Computational Structural Mechanics And Dynamics

Assignment – 1

Submitted by:

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Assignment-1

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Computational Mechanics

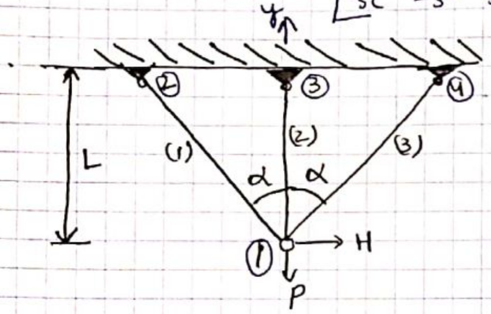
(a) $\bar{u}^e = T^e u^e$; $f^e = (T^e)^T \bar{f}^e$

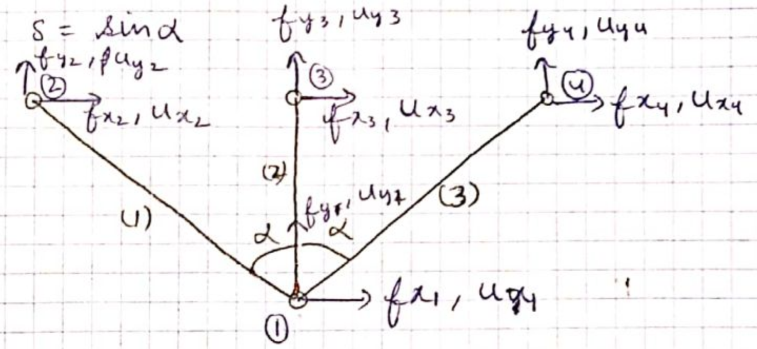
$K^e = (T^e)^T \bar{K}^e T^e$

$$K^e = \frac{E^e A^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$C = \cos\alpha$

$s = \sin\alpha$





$$\frac{L}{h} = \sin(90-\alpha) \Rightarrow h = \frac{L}{\cos\alpha} = \frac{L}{c}$$

$$L^{(1)} = L/c = L^{(3)} \qquad L^{(2)} = L$$

$$f^{(e)} = K^{(e)} u^{(e)} \mapsto lf$$

Element (1)

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EA}{L/c} \begin{array}{c} \overbrace{}^{1} \\ \begin{bmatrix} s^2 & -sc & -s^2 & sc \\ -sc & c^2 & sc & -c^2 \\ -s^2 & sc & s^2 & -sc \\ sc & -c^2 & -sc & c^2 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

~~Element (2)~~

$$K^{(1)} = \frac{EA}{L} \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} cs^2 & -sc^2 & -cs^2 & sc^2 \\ -sc^2 & c^3 & sc^2 & -c^3 \\ -cs^2 & sc^2 & cs^2 & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix} & \begin{array}{c} 1 \\ \\ 2 \\ \end{array} \end{array}$$

Element (2)

$$K^{(2)} = \frac{EA}{L} \begin{array}{cc} \begin{array}{cc} 1 & 3 \end{array} & \\ \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] & \begin{array}{c} 1 \\ \\ 3 \\ \end{array} \end{array}$$

Element (3)

$$K^{(3)} = \frac{EA}{L/c} \begin{array}{cc} \begin{array}{cc} 1 & 3 \end{array} & \\ \left[\begin{array}{cc|cc} s^2 & sc & -s^2 & sc \\ sc & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & sc \\ -sc & -c^2 & sc & c^2 \end{array}\right] & \begin{array}{c} 1 \\ \\ 4 \\ \end{array} \end{array}$$

$$= \frac{EA}{L} \begin{array}{cc} \begin{array}{cc} 1 & 4 \end{array} & \\ \left[\begin{array}{cc|cc} cs^2 & sc^2 & -cs^2 & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -cs^2 & -sc^2 & cs^2 & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{array}\right] & \begin{array}{c} 1 \\ \\ 4 \\ \end{array} \end{array}$$

