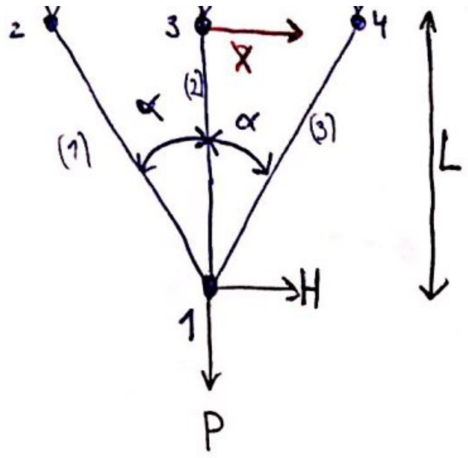
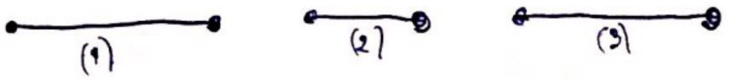


E, A

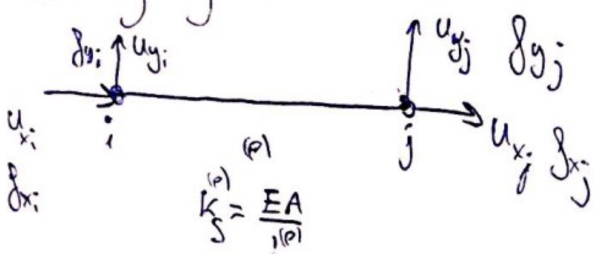
$(1)\ (3) \longrightarrow \dfrac{L}{\cos\alpha}$

starting with the first element

Localizing



considering a general element (e)



$k_s^{(e)} = \frac{EA}{L^{(e)}}$

$$F = k_s^{(e)} d = \frac{EA}{L^{(e)}} d \quad \left[\begin{array}{l} \text{where } d = u'_{x_j} - u'_{x_i} \\ \text{and } F = f'_{x_j} = -f'_{x_i} \end{array} \right.$$

Then:

$$\vec{f}' = \frac{EA}{L^{(e)}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u'_{x_i} \\ u'_{y_i} \\ u'_{x_j} \\ u'_{y_j} \end{pmatrix} \implies \vec{f}' = \bar{\bar{k}}' \vec{u}'$$

Globalizing for each element:

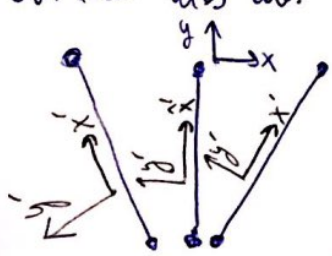
$$K^{(e)} = \left(T^{(e)}\right)^T K^{(e)'} T^{(e)}$$

which substituting the expressions of T^T and T becomes:

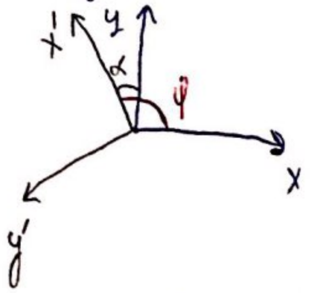
$$K^{(e)} = \frac{EA}{L^{(e)}} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

here $c = \cos\varphi$, $s = \sin\varphi$ (not α) yet

our local axes are:



starting with the first element



$$c = \cos\varphi = \cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$$

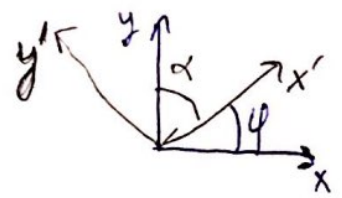
$$s = \sin\varphi = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$

it can be confusing but now we will refere to $\left[\begin{array}{l}\cos\alpha = c\\ \sin\alpha = s\end{array}\right.$

so element stiffness matrix for element (1):

$$K^{(1)} = \frac{EA\cos\alpha}{L}\begin{pmatrix} +s^2 & -sc & -s^2 & sc \\ -sc & c^2 & sc & -c^2 \\ -s^2 & sc & s^2 & -sc \\ sc & -c^2 & -sc & c^2 \end{pmatrix} = \frac{EA}{L}\begin{pmatrix} cs^2 & -sc^2 & -cs^2 & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -cs^2 & sc^2 & cs^2 & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{pmatrix}$$

element (3)



$$\varphi = \frac{\pi}{2}-\alpha$$

$$c = \cos\varphi = \cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$$

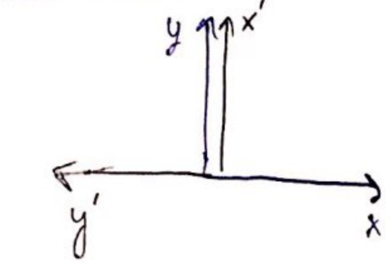
$$s = \sin\varphi = \sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$$

$$K^{(3)} = \frac{EA\cos\alpha}{L}\begin{pmatrix} s^2 & sc & -s^2 & -sc \\ sc & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & sc \\ -sc & -c^2 & sc & c^2 \end{pmatrix}$$

here $\left[\begin{array}{l}c = \cos\alpha\\ s = \sin\alpha\end{array}\right.$

$$K^{(3)} = \frac{EA}{L}\begin{pmatrix} cs^2 & sc^2 & -cs^2 & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2 & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{pmatrix}$$

