Computational Structural Mechanics and Dynamics

Assignment 7

Plate elements

By

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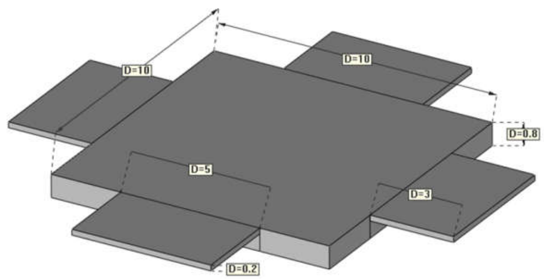
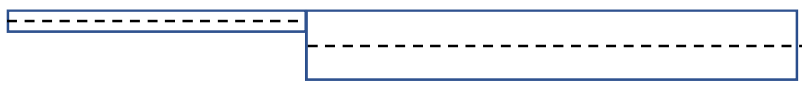
Master in numerical method in engineering

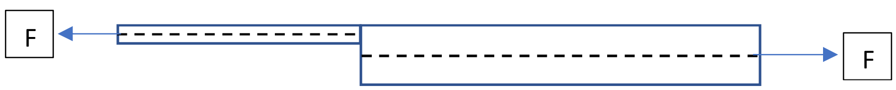
Assignment 7.1: Plate assembly

What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will use for solving the following problem:

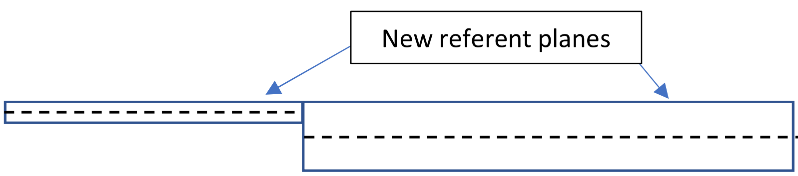


This problem presents an offset between the middle planes of each plate. It must take into account that offset induces moments in the structure when a load is applied along the neutral axis of each plate.

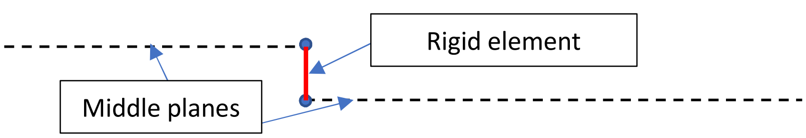




To leading with this problem, some strategies can be used. As a first strategy, it can be defined a new coordinate system on top of each plate faces and then define a new reference plane coincident with top face of each plate. But, take care when the stiffness matrices are computed.



Another strategy, rigid elements can be used in order to link the middle planes of the plate elements.



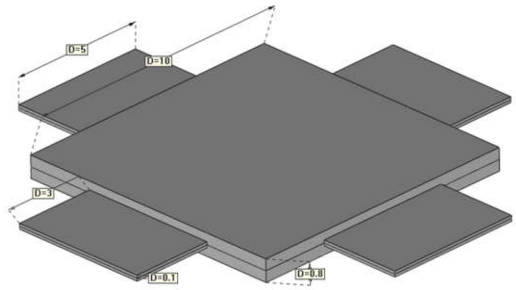
In case of using the strategies described above, plates elements based on Kirchhoff or Mindlin/Reissner's theory can be used. For instance, if it is used the first strategy then it will be necessary to use plate elements based on The Mindlin/Reissner's theory, especially, when a sudden change in thickness appears or transversal shear effect appears.

If it is not expected transverse shear effect or the applied loading is such that produces pure-bending, plate elements based on Kirchhoff theory can be applied.

In case of transversal shear effects are important, plate elements based on The Mindlin/Reissner's theory must be used taking into account of using different technics to avoid the locking effects when thin plates are modelled whit these elements.

Assignment 7.2: Plate assembly

What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will use for solving the following problem:



In this case, the middle plane of each plate match, so, do not exist offset problems as the previous geometry, then, Several strategies could be used:

For neglected shear deformation or pure bending:

If It consider, as a first approximation, that all plates have a relation between thickness/width ≤ 0.1 and If shear deformation is not expected, then, thin plate elements, whose formulation is based on the Kirchhoff's theory, can be used. Also, it can be used plates elements based on The Mindlin/Reissner's theory, especially, near sudden changes in thickness.

If all the plates are considered as thin plates, then, it can be used Discrete Kirchhoff thin plates elements.

