Computational Structural Mechanics and Dynamics

Assignment 6

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Assignment 6.a

Write a Program In Mat Lab for the Timoshenko 2 Nodes Beam element with reduced integration for the shear stiffness matrix

$$K_b^{(e)} = \left(\frac{EI}{l}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \text{(The point interpolation is exact for } K_b^{(e)}\text{)}$$

$$K_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \dots & \frac{(l^{(e)})^2}{4} & \frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \dots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & & \dots & \frac{(l^{(e)})^2}{4} \end{bmatrix} \text{(Reduced integration)}$$

Hint: For stress evaluation make gaus1 = gaus2 = 0.0

<u>Solution</u>

There are two main theories to approach the mechanical bending behavior of beams: Euler-Bernoulli and Timoshenko. The former is designed for slender beams, the later for thick beams. Timoshenko's theory allows the that x-section of the deformed beam to be non- orthogonal to the original axis of the beam.The Timoshenko's method leads so to a different element stiffness matrix, split in a bending stiffness matrix and a shear one:

$$K^e a^e - f^e = q^e$$
$$K^e = K_b^e + K_s^e$$

Where the shear matrix is given by:

$$K_s^e = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l}{2} & -1 & \frac{l}{2} \\ & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \\ & & 1 & -\frac{l}{2} \\ sym & & & \frac{l^2}{4} \end{bmatrix}$$

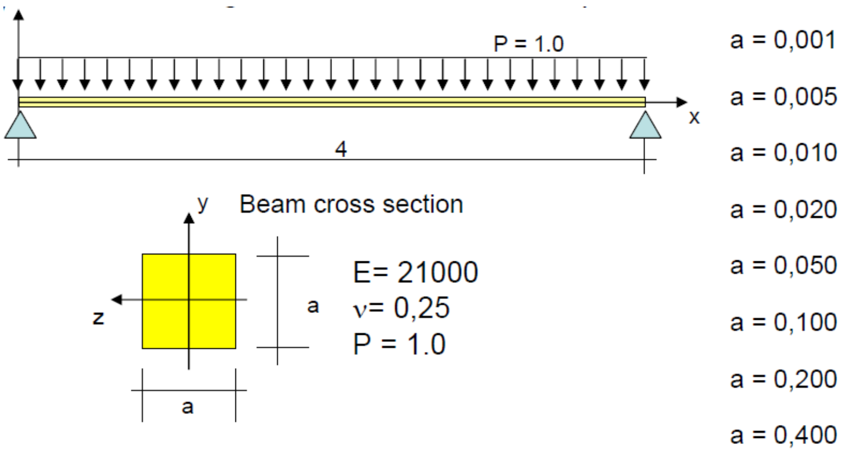
In this assignment, Timoshenko's method has to be implemented. However, the Stress_Beam_Timoshenko_v1_3.m is already provided, all that is needed is to change the K_s matrix (share) to the following:

```
K_s = [ 1 , len/2 , -1 , len/2 ;
len/2 , len^2/3 , -len/2 , len^2/6 ;
-1 , -len/2 , 1 , -len/2 ;
len/2 , len^2/6 , -len/2 , len^2/3 ];
```

Assignment 6.b

Solve the following problem with a 64 element mesh with the

- 2 nodes Euler Bernoulli element
- 2 nodes Timoshenko Full Integrate element
- 2 nodes Timoshenko Reduce Integration element.
- Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship



Beam cross section
E= 21000
ν = 0,25
P = 1.0

Solution

We are asked to model the 64 element beam shown above using three models:

- Euler-Bernoulli
- Timoshenko
- Timoshenko Reduced Integration

While using multiple values of the beam width, which affects for the x-sectional area and the moment of inertia calculation, therefore affecting both the bending and shear stiffness matrices:

$$K_b = \int_{l^e} B_b^T (E\mathbf{I}) B_b dx$$
$$K_s = \int_{l^e} B_s^T (G\alpha\mathbf{A}) B_s dx$$

This beam is 4m long and simply supported on its edges (X-Constraint), with a downward uniform load of 1N/m along the element. In addition, the material's properties are the Young's Modulus E = 21000MPa and the poisson's ratio = 0:25.

A Matlab code was implemented for each of the models and the calculation was made for the suggested values of a/L. The results can be seen in Figures 1, 2 and 3.

Observations and comments

- The first thing to notice is that for thick beams (a/L>0.02) the three models deliver practically the same results by all metrics, with the exception of shear in the Euler-Bernoulli model (which can’t evaluate them).
- Displacements for slender beams (a/L<0.02) are almost identical for Euler-Bernoulli and Timoshenko reduced, and largely correct. On the other hand, Timoshenko’s fully integrated model underestimates displacement due to the shear locking effect.

- The momentum vs a/L plot shows how the Timoshenko reduced integration can produce correct results even when the beam is thin, the Timoshenko with full integration doesn't
- Evaluating the shear is only possible with the Timoshenko approach.

The two-noded Timoshenko beam element with the reduced integration has the best overall performance, predicting correct displacements for thick and slender bar as well as being able to predict shear stresses. It's the best overall method.

