

# Computational Structural Mechanics and Dynamics

## Assignment 7

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- a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate.

Use the 5x5 Mesh.

$t = 0,001$

$t = 0,010$

$E = 10.92$

$\nu = 0,3$

$Q = 1.0$

$t = 0,020$

$t = 0,100$

$t = 0,400$

Discusses  
the results  
observed.

### Solution:

MCZ and RM elements are tested on a 5x5 grid (with the above stated properties) for different thicknesses. The model is pre- and post-processed in GiD while the simulation was run on Matlab with the respective MCZ and RM codes (downloaded from the MAT-FEM website). The prescribed conditions are zero displacements at the bottom nodes, constant -5kN force on nodes at to (in the y direction) except nodes 1 and 16, which are assigned -2.5kN (otherwise the stresses are not symmetric).

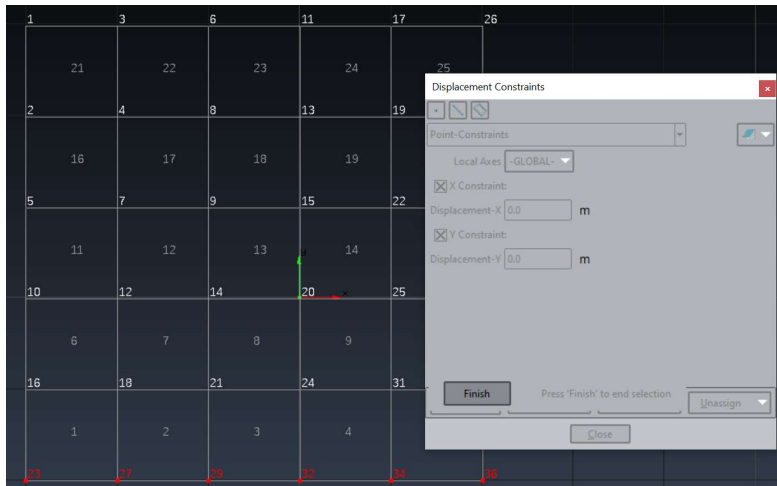


Figure 1 constructed model, with zero displacement being assigned in the bottom nodes

Displacements and moments (maximum absolute values) for both the RM and MZC elements are plotted below for the different thickness values required.

Max displacement			Moment		
t	MZC	RM	t	MZC	RM
0.001	2.17E+09	2.05E+09	0.001	4.27E-02	3.98E-02
0.01	2.17E+06	2.05E+06	0.01	4.27E-02	3.99E-02
0.02	2.72E+05	2.56E+05	0.02	4.27E-02	4.01E-02
0.1	2.17E+03	2.07E+03	0.1	4.27E-02	4.39E-02
0.4	3.39E+00	3.54E+00	0.4	4.27E-02	4.86E-02