$$K = K^{(1)} + K^{(2)} + K^{(3)} = \begin{bmatrix} K_{11}^{(1)} + K_{11}^{(2)} + K_{11}^{(3)} & K_{12}^{(1)} & K_{13}^{(2)} & K_{14}^{(3)} \\ K_{21}^{(1)} & K_{22}^{(1)} & 0 & 0 \\ K_{31}^{(2)} & 0 & K_{33}^{(2)} & 0 \\ K_{41}^{(3)} & 0 & 0 & K_{44}^{(3)} \end{bmatrix}$$

$$
K = \frac{EA}{L}\begin{bmatrix}
2cs^2 & 0 & -cs^2 & sc^2 & 0 & 0 & -cs^2 & -sc^2 \\
0 & 2c^3+1 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\
-cs^2 & sc^2 & cs^2 & -sc^2 & 0 & 0 & 0 & 0 \\
sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
-cs^2 & -sc^2 & 0 & 0 & 0 & 0 & cs^2 & sc^2 \\
-sc^2 & -c^3 & 0 & 0 & 0 & 0 & sc^2 & c^3
\end{bmatrix}
$$

$$
K = \frac{EA}{L}\begin{bmatrix}
2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\
 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\
 & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\
 & & & c^3 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 \\
 & \text{symm} & & & & & cs^2 & c^2s \\
 & & & & & & c^2s & c^3
\end{bmatrix}
$$

From the above matrix, it can be seen that 5th row and column contain only 0 because 5th row and column represents x-direction component of 3rd node and bar(2) is vertical. ~~Therefore~~ Now, Bar (2) is loaded ~~as~~ axially and axial direction in this case is along y-axis. So, Row 5th and Column 5th are zeroes.

$$f = K u$$

$$
\begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
= \frac{EA}{L}\begin{bmatrix}
2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\
 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\
 & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\
 & & & c^3 & 0 & 0 & 0 & 0 \\
 & & & & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 \\
 & & & & & & cs^2 & c^2s \\
 & & & & & & & c^3
\end{bmatrix}
\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}
$$

(b) 2, 3 and 4 Nodes are fixed $\therefore$ displacement will be 0 at these nodes.

$$u_{x2} = u_{y2} = u_{x3} = u_{y3} = u_{x4} = u_{y4} = 0$$

$\Rightarrow$ There will be no contribution to global force vector due to displacement BCs.

$\therefore$ Modified Stiffness matrix is

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

(c) $\frac{EA}{L}(2cs^2)\,u_{x1} = H \quad \Rightarrow u_{x1} = \left(\frac{L}{EA}\right)\left(\frac{H}{2cs^2}\right)$

$\frac{EA}{L}(1+2c^3)\,u_{y1} = -P \Rightarrow u_{y1} = \left(\frac{L}{EA}\right)\left(\frac{-P}{1+2c^3}\right)$

$\alpha \to 0 \Rightarrow c \to 1$ and $s \to 0 \Rightarrow cs^2 \to 0$

$\therefore u_{x1} \to \infty$ and $u_{y1} \to \frac{-PL}{3EA}$

When $\alpha \to 0$, nodes 2, 3 and 4 tends to overlap, so instead of 3 bar we can consider a single bar with very high stiffness indicating high resistance to motion ~~for~~ due to external force. But, there will be no resistance to horizontal force H (x-axis) as bars(1) and (3) are now vertical, ~~therefore~~ ~~No~~ $\therefore u_{x1}$ blows up.

$\alpha \to \pi/2 \Rightarrow c \to 0$ and $s \to 1 \Rightarrow cs^2 \to 0$

$\therefore u_{x1} \to \infty$ and $u_{y1} = \left(\frac{L}{EA}\right)(-P)$

When $\alpha \to \frac{\pi}{2}$, length of bar (1) and bar (3) will tend to infinity. Therefore, Nodes 1 and 3 will tend to be coincident. So, u_{x1} will ~~be~~ tend to infinity as ~~However, and~~ stiffness tends to 0 and makes no physical sense.