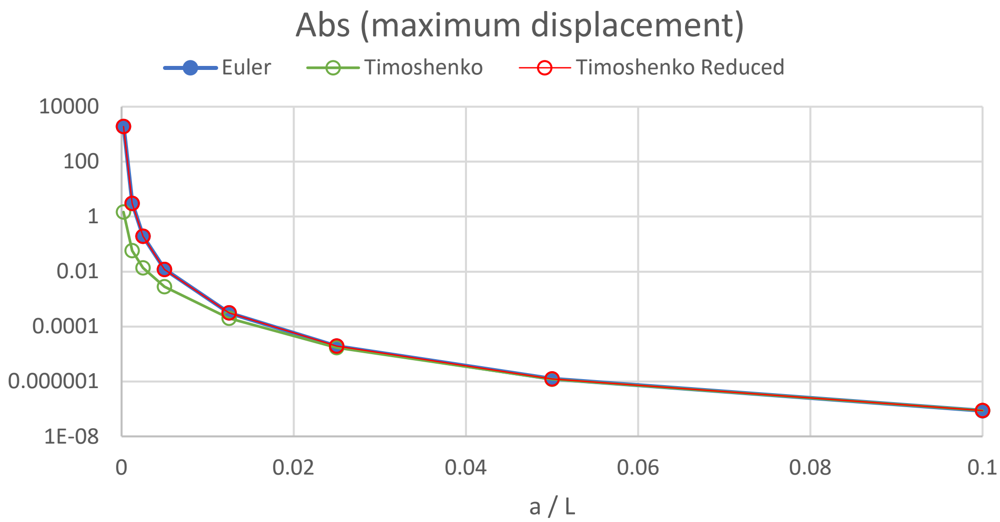


Figure 1 Absolute value of maximum displacement vs a/L

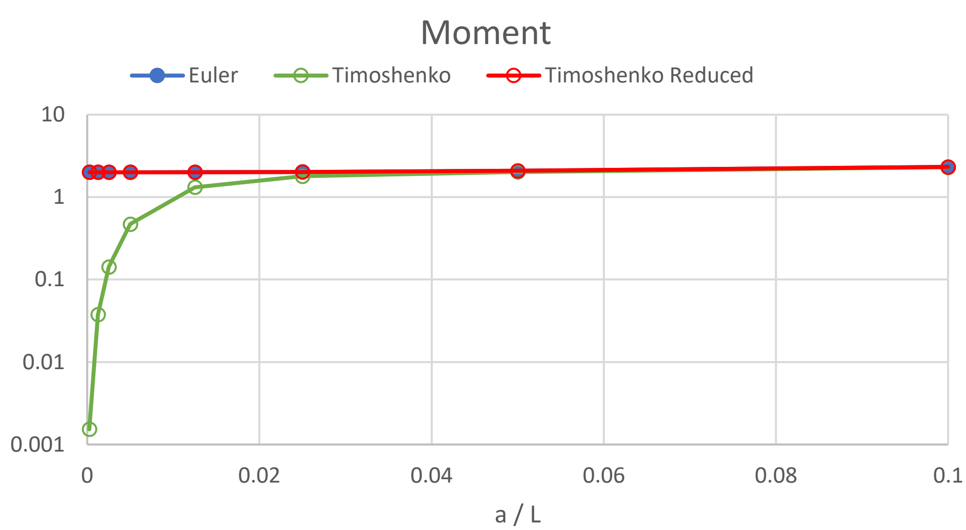


Figure 2 Moment vs a/L

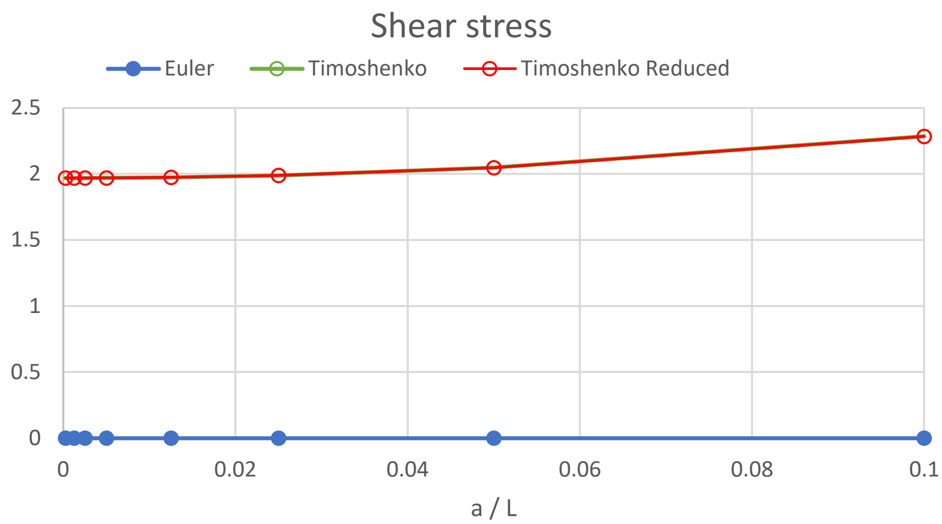


Figure 3 shear stress vs a/L.