Direct Stiffness Method

Computational Structural Mechanics and Dynamics

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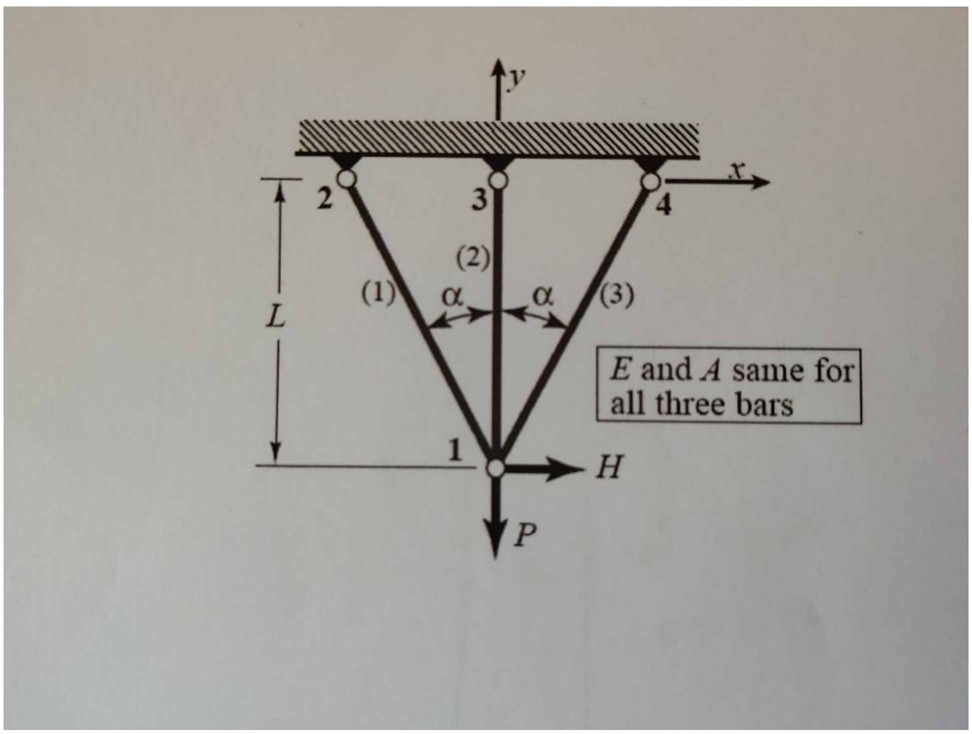
Master in Numerical Methods of Engineering

Assignment 1: Direct Stiffness Method



Part A: The truss problem is well defined in the figure above, the Elasticity of all bars is E and the area of all bars is A.

The stiffness matrices for individual Trusses are as follows:

For truss element 1: The angle made by the element wrt the x axis is $\phi = 90 + \alpha$, therefore $\cos(\phi) = -\sin(\alpha) = c$ and $\sin(\phi) = \cos(\alpha) = s$. The length of this element $L^e = L/\cos(\alpha)$.

The stiffness matrix is given by

$$K_1 = \frac{EA}{L}\begin{bmatrix} cs^2 & -sc^2 & -cs^2 & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -cs^2 & sc^2 & cs^2 & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix}$$

For truss element 2: The angle made by the element wrt the x axis is $\phi = 90$, therefore $\cos(\phi) = 0$ and $\sin(\phi) = 1$. The length of this element $L^e = L$.

$$K_2 = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

For truss element 1: The angle made by the element wrt the x axis is $\phi = 90 - \alpha$, therefore $\cos(\phi) = \sin(\alpha) = c$ and $\sin(\phi) = \cos(\alpha) = s$. The length of this element $L^e = L/\cos(\alpha)$.

$$K_3 = \frac{EA}{L}\begin{bmatrix} cs^2 & sc^2 & -cs^2 & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -cs^2 & -sc^2 & cs^2 & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix}$$

Now we expand these matrices according to the direct stiffness method and add them

$f = f^{(1)} + f^{(2)} + f^{(3)} = (K_1 + K_2 + K_3)u = Ku$

The final stiffness matrix is as follows

$$K = \frac{EA}{L}\begin{pmatrix} 2cs^2 & 0 & -cs^2 & sc^2 & 0 & 0 & -cs^2 & -sc^2 \\ 0 & 1+2c^3 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ -cs^2 & sc^2 & cs^2 & -sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -sc^2 & 0 & 0 & 0 & 0 & cs^2 & sc^2 \\ -sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3 \end{pmatrix}$$

The fifth row and column represent the x displacement of the node three. Since node three is not connected horizontally, it cannot have any horizontal displacement.