(d) $\bar{u}^e = T^e u^e$ $\quad T^e = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$

Element (1) $\quad \theta = \alpha + 90°$

$$\bar{u}^{(1)} = T^{(1)} u^{(1)} \Rightarrow \begin{bmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x2} \\ \bar{u}_{y2} \end{bmatrix} = \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -s u_{x1} + c \cdot u_{y1} \\ -c \cdot u_{x1} - s \cdot u_{y1} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(1)} = \bar{u}_{x2} - \bar{u}_{x1} = 0 - (-s \cdot u_{x1} + c \cdot u_{y1}) = s \cdot u_{x1} - c \cdot u_{y1}$$

$$= s \cdot \left(\frac{L}{EA}\right)\left(\frac{H}{2cs^2}\right) - c \cdot \left(\frac{L}{EA}\right)\left(\frac{-P}{1+2c^3}\right)$$

$$= \left(\frac{L}{EA}\right)\left[\frac{H}{2cs} + \frac{Pc}{1+2c^3}\right]$$

$$F^{(1)} = \frac{EAc}{L} \times \frac{L}{EA}\left[\frac{H}{2cs} + \frac{Pc}{1+2c^3}\right] = \frac{H}{2s} + \frac{Pc^2}{1+2c^3}$$

Element (2) $\quad \theta = 90°$

$$\bar{u}^{(2)} = T^{(2)} u^{(2)} \Rightarrow \begin{bmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x3} \\ \bar{u}_{y3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x3} = 0 \\ u_{y3} = 0 \end{bmatrix} = \begin{bmatrix} u_{y1} \\ -u_{x1} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(2)} = \bar{u}_{x3} - \bar{u}_{x1} = 0 - u_{y1} = \left(\frac{L}{EA}\right)\left(\frac{P}{1+2c^3}\right)$$

$$F^{(2)} = \frac{EA}{L}\left(\frac{L}{EA}\right)\left(\frac{P}{1+2c^3}\right) = \frac{P}{1+2c^3}$$

Element (3) $\quad \theta = 90 - \alpha$

$$\bar{u}^{(3)} = T^{(3)} u^{(3)} \Rightarrow \begin{bmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x4} \\ \bar{u}_{y4} \end{bmatrix} = \begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} s \cdot u_{x1} + c u_{y1} \\ -c u_{x1} + s u_{y1} \\ 0 \\ 0 \end{bmatrix}$$

$$d^{(3)} = \bar{u}_{x4} - \bar{u}_{x1} = 0 - (s \cdot u_{x1} + c u_{y1})$$

$$= -s\left(\frac{L}{EA}\right)\left(\frac{H}{2cs^2}\right) + c\left(\frac{L}{EA}\right)\left(\frac{P}{1+2c^3}\right)$$

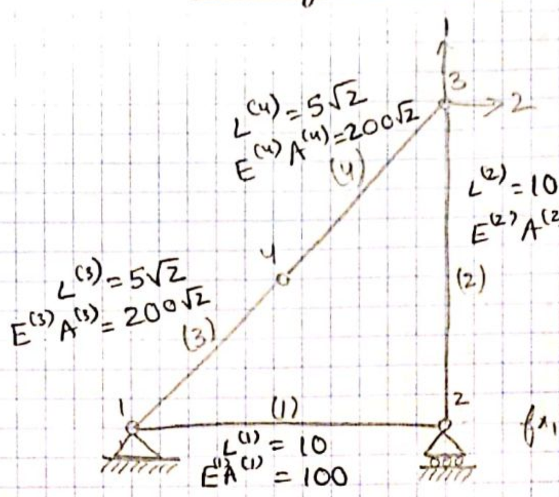
$$= \frac{L}{EA}\left[\frac{-H}{2cs} + \frac{Pc}{1+2c^3}\right]$$

