
Assignment 8 - Shell FEM modelling

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This assignment will focus on analysis of the the stress state, from a FEM calculation, of the proposed concrete structure under self-weight seen in figure (1).

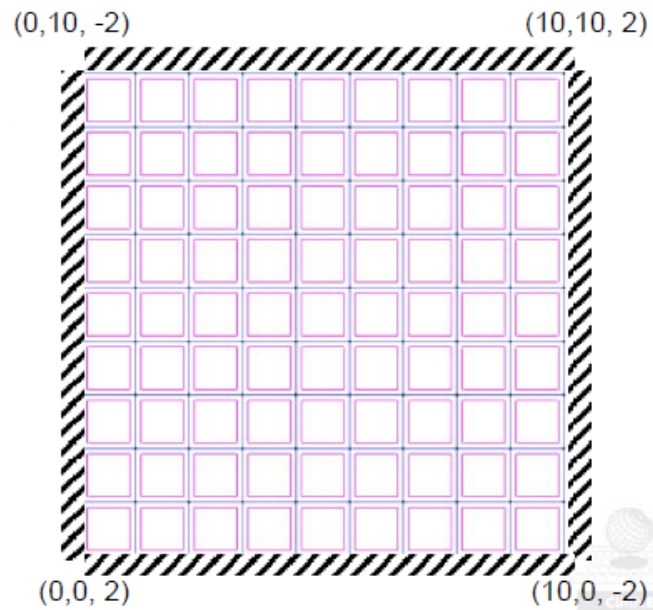


Figure 1: Concrete structure

The geometry is discretized with an structured mesh with quadrilateral elements as seen in figure (2) and the boundary conditions are such that all boundary nodes are clamped (i.e., zero displacements and rotations).

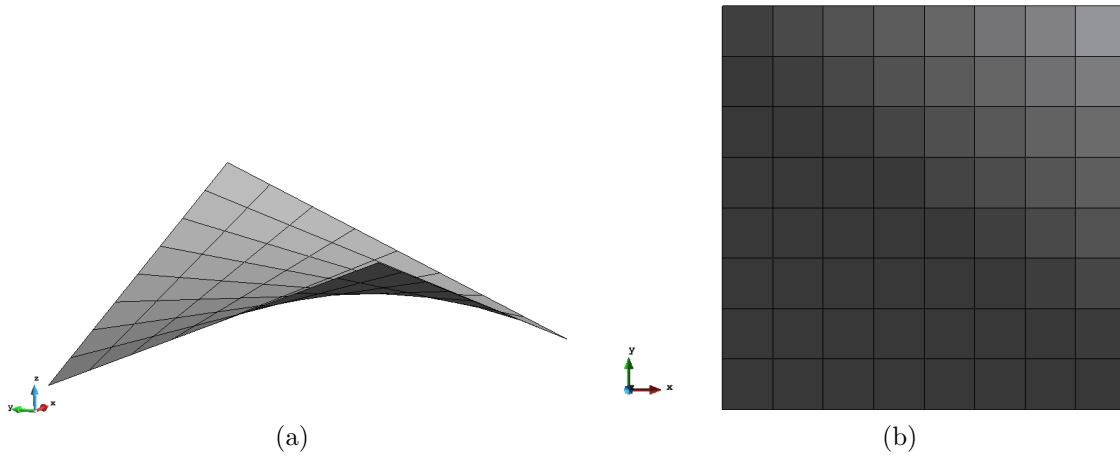


Figure 2: Sample mesh 8x8 elements:(a) 3D arbitrary view; (b) top view.

Before drawing some conclusions towards the stress state of the structure, a mesh convergence test on the z displacement (w) and in the membrane load norm $|N|$ at the middle structure point $(5,5,0)$ is carried out, as seen in figure (3). As it can be seen, a reasonable convergence and asymptotic behavior for displacement and membrane stress is attained for a mesh density of around 1k nodes.

