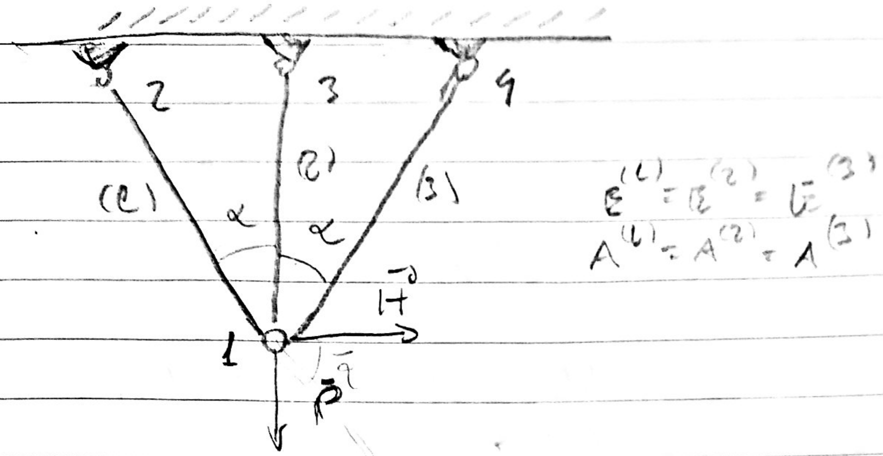
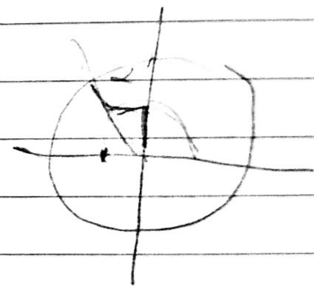
A L

$f = \frac{EA}{L}(u_{x1} - u_x$

Assignment 1.1



1. Find master stiffness equations.
   - 1.1 • member stiffness relation (local coordinates)
   - 1.2 • member stiffness matrix in global coordinates.
   - 1.3. master stiffness equations



1.1
$$\begin{pmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{pmatrix}$$

$\cos(\alpha + 90^\circ) = -$

$\sin(\alpha + 90^\circ) = \cos$

$c \to -s$

$s \to c$

1.2
$$K^{(3)} = \frac{EA.C}{L} \begin{pmatrix} s^2 & sc & -s^2 & -sc \\ sc & c^2 & -sc & -c^2 \\ -s^2 & -sc & s^2 & sc \\ -sc & -c^2 & sc & c^2 \end{pmatrix}$$

$$K^{(2)} = \frac{EA}{L} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$\cos\alpha$

$c \to$

$$K^{(1)} = \frac{EA.C}{L} \begin{pmatrix} s^2 & -sc & -s^2 & sc \\ -sc & c^2 & sc & -c^2 \\ -s^2 & sc & s^2 & -sc \\ sc & -c^2 & -sc & c^2 \end{pmatrix}$$

$$f^{e^T} = [\, f^e_{x1} \;\; f^e_{y1} \;\; f^e_{x2} \;\; f^e_{y2} \;\; f^e_{x3} \;\; f^e_{y3} \;\; f^e_{x4} \;\; f^e_{y4} \,]$$

$$f^T = [\, f_{x1} \;\; f_{y1} \;\; f_{x2} \;\; f_{y2} \;\; f_{x3} \;\; f_{y3} \;\; f_{x4} \;\; f_{y4} \,]$$

where $\underline{f} = f^{(1)} + f^{(2)} + f^{(3)}$

as for instance, $f_{x1} = f^{(1)}_{x1} + f^{(2)}_{x1} + f^{(3)}_{x1}$

$$\begin{pmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \\ f_{x3}^{(1)} \\ f_{y3}^{(1)} \\ f_{x4}^{(1)} \\ f_{y4}^{(1)} \end{pmatrix} = \frac{EAC}{L} \begin{pmatrix} S^2 & -SC & -S^2 & SC & 0 & 0 & 0 & 0 \\ -SC & C^2 & SC & -C^2 & 0 & 0 & 0 & 0 \\ -S^2 & SC & S^2 & -SC & 0 & 0 & 0 & 0 \\ SC & -C^2 & -SC & C^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$\begin{pmatrix} f_{x1}^{(2)} \\ f_{y1}^{(2)} \\ f_{x2}^{(2)} \\ f_{y2}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \\ f_{x4}^{(2)} \\ f_{y4}^{(2)} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

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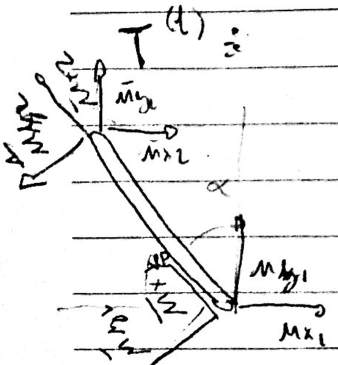
$$\begin{pmatrix} f_{x1}^{(3)} \\ f_{y1}^{(3)} \\ f_{x2}^{(3)} \\ f_{y2}^{(3)} \\ f_{x3}^{(3)} \\ f_{y3}^{(3)} \\ f_{x4}^{(3)} \\ f_{y4}^{(3)} \end{pmatrix} = \frac{EAC}{L} \begin{pmatrix} S^2 & SC & 0 & 0 & 0 & 0 & -S^2 & -SC \\ SC & C^2 & 0 & 0 & 0 & 0 & -SC & -C^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -S^2 & -SC & 0 & 0 & 0 & 0 & S^2 & SC \\ -SC & -C^2 & 0 & 0 & 0 & 0 & SC & C^2 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$\underline{f} = f^{(1)} + f^{(2)} + f^{(3)} = (K_1 + K_2 + K_3)\,\underline{u}$$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 2CS^2 & 0 & -2CS^2 & SC^2 & 0 & 0 & -CS^2 & -SC^2 \\ 0 & (2C^3+1) & SC^2 & -C^3 & 0 & -1 & -SC^2 & -C^3 \\ -CS^2 & SC^2 & CS^2 & -SC^2 & 0 & 0 & 0 & 0 \\ SC^2 & -C^3 & -SC^2 & C^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -CS^2 & -SC^2 & 0 & 0 & 0 & 0 & CS^2 & SC^2 \\ -SC^2 & -C^3 & 0 & 0 & 0 & 0 & SC^2 & C^3 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$K^{(e)} = (T^e)^T