_3}{2\sqrt{2}} \end{bmatrix}$$

(b) $K_{bar} = K_{tria}$?

As there is no similarity on basis of values of the cross-section for which the stiffness matrix of bar and triangle are equivalent.

To make both the matrix similar to some extent by making $A_1 = A_2 = a/2$ & $A_3 = \sqrt{2}\, a$.
We can make them more similar but not equal.

(c) Physical Significance, why $K_{tri} \neq K_{bar}$?

(i) Turner triangles are closed form (whole surface). Its stiffness matrix takes care of stresses in plane stress problems.
Also, its able to formulate the strains & displacements in the middle of the element as that shape function is also considered.

(ii) Bar triangle is open form (only joint to each other to form a triangle). Its stiffness matrix takes care of trusses.
Its not able to formulate the strains & displacements in the middle of the element, because ~~only displacements~~ the shape function takes into account displacement along the axis of each element.

(d) Considering $\nu \neq 0$,

When $\nu \neq 0$, the results ~~are~~ obtained ~~are~~ doesn't differ that much. But lateral displacement comes into the picture, which is not ~~there~~ present when $\nu = 0$.

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