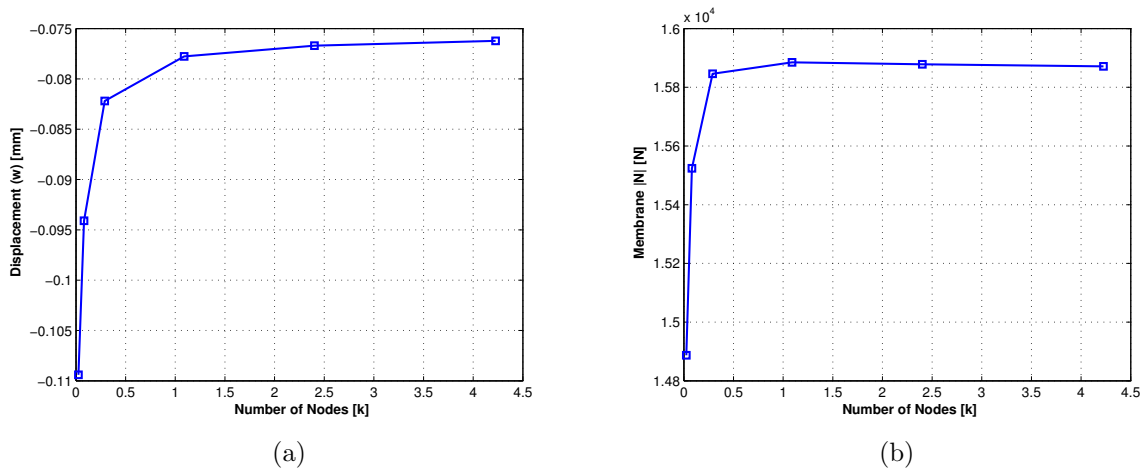


Figure 3: Mesh convergence at point $(5,5,0)$: (a) Displacement in z direction (w); (b) Norm of Membrane loads $|N|$.

Figure (4) shows the results for the displacement (w) and deformed shape for a mesh with 4k nodes (64×64 Quadrilaterals). Besides a 1k nodes mesh is converged, from here all results will refer to the 4k nodes mesh.

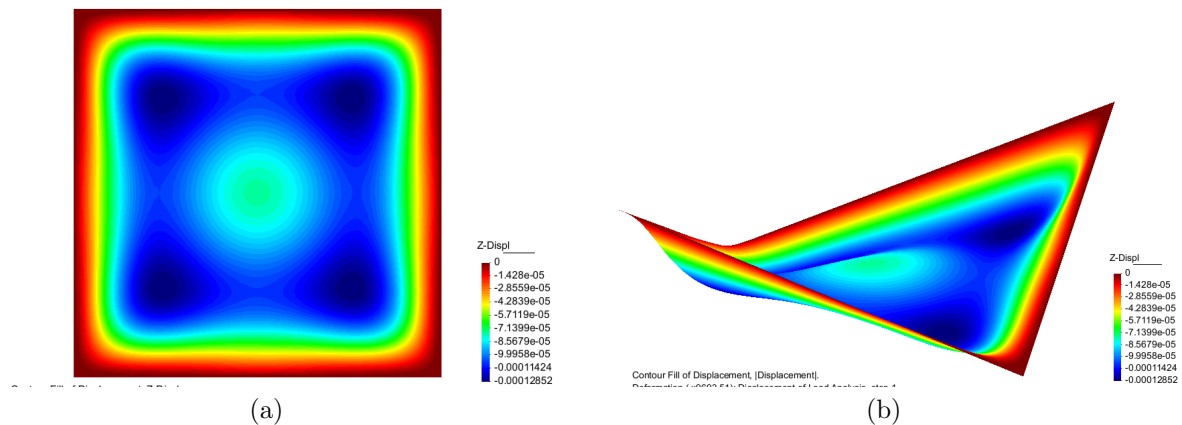


Figure 4: Displacement in z direction (w):(a) Undeformed shape; (b) Deformed Shape.

It can be noticed that due to the structure shape, the maximum displacement does not happen at the middle structure point but in four points located symmetrically with respect to the middle point and four structure corner, as better observed in figure (4) (a).

In order to better understand the internal stresses developed within the structure the rotation and displacements needs to be taken into account. Those displacements and rotations combined with each respective stress will make up the internal work and will balance with the external work done by the body force.

Results for displacements (u, v and w) are shown in Figure (5).

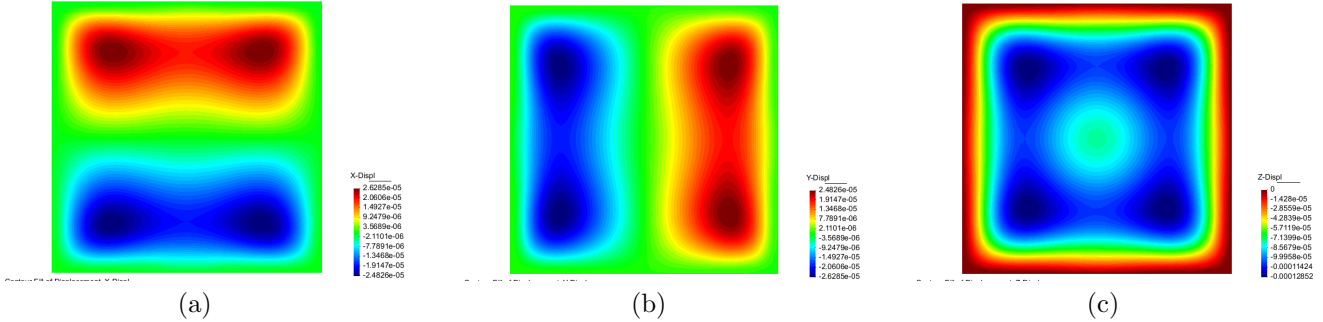


Figure 5: Displacements:(a) u; (b) v;(c) w.