Element (2)



$\varphi = 90^\circ$

$c = \cos\varphi = \cos 90^\circ = 0$

$s = \sin\varphi = \sin 90^\circ = 1$

$$k^{(2)} = \frac{EA}{L}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

Now we will consider the expanded Matrix!!

$$\vec{u} = \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$k^{(1)}_{ext} = \frac{EA}{L}\begin{pmatrix} cs^2 & -sc^2 & -cs^2 & sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & c^3 & sc^2 & -c^3 & 0 & 0 & 0 & 0 \\ -cs^2 & sc^2 & cs^2 & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$k^{(2)}_{ext} = \frac{EA}{L}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$k^{(3)}_{ext} = \frac{EA}{L}\begin{pmatrix} cs^2 & sc^2 & 0 & 0 & 0 & 0 & -cs^2 & -sc^2 \\ sc^2 & c^3 & 0 & 0 & 0 & 0 & -sc^2 & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2c & -sc^2 & 0 & 0 & 0 & 0 & s^2 & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{pmatrix}$$

Then the global stiffness matrix reads: $k = k^{(1)}_{ext} + k^{(2)}_{ext} + k^{(3)}_{ext}$

$$k = \frac{EA}{L}\begin{pmatrix} 2cs^2 & 0 & -cs^2 & sc^2 & 0 & 0 & -cs^2 & -sc^2 \\ 0 & 2c^3+1 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ -cs^2 & sc^2 & cs^2 & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -s^2c & -sc^2 & 0 & 0 & 0 & 0 & s^2 & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{pmatrix}$$

The only forces applied are in the node 1

$f_{x_1} = H \qquad f_{y_1} = -P \qquad f_{x_i} = f_{y_i} = 0 \qquad i = 2,3,4$

$$\vec{f} = \begin{pmatrix} H \\ -P \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The reason why there is a row and column with 0 is because node 3 if displaced infinitesimally in the $\hat{x}$ direction it will produce no effect!

exercise 2

B.C: $\boxed{U_{x_2} = U_{x_3} = U_{x_4} = U_{y_2} = U_{y_3} = U_{y_4} = 0}$ and $\boxed{F_{x_1} = H \qquad F_{y_1} = -P}$

then:

$$\boxed{\begin{pmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{pmatrix}\begin{pmatrix} U_{x_1} \\ U_{y_1} \end{pmatrix} = \begin{pmatrix} H \\ -P \end{pmatrix}}$$

exercice 3

$$\boxed{U_{x_1} = \frac{H}{2cs^2}} \qquad \boxed{U_{y_1} = \frac{-P}{1+2c^3}}$$

U_{x_1} "blow up" because when $\alpha \to 0$ $s \to 0$ then

$U_{x_1} \to \infty$ if $H \neq 0$

exercice 4

Starting by the end we will focus on by the forces "blow up" when $\alpha \to 0$

The forces are: $F^{(3)} = -\frac{H}{2s} + \frac{Pc^2}{(1+2c^3)}$

when $\alpha \to 0$ $\sin\alpha \to 0 \implies F \to \infty$ so thats why it blows up

Axial Forces:

Element (1)

$$u^{(1)} = \begin{pmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \end{pmatrix} = U^{(1)} = \begin{pmatrix} H/2cs^2 \\ -P/_{1+2c^3} \\ 0 \\ 0 \end{pmatrix}$$

we move to the local axis now:

$$\vec{u}'_{\text{local}} = T\, u_{\text{global}}$$

$$T^{(1)} = \begin{pmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{pmatrix} \quad \text{with} \quad \left[\begin{aligned} c &= \cos\varphi = -\sin\alpha \\ s &= \sin\varphi = \cos\alpha \end{aligned}\right.$$

$$T^{(1)} = \begin{pmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{pmatrix}$$

$$\vec{u}' = \begin{pmatrix} u'_{x1} \\ u'_{y1} \\ u'_{x2} \\ u'_{y2} \end{pmatrix} = \begin{pmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{pmatrix} \begin{pmatrix} H/2cs^2 \\ -P/1+2c^3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{H}{2cs} - \frac{Pc}{1+2c^3} \\ -\frac{H}{2s^2} + \frac{Ps}{1+2c^3} \\ 0 \\ 0 \end{pmatrix}$$

$$F^{(1)} = \frac{EA}{L} c \left[\frac{H}{2cs} + \frac{Pc}{1+2c^3} \right] = \boxed{\frac{EA}{L} \left(\frac{H}{2s} + \frac{Pc^2}{1+2c^3} \right)}$$