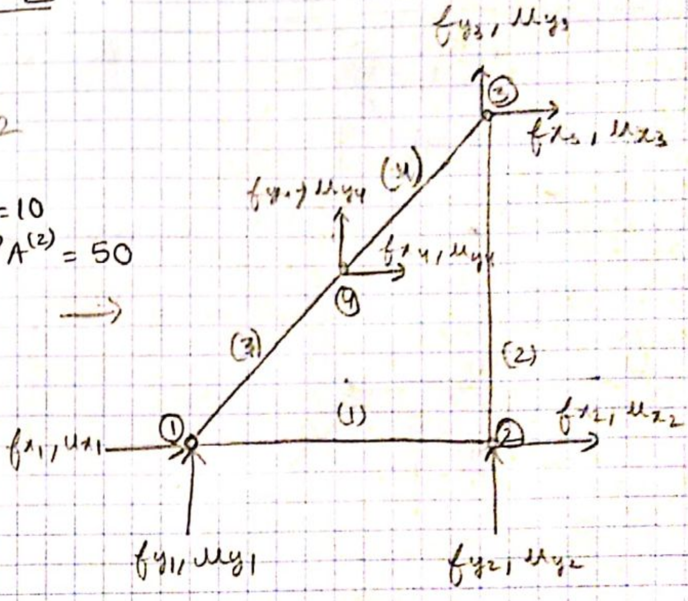
$$F^{(3)} = \frac{EA}{L} c \times \frac{L}{EA}\left[\frac{-H}{2cs} + \frac{Pc}{1+2c^3}\right] = \frac{-H}{2s} + \frac{Pc^2}{1+2c^3}$$

If $\alpha \to 0$ and $H \neq 0$, $s \to 0$ therefore $F^{(1)} \to \infty$ and $F^{(3)} \to \infty$

when $\alpha \to 0$, bars 1, 2 and 3 are coincident, i.e., a vertical truss member. Truss members can only allow axial loading. When a horizontal force H is applied, the truss member does take it into account and hence $F^{(1)}$ and $F^{(3)}$ blows up

Assignment -2





$$K^{(e)} = \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$$K^{(1)} = 10 \begin{array}{c} \begin{array}{cc} \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \\ 2 \end{array} \qquad K^{(2)} = 5 \begin{array}{c} \begin{array}{cc} \quad 2 \quad & \quad 3 \quad \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ \\ 3 \end{array}$$

$$K^{(3)} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} = 20 \begin{array}{c} \begin{array}{cc} \quad 1 \quad & \quad 4 \quad \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \\ 4 \end{array}$$

$$K^{(4)} = 20 \begin{array}{c} \begin{array}{cc} \quad 3 \quad & \quad 4 \quad \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ \\ 4 \end{array}$$

$$\text{Global } K = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & 20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & 20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix}$$

Now $u_{x_1} = u_{y_1} = 0 \qquad u_{y_2} = 0$

$f_{x_3} = 2 \qquad f_{y_3} = 1$

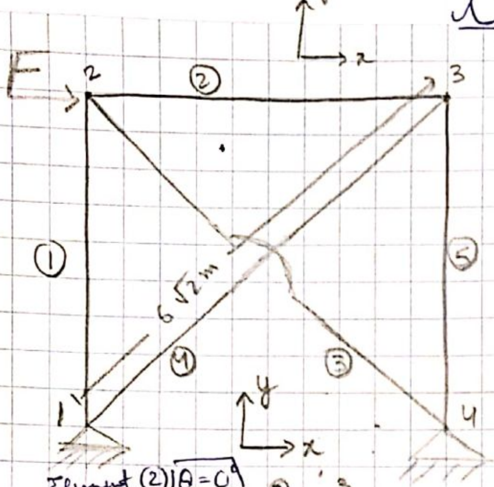
$$K \, u = f$$

Applying these B.C., we get

$$\begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

- This Master Stiffness matrix is singular as its determinant is zero.
- Physically, truss members can't take shear loads and bending moment. and used in a completely triangulated trusses. So, matrix is singular and this 4-noded truss is unstable.

Classwork



$L = 6m$, $E = 200\ GPa = 20 \times 10^9\ Pa$

$F = 80\ kN = 8 \times 10^4\ N$, $A = 6\ cm = 6 \times 10^{-2}\ m$

Element (1) $\theta = 90^\circ$

$$k^{(1)} = \frac{2\times10^9 \times 6\times10^{-2}}{6}\begin{bmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix} = 2\times10^7 \begin{array}{c|c} \begin{array}{cc}1&2\end{array} & \\ \begin{bmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix} & \begin{array}{c}1\\ \\2\\ \end{array}\end{array}$$

Element (2) $\boxed{\theta = 0^\circ}$

$$k^{(2)} = 2\times10^7 \begin{array}{cc} \begin{array}{cc}2&3\end{array} & \\ \begin{bmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{bmatrix} & \begin{array}{c}2\\ \\3\\ \end{array}\end{array}$$