For shear deformation effect

If shear deformation is expected, then it will necessary to use thick/thin elements, whose formulation follows The Mindlin/Ressner's theory. But, some consideration must be taken into account. First of all, in order to avoid the locking effect, in the plates whose thickness is equal to 0.1, it will be necessary to use reduced integration for the transversal shear stiffness matrix (K_s) and full integration for the bending stiffness matrix (K_b) or transversal shear strains method, and second, for the thick plates (thickness = 0.8) will not be necessary take into account the remark written above.

Assignment 7.3: Patch Test

In this section it will apply the so-called displacement patch test. The procedure will be as follows. Pick a patch. Evaluate the displacement field at the external nodes of the patch and apply as prescribed displacements. Set forces at interior DOFs to zero. Solve for the displacement components of the interior nodes (one in the example). These should agree with the value of the displacement field at that node. Recover the strain field over the elements: all components should vanish identically at any point.

The Figure below shows the mesh used for the Patch test. Each element and node have been numbered with a global numeration.

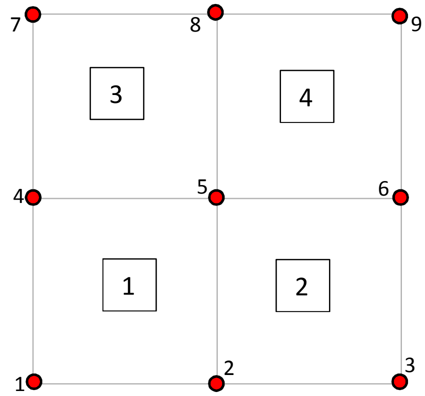


Figure 1: Mesh for Patch test.

Nodal displacements were prescribed. The displacements of the node 5 was not prescribed.

In order to see if plate elements used next can pass the patch test satisfactory, a set of displacement functions will be chosen.

$$w = \frac{1}{2}(x^2 + y^2 + xy)$$

$$\theta_x = x + \frac{1}{2}y$$

$$\theta_y = y + \frac{1}{2}x$$

The Table 1 shows different nodal displacements values to be used as an input in the patch test. Each value was computed using the displacement function written above.

MCZ and RM plate will be tested by using the Patch Test.

Table 1: Different cases to be tested in the patch test.

Node number	X	Y	w	θ_x	θ_y
1	0	0	0	0	0
2	0.5	0	0.125	0.5	0.25
3	1	0	0.5	1	0.5
4	0	0.5	0.125	0.25	0.5
5	0.5	0.5	0.375	0.75	0.75
6	1	0.5	0.875	1.25	1
7	1	0	0.5	0.5	1
8	0.5	1	0.875	1	1.25
9	1	1	1	1.5	1.5

The displacement value of node number 5 were computed in order to compare with the results computed by fem.

<u>Results of Patch Test for MCZ plate elements</u>

The Table 2 shows the results of the node 5 obtained by applying the Patch Test.

Table 2: Results obtained.

node	Calculated by	DOF			Strain		
		w	θ_x	θ_y	$\hat{\varepsilon}_x$	$\hat{\varepsilon}_y$	$\hat{\gamma}_{xy}$
5	FEM	0,3751	0,7493	0,7500	-1,0036	-0,9976	-1,000
	Analytically	0.375	0.750	0.750	-1	-1	-1

Where:

$$\hat{\varepsilon}_x = -\frac{\partial^2 w}{\partial x^2};\ \hat{\varepsilon}_y = -\frac{\partial^2 w}{\partial y^2};\ \gamma_{xy} = -2\frac{\partial^2 w}{\partial xy}$$

As it can be seen the element passed the Patch test.

Results of Patch Test for RM plate elements

The **¡Error! No se encuentra el origen de la referencia.** shows the results of the node 5 obtained by applying the Patch Test.

Table 3: Results obtained.

node	Calculated by	DOF			Strain				
		w	θ_x	θ_y	$\hat{\varepsilon}_x$	$\hat{\varepsilon}_y$	$\hat{\gamma}_{xy}$	$\hat{\gamma}_{xz}$	$\hat{\gamma}_{yz}$
5	FEM	0,375	0,750	0,750	-1,000	-1,000	-1,000	-2,5E-4	-2,77E-17
	Analytically	0.375	0.750	0.750	-1	-1	-1	0	0

Where:

$$\hat{\varepsilon}_x = -\frac{\partial \theta_x}{\partial x};\ \hat{\varepsilon}_y = -\frac{\partial \theta_y}{\partial y};\ \gamma_{xy} = -\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right);$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} - \theta_x;\ \gamma_{yz} = \frac{\partial w}{\partial y} - \theta_y$$

As it can be seen the element passed the Patch test. But there is a difference between the result of $\hat{\gamma}_{xz}$, obtained by the analytical expression written above, and the result obtained by FEM.