$\boxed{F^{(1)} \text{ also "blows up" for the same reason!!}}$

element 3

$$T^{(3)} = \begin{pmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{pmatrix}$$

$c = \cos\varphi = \sin\alpha$

$s = \sin\varphi = \cos\alpha$

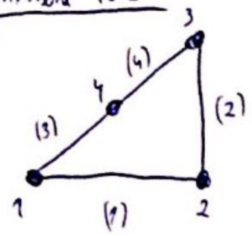
$$T^{(3)} = \begin{pmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{pmatrix}$$

$$U^{(3)} = \begin{pmatrix} U_{x1} \\ U_{y1} \\ U_{x4} \\ U_{y4} \end{pmatrix} = \begin{pmatrix} \frac{H}{2cs^2} \\ \frac{-P}{1+2c^3} \\ 0 \\ 0 \end{pmatrix}$$

$$U' = \begin{pmatrix} U'_{x1} \\ U'_{y1} \\ U'_{x4} \\ U'_{y4} \end{pmatrix} = \begin{pmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{pmatrix} \begin{pmatrix} H/2cs^2 \\ -P/1+2c^3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} H/2cs - \frac{Pc}{1+2c^3} \\ \frac{-H}{2s^2} - \frac{Ps}{1+2c^3} \\ 0 \\ 0 \end{pmatrix}$$

with that $F^{(3)} = \frac{EA}{L} c \left[-\frac{H}{2cs} + \frac{Pc}{1+2c^3} \right] = \boxed{\frac{EA}{L}\left(-\frac{H}{2s} + \frac{Pc^2}{1+2c^3} \right) = F^{(3)}}$

Assignment 1.2



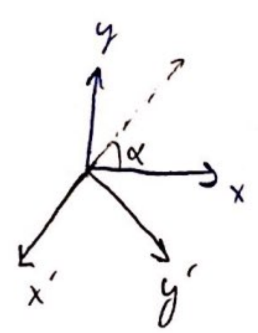
copying $k^{(1)}$ and $k^{(2)}$:

$$k^{(1)} = \begin{pmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad k^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$k^{(3)}$ will be the same but instead of element lenght L we have $L \to L/2$

$$k^{(3)} = \begin{pmatrix} 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 & -0{,}5 & -0{,}5 \\ 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 & -0{,}5 & -0{,}5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0{,}5 & -0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 \\ -0{,}5 & -0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 \end{pmatrix} 10$$

$k^{(4)}$:



$$\varphi = \alpha + \pi$$

$$\begin{cases} S = \sin(\alpha+\pi) = -\sin\alpha \\ C = \cos(\alpha+\pi) = -\cos\alpha \end{cases}$$

$$K^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 & -5 & -5 \\ 0 & 0 & 0 & 0 & 5 & 5 & -5 & -5 \\ 0 & 0 & 0 & 0 & -5 & -5 & 5 & 5 \\ 0 & 0 & 0 & 0 & -5 & -5 & 5 & 5 \end{pmatrix}$$

$$K_{global} = K^{(1)} + K^{(2)} + K^{(3)} + K^{(4)}$$

$$K_{total} = \begin{pmatrix} 15 & 5 & -10 & 0 & 0 & 0 & -5 & -5 \\ 5 & 5 & 0 & 0 & 0 & 0 & -5 & -5 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 & -5 & -5 \\ 0 & 0 & 0 & -5 & 5 & 10 & -5 & -5 \\ -5 & -5 & 0 & 0 & -5 & -5 & 10 & 10 \\ -5 & -5 & 0 & 0 & -5 & -5 & 10 & 10 \end{pmatrix}$$

applying B.C: $\left[\begin{array}{l} U_{x_1} = U_{y_1} = U_{y_2} = 0 \\ f_{x_2} = 0 \quad f_{x_3} = 2 \quad f_{y_3} = 1 \end{array}\right.$

The reduced Master Stiffness Matrix reads:

$$K_{reduced} = \begin{pmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 5 & 5 & -5 & -5 \\ 0 & 5 & 10 & -5 & -5 \\ 0 & -5 & -5 & 10 & 10 \\ 0 & -5 & -5 & 10 & 10 \end{pmatrix}$$

the equation to solve is

$$\hat{K}\hat{u} = \hat{f}$$

given that 4th and 5th rows are linearly dependent the determinant is 0 so the stiffness matrix is not invertible. Then the system can't be solved and is said to "blow up"

Physically, the problem is that the structure is not "stable" it can't sustain itself

to make it possible the 4th node should be constrained so that can't happend.