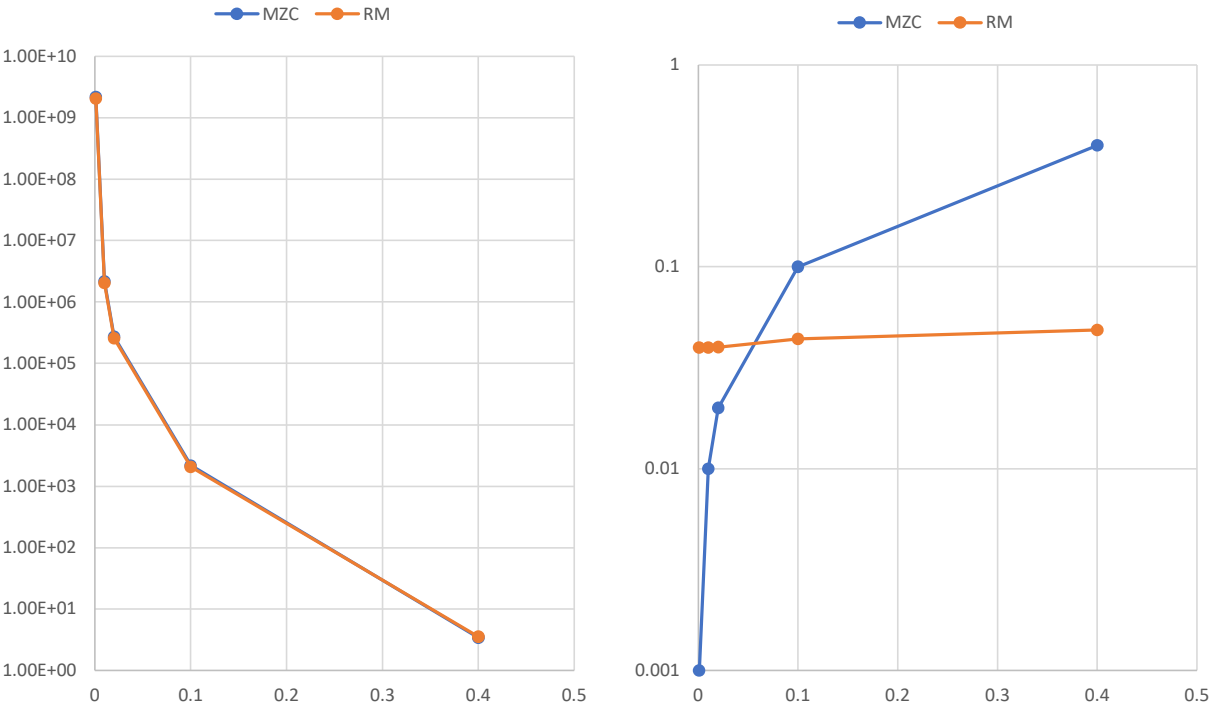


Figure 2 Maximum displacements and moments as a function of plate thickness

For both elements, displacement decreases as thickness increases. For low values of thickness, both elements exhibit practically the same value, with some difference appearing at high thickness values. The RM element shows lower displacement values than the MZC element because of the shear blocking effect at lower thicknesses

Regarding moments, MZC element exhibits constant values for all thickness. Conversely, the RM element shows small momentum for thin plates and they increase with thickness.

b) Define and verify a patch test mesh for the MCZ element.

Discusses the results observed.

Solution:

The purpose of the patch test is to check if the Finite Element solution honors the exact solution (for a simple problem that we know the solution to). This test is performed using a specific node and all the nodes surrounding it. On the later, a known displacement prescribed, the same for all nodes. No conditions are prescribed for the central node. The test consists of checking that the central node does not experience displacements different from those prescribed on all the surrounding nodes. It is considered failed if the displacements differ by a margin larger than the tolerances required for the problem.

In this case, the patch test is done on an MZC element on one of the central nodes of the 5x5 square mesh. The patch test as carried around node 9. All the surrounding nodes were prescribed a displacement of 0.1 upward in the y direction. All the outer nodes in the body are prescribed with zero displacement (in all directions)

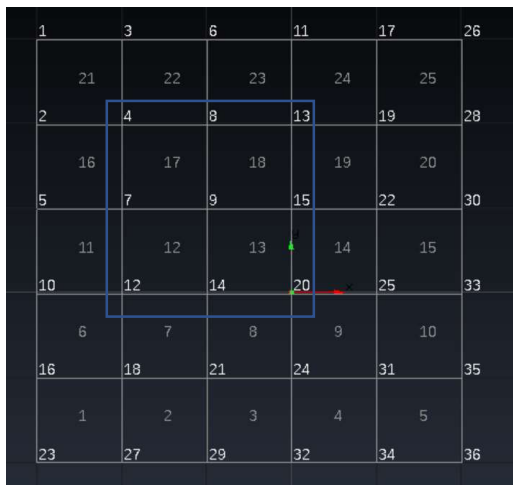


Figure 3 location around node 9 used for the patch test

The material properties used are the same as in the prior section with a thickness 0.1.

In order to satisfy the patch test the middle node should also exhibit a displacement as 0.1 positive in the y direction and zero in the x direction, just like all its surrounding nodes.