The f vector is given by f = [$H \quad -P \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$]^T

Part B.

After applying the BCs we see that since points 2,3 and 4 are fixed their displacement is zero in both directions. Therefore the system reduces to a system of linear equations.

$2cs^2 \frac{EA}{L} u_{x1} = H$ and $(1 + 2c^3) \frac{KA}{L} u_{y1} = -P$

Thus the value of $u_{x1} = \frac{HL}{2EAcs^2}$ and $u_{y1} = \frac{-PL}{EA(1+2c^3)}$

Part C.

When α = 0; cos(α) = 1 and sin(α) = 0 therefore, if H is nonzero the value of u_{x1} approaches to infinity as α approaches zero. In this case, all the bars would align on each other this structure would thus offer maximum resistance to any displacement in the y direction, and the x displacement blows up since a truss element has no resistance for a lateral load. The value of $u_{y1} = \frac{-PL}{3EA}$.

When α = $\frac{\pi}{2}$; cos(α) = 0 and sin(α) = 1 in this case the points 1 and 3 would overlap on top of each other the length of that bar segment would approach zero therefore, u_{x1} approaches zero. The value of $u_{y1} = \frac{-PL}{EA}$

Part D: Calculating the local displacement, then the local elongation and then the axial forces.

<u>Element 1:</u> $u'_{1x} = \frac{-sHL}{2EAcs^2} + \frac{-PL}{EA(1+2c^3)}$; $u'_{2x} = 0$; $d_1 = \frac{HL}{2EAcs} + \frac{PLc}{EA(1+2c^3)}$

Therefore, $F_1 = \frac{H}{2s} + \frac{Pc^2}{(1+2c^3)}$

<u>Element 2:</u> $u'_{1x} = \frac{-PL}{EA(1+2c^3)}$; $u'_{2x} = 0$; $d_2 = = \frac{PL}{EA(1+2c^3)}$

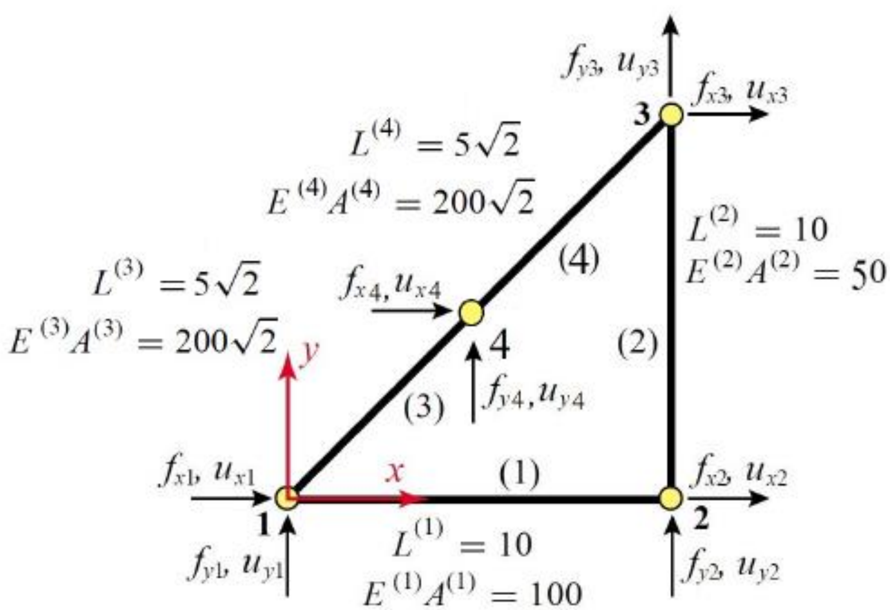
Therefore, $F_2 = \frac{P}{(1+2c^3)}$

Element 3: $u'_{1x} = \frac{sHL}{2EAcs^2} - \frac{cPL}{EA(1+2c^3)}$; $u'_{2x} = 0$; $d_1 = \frac{-HL}{2EAcs} + \frac{PLc}{EA(1+2c^3)}$