Results for Rotations (θ_x and θ_y) are shown in Figure (6).

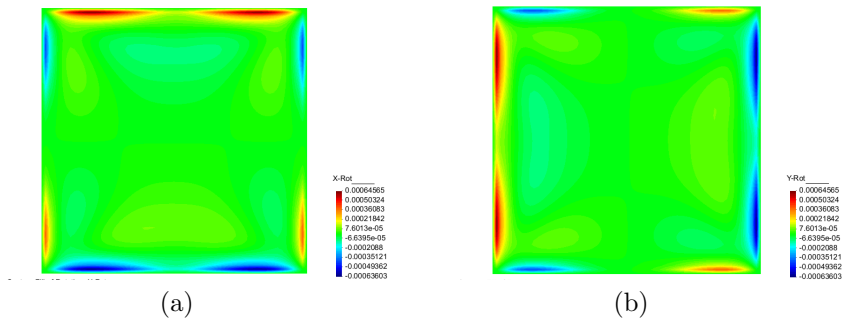


Figure 6: Rotations:(a) θ_x ; (b) θ_y .

Results for bending moment loads are shown in Figure (7).

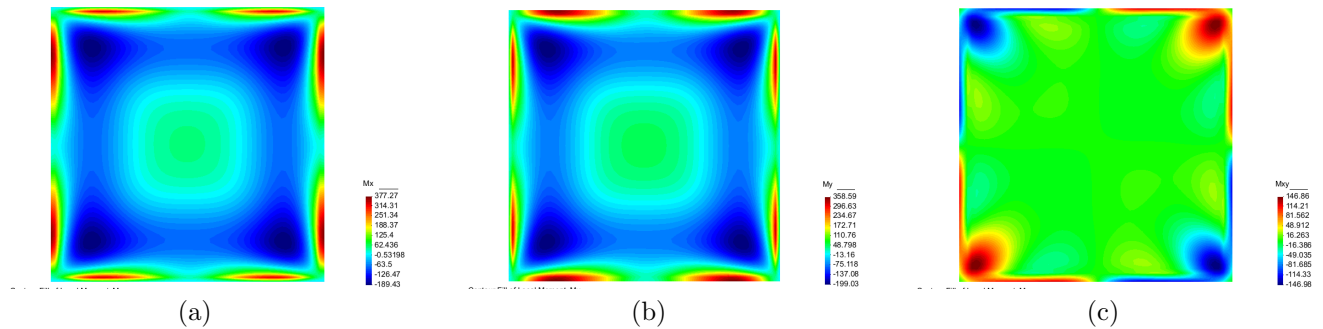


Figure 7: Bending Moment:(a) M_x ; (b) M_y ;(c) M_{xy} .

The bending moments, shown in Figure (7), are related with the variation of the local rotation angle such that the higher the angle change (i.e., variation of θ_x and θ_y with respect to spatial coordinates) the higher will be the stresses due to bending (Here evaluated as moments M_x , M_y and M_{xy}). Thus, the regions where the bending moment M_x and M_y are maximum, are the regions where the variation of rotation angles θ_x and θ_y are higher, as seen in Figure (6). The maximum bending moment lies naturally in the boundaries (support) where the distance from the load "barycenter" is higher. As an antisymmetric body force field combined with an uniform and homogeneous structure has its resultant in the structure center of mass.

It can be also noticed from Figure (7) (a) and (b) that the bending moment M_x and M_y are positive in the boundaries and at the center of the structure. This means tractive stress at those regions, which is in accordance with the load direction and structure form. Further, the points with the minimum bending, are the points with a compressive load and are the points which correspond to the maximum z displacement seen in Figure (5) (c). Regarding M_{xy} , it is a combination of the cross derivative of θ_x and θ_y , and thus is higher where those variations are higher. Due to the structure shape, the M_{xy} will be positive in the corners (0,0,2) and (10,10,2) and negative at the other two corners.

Results for membrane loads are shown in Figure (8).

