Computational Structural Mechanics and Dynamics

Assignment 5
Isoparametric Representation
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Assignment 5.1

1. Determine the coefficients $a_0 ... c_2$ of the shape functions written above:

$$N_1^e = a_0 + a_1\xi + a_2\xi^2$$

$$N_2^e = b_0 + b_1\xi + b_2\xi^2$$

$$N_3^e = c_0 + c_1\xi + c_2\xi^2$$

The condition that it will be used is $N_i^e(\xi = -1, 0 \; or \; 1) = 1$

$$N_1^e(\xi = -1) = a_0 - a_1 + a_2 = 1 \quad N_1^e(\xi = -0) = a_0 = 0 \quad N_1^e(\xi = 1) = a_0 + a_1 + a_2 = 0$$

$$N_2^e(\xi = -1) = b_0 - b_1 + b_2 = 0 \quad N_2^e(\xi = -0) = b_0 = 0 \quad N_2^e(\xi = 1) = b_0 + b_1 + b_2 = 1$$

$$N_1^e(\xi = -1) = c_0 - c_1 + c_2 = 0 \quad N_1^e(\xi = -0) = c_0 = 1 \quad N_1^e(\xi = 1) = c_0 + c_1 + c_2 = 0$$

It can be seen it has obtained a system of 9 unknown with 9 equation. After some algebra it can be obtained the value of $a_0 ... c_2$.

$$a_0 = 0; \; a_1 = -1/2; \; a_2 = 1/2$$

$$b_0 = 0; \; b_1 = 1/2; \; b_2 = 1/2$$

$$c_0 = 1; \; c_1 = 0; \; c_2 = -1$$

Finally, the shape functions are:

$$N_1^e = -1/2\xi + 1/2\xi^2 = 1/2\xi(\xi - 1)$$

$$N_2^e = 1/2\xi + 1/2\xi^2 = 1/2\xi(\xi + 1)$$

$$N_3^e = 1 - \xi^2 = 1 - \xi^2$$

2. Prove that $N_1^e + N_2^e + N_3^e = 1$

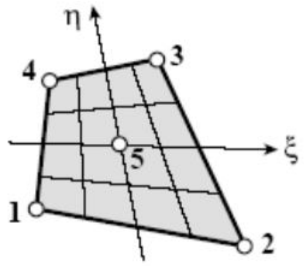
$$N_1^e + N_2^e + N_3^e = -1/2\xi + 1/2\xi + 1/2\xi^2 + 1/2\xi^2 - \xi^2 + 1 = 1$$

3. Calcute thier derivatives respect to the natural coordinates.

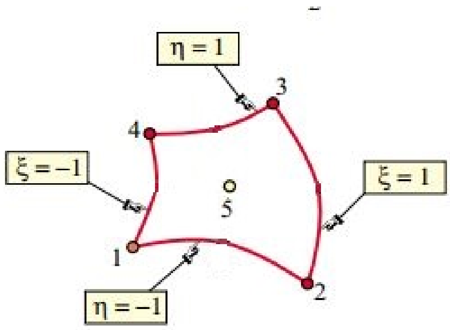
$$\frac{dN_1}{d\xi} = \xi - 1/2 \quad \frac{dN_2}{d\xi} = \xi + 1/2 \quad \frac{dN_3}{d\xi} = -2\xi$$

Assignment 5.2

1. Find the shape functions of the quadrilater shown above:



As a hint it will be used the line-product method in order to obtain the five shape fuctions. N_9^e shape function



$$N_9^e = c_9 L_{1-2} L_{2-3} L_{3-4} L_{4-1} = c_9(\eta+1)(\xi-1)(\eta-1)(\xi+1)$$
$$N_9^e = c_9(1-\eta^2)(1-\xi^2)$$

Now, the condition is $\eta = 0, \xi = 0$

$$N_9^e = c_9(1-0^2)(1-0^2) = 1$$

Finally, $c_9 = 1$

In order to obtain the others shape functions it will be used the condition written above:

$$N_i = N_i^e + \alpha N_9^2$$

For N_1^e

$$N_1^e = 1/4(1-\eta)(1-\xi) + \alpha(1-\eta^2)(1-\xi^2)$$

For N_2^e

$$N_2^e = 1/4(1-\eta)(1+\xi) + \alpha(1-\eta^2)(1-\xi^2)$$

For N_3^e

$$N_3^e = 1/4(1+\eta)(1+\xi) + \alpha(1-\eta^2)(1-\xi^2)$$

For N_4^e

$$N_4^e = 1/4(1+\eta)(1-\xi) + \alpha(1-\eta^2)(1-\xi^2)$$

In this case the condition is that $N_1(\eta = 0, \xi = 0) = N_2(\eta = 0, \xi = 0) = N_3(\eta = 0, \xi = 0) = N_4(\eta = 0, \xi = 0) = 0$. If that condition is fulfiled, then, it will be possible to obtain α.

$$N_1^e = 1/4(1-0)(1-0) + \alpha(1-0^2)(1-0^2) = 0$$

$$N_2^e = 1/4(1-0)(1+0) + \alpha(1-0^2)(1-0^2) = 0$$
$$N_3^e = 1/4(1+0)(1+0) + \alpha(1-0^2)(1-0^2) = 0$$
$$N_4^e = 1/4(1+0)(1-0) + \alpha(1-0^2)(1-0^2) = 0$$

After some algebra, then it get:

$$\alpha = -1/4$$

And the shape function will be:

$$N_1^e = 1/4(1-\eta)(1-\xi) + 1/4(1-\eta^2)(1-\xi^2)$$
$$N_2^e = 1/4(1-\eta)(1+\xi) + 1/4(1-\eta^2)(1-\xi^2)$$
$$N_3^e = 1/4(1+\eta)(1+\xi) + 1/4(1-\eta^2)(1-\xi^2)$$
$$N_4^e = 1/4(1+\eta)(1-\xi) + 1/4(1-\eta^2)(1-\xi^2)$$

It can be seen that $N_1 + N2 + N_3 + N4 + N5 = 1$

Assignment 5.3

1. Which minimum integration rules of Gauss-product type gives a rank sufficient stiffness matrix for these elements:
   1. the 8-node hexahedron
   2. the 20-node hexahedron
   3. the 27-node hexahedron
   4. the 64-node hexahedron

The condition to be satisfy in order to avoid a rank deficient will be:

$$r = min(n_f - n_r, n_E * n_G)$$

$$rd = (n_f - n_r) - r = 0$$

Where: r: rank n_f: dimension multiply by the number of nodes
n_r: number of rigid body motions
n_E: number of components of the constitutive matrix
n_G: number of Gauss points
rd: deficient rank

1. the 8-node hexahedron

$$r = min(3*8-6, 6*x)$$
$$rd = (18) - 18 = 0$$

Finally x=3. It will be necessary 3 Gauss point in order to avoide a deficient rank

2. the 20-node hexahedron

$$r = min(3*20-6, 6*x)$$
$$rd = (54) - 54 = 0$$

Finally x=9. It will be necessary 9 Gauss point in order to avoide a deficient rank

3. the 27-node hexahedron

$$r = min(3 * 27 - 6, 6 * x)$$
$$rd = (75) - 75 = 0$$

Finally x=13. It will be necessary 13 Gauss point in order to avoide a deficient rank

4. the 64-node hexahedron

$$r = min(3 * 64 - 6, 6 * x)$$
$$rd = (186) - 186 = 0$$

Finally x=31. It will be necessary 31 Gauss point in order to avoide a deficient rank