Element (5) $\theta = 90^\circ$

$$k^{(5)} = 2\times10^7 \begin{array}{cc} \begin{array}{cc}3&4\end{array} & \\ \begin{bmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix} & \begin{array}{c}3\\ \\4\\ \end{array}\end{array}$$

Element 3 $\theta = 135^\circ$

$$k^{3} = \frac{2\times10^{+7}}{\sqrt{2}} \times \frac{1}{2}\begin{bmatrix}1&-1&-1&1\\-1&1&1&-1\\-1&1&1&-1\\1&-1&-1&1\end{bmatrix} = \frac{10^7}{\sqrt{2}} \begin{array}{cc} \begin{array}{cc}2&4\end{array} & \\ \begin{bmatrix}1&-1&-1&1\\-1&1&1&-1\\-1&1&1&-1\\1&-1&-1&1\end{bmatrix} & \begin{array}{c}2\\ \\4\\ \end{array}\end{array}$$

Element 4 $\theta = 45^\circ$

$$k^{(4)} = \frac{2\times10^7}{\sqrt{2}} \times \frac{1}{2}\begin{bmatrix}1&1&-1&-1\\1&1&-1&-1\\-1&-1&1&1\\-1&-1&1&1\end{bmatrix} = \frac{10^7}{\sqrt{2}} \begin{array}{cc} \begin{array}{cc}1&3\end{array} & \\ \begin{bmatrix}1&1&-1&-1\\1&1&-1&-1\\-1&-1&1&1\\-1&-1&1&1\end{bmatrix} & \begin{array}{c}1\\ \\3\\ \end{array}\end{array}$$

Global Matrix

$$K = 10^7 \times \begin{bmatrix}
0.707 & 0.707 & 0 & 0 & -0.707 & -0.707 & 0 & 0\\
0.707 & 2.707 & 0 & -2 & -0.707 & -0.707 & 0 & 0\\
0 & 0 & 2.707 & -0.707 & -2 & 0 & -0.707 & 0.707\\
0 & -2 & -0.707 & 2.707 & 0 & 0 & 0.707 & -0.707\\
-0.707 & -0.707 & -2 & 0 & 2.707 & 0.707 & 0 & 0\\
-0.707 & -0.707 & 0 & 0 & 0.707 & +2.707 & 0 & -2\\
0 & 0 & -0.707 & +0.707 & 0 & 0 & 0.707 & -0.707\\
0 & 0 & +0.707 & -0.707 & 0 & -2 & -0.707 & 2.707
\end{bmatrix}$$

B.C.s

$u_{x_1} = u_{y_1} = u_{x_4} = u_{y_4} = 0$; $f_{x_2} = 8\times10^4 N$ $f_{y_2} = f_{x_3} = f_{y_3} = 0$

Imposing these B.C.s, we get. $f = Ku$

$$\begin{bmatrix}8\times10^4\\0\\0\\0\end{bmatrix} = 10^7\begin{bmatrix}2.707 & -0.707 & -2 & 0\\-0.707 & 2.707 & 0 & 0\\-2 & 0 & 2.707 & 0.707\\0 & 0 & 0.707 & 2.707\end{bmatrix}\begin{bmatrix}u_{x_2}\\u_{y_2}\\u_{x_3}\\u_{y_3}\end{bmatrix}$$

$2.707\,u_{x_2} - 0.707\,u_{y_2} - 2u_{x_3} = 8\times10^{-3}$

$-0.707u_{x_2} + 2.707\,u_{y_2} = 0$

$-2\,u_{x_2} + 2.707\,u_{x_3} + 0.707\,u_{y_3} = 8\times10^{-3}$

$0.707\,u_{x_3} + 2.707\,u_{y_3} = 0$

Solving the matrix we get

$$\begin{bmatrix}u_{x_2}\\u_{y_2}\\u_{x_3}\\u_{y_3}\end{bmatrix} = \begin{bmatrix}8.54\times10^{-3}\\2.23\times10^{-3}\\6.77\times10^{-3}\\-1.77\times10^{-3}\end{bmatrix}$$