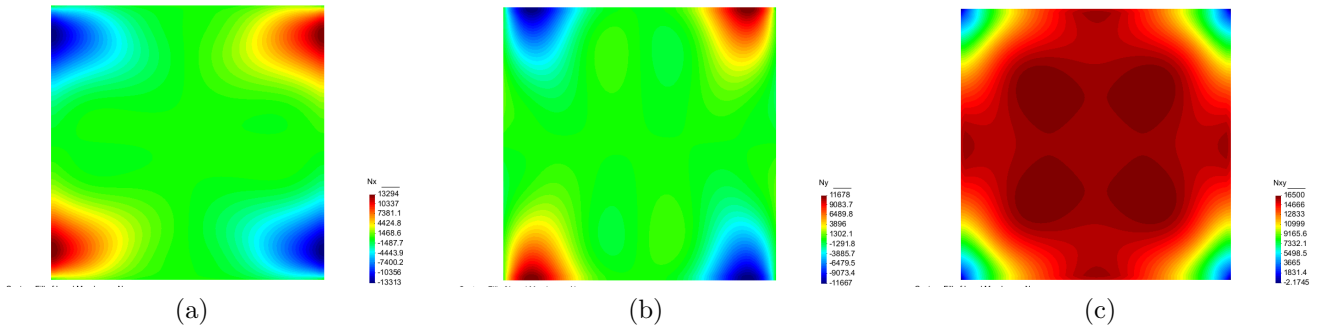


Figure 8: Membrane loads:(a) N_x ; (b) N_y ;(c) N_{xy} .

The membrane loads, shown in Figure (8), are related with the variation of the local axial displacements (elongation) such that the higher displacements variation (i.e., variation of u_o and v_o with respect to spatial coordinates) the higher will be the stresses due to membrane effects (Here evaluated as moments N_x , N_y and N_{xy}). Thus, the regions where the membrane internal loads N_x and N_y are maximum are the regions where the variation of displacements u_o and v_o are higher. As $u_o = u + z\theta_x$ and $v_o = v + z\theta_y$, the variation of u_o and v_o will be composed by the variation of u and θ_x and v and θ_y respectively. Thus, figures (5) and (6) should be considered.

Given the aforementioned, the locations with maximum membrane loads N_x and N_y are the places where the combination of linear displacements and rotation variation are higher as observed in figures (5) and (6). From Figure 8 the locations for maximum N_x and N_y are near the structure corners. It can be further noticed that in the corners (0,0,2) and (10,10,2) the loads N_x and N_y are tractive and in the corners (10,0,-2) and (0,10,-2) they are compressive. This is explained by the shape of the structure, supports type and the direction of the body force field which tends to move the structure downwards, thus points upwards the middle structure plane (parallel to x-y) will be pulled-down and points downwards this symmetry plane will be pushed.

Results for shear forces loads are shown in Figure (9).

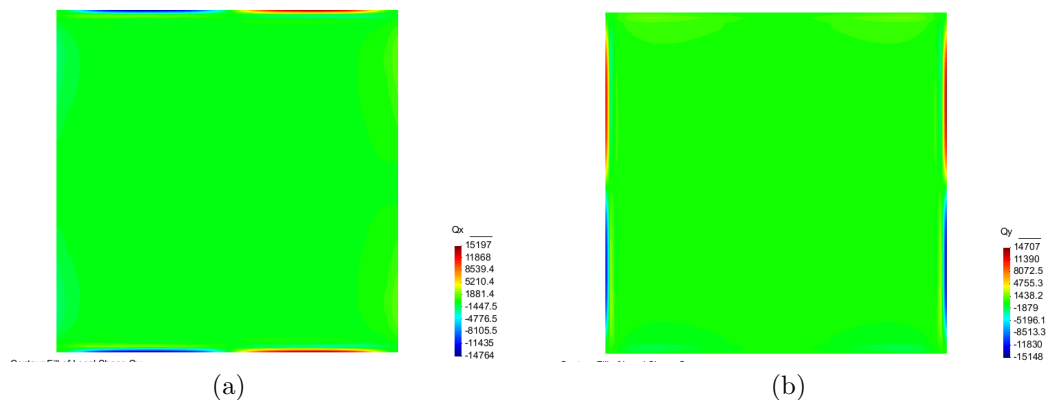


Figure 9: Shear forces:(a) Q_x ; (b) Q_y .

The shear forces Q_x and Q_y are related with the variation of w_o with respect to x and y respectively. Also, θ_x and θ_y components influences the shear forces Q_x and Q_y respectively. Thus with the help of Figures (5) (c) and Figure (6) the results in figure (9) can be understood. Looking at Figure(5) (c) it is clear that the variation of w with respect to x is higher at the two side edges while the variation of w with respect to y is higher near upper and lower edges. This result, in combinations with the rotations, seen in Figure (6) leads to the shear forces found in Figure (9).