Therefore, $F_3 = -\frac{H}{2s} + \frac{Pc^2}{(1+2c^3)}$

When α approaches 0, F_1 and F_3 blow up if $H \neq 0$ because the Truss element has no resistance to displacement in the x direction in this case. The part which blows corresponds to the horizontal component of the force. Since there is no such component in element two, it does not blow up in this case.

Assignment 2:



The stiffness matrices are as follows

$$K_1 = 10\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad K_2 = 5\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \qquad K_3 = K_4 = 20\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

After expanding these, we look at the reduced system of equations which is as follows:

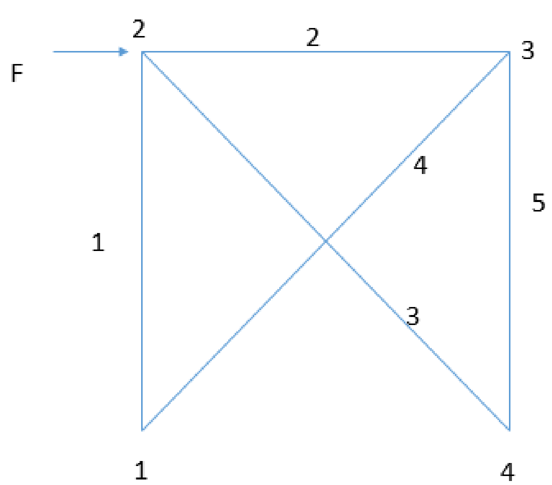
$$\begin{pmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & -5 & 20 & 25 & -20 & -20 \\ 0 & 0 & -20 & -20 & 40 & 40 \\ 0 & 0 & -20 & -20 & 40 & 40 \end{pmatrix} \begin{matrix} \mathrm{ux2} \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{matrix} = \begin{matrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{matrix}$$

This matrix is singular because its fourth and fifth rows and columns are equal to each other. This is because we have the bar properties which are constant along the length and the loads applied are at point nodes. Because of these pre-conditions we only have linear axial displacement. Adding more nodes will not affect the solution. The minimum discretization of truss is enough to find a solution in such cases.

Class Work assignment:

A schematic of the Truss structure is shown below

L = 6m; A = 6cm*cm ; E = 200GPa ; F = 80kN



The stiffness matrices are given by

$K_1 = K_5 = 2\text{x } 10^7 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$ $K_3 = 2\text{x } 10^7 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$K_2 = 0.707\text{x } 10^7 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$ $K_5 = 0.707\text{x } 10^7 \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

Now, the nodes 1 and 4 are fixed therefore we global u is as follows

$U = [0, 0, u_{x2}, u_{y2}, u_{x3}, u_{y3}, 0, 0]^T$

And the global reaction force F is as follows

$F = [f_{x1}, f_{y1}, 80000, 0, 0, 0, f_{x4}, f_{y4}]^T$

The reduced system of equations is as follows has the following stiffness matrix

$$K = 10^7 \begin{bmatrix} 2.707 & -0.707 & -2 & 0 \\ -0.707 & 2.707 & 0 & 0 \\ -2 & 0 & 2.707 & 0.707 \\ 0 & 0 & 0.707 & 2.707 \end{bmatrix}$$

Solving this system we get the values of displacements as

$$U = \begin{matrix} 0.0085 \\ 0.0022 \\ 0.0068 \\ -0.0018 \end{matrix}$$

Therefore the displacements are as follows: u_{x2} = 0.0085m, u_{y2} = 0.0022m, u_{x3} = 0.0068m and

u_{y3} = -0.018 m.