After generating the geometry of the patch in GiD, the .mat file is obtained and simulated using the MZC plate Finite Element code. The results are again post-processed by using GiD

Simple visual inspection of the simulation suggests the patch test criteria is met

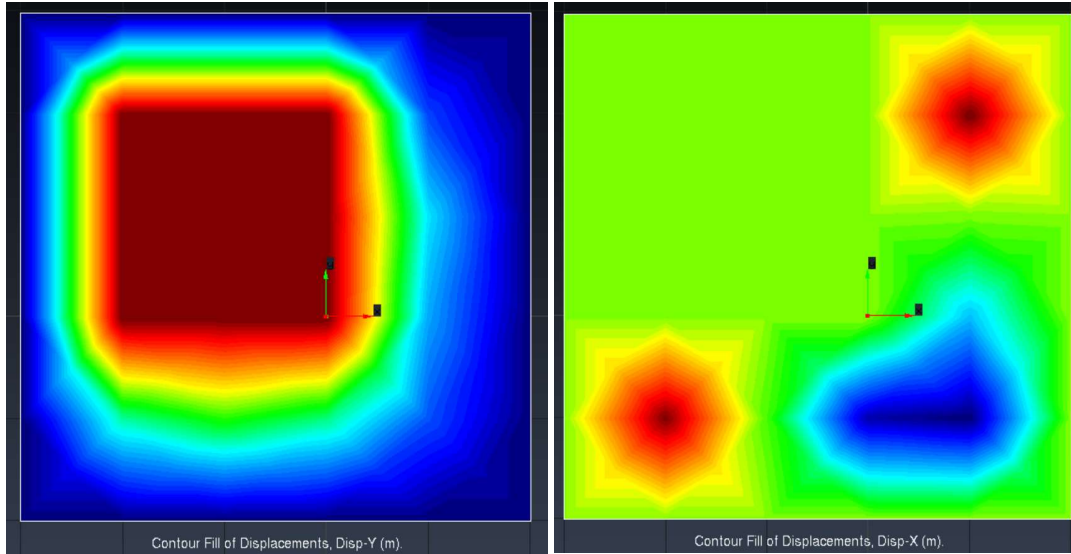


Figure 4 Patch test displacements in the Y and X direction (from left to right)

The displacement experienced by the central node (9) are within  $10^{-13}$  of the value it should be (0.1). Unless extreme accuracy is required for the application, I would state that the patch test is passed.

Some comments:

- When preparing this assignment I found this interesting quote: 'It was long conjectured by engineers that passing the patch test is sufficient for the convergence of the finite element, that is, to ensure that the solutions from the finite element method converge to the exact solution of the partial differential equation as the finite element mesh is refined. *However, this is not the case, and the patch test is neither sufficient nor necessary for convergence.*' (Bathe, Klaus-Jürgen (June 1995). Finite Element Procedures (2 ed.). Prentice Hall. ISBN 0-9790049-0-X.). It seems the patch test is more of a qualitative indicator than a rigorous mathematical sufficient condition.
- It appears that the MZC element does not normally pass the patch test with irregular meshes. Below is an example of central node displacements differing from displacement in surrounding nodes for an irregular quad mesh.

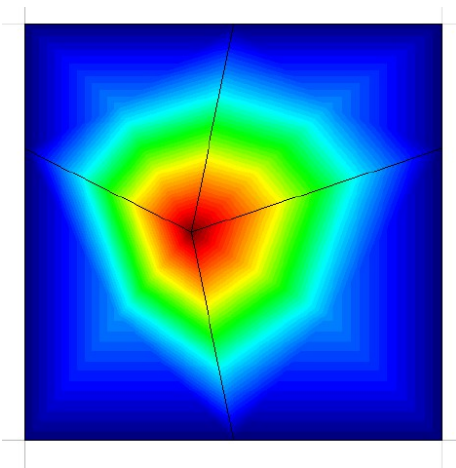


Figure 5 displacements in a patch test done with MCZ elements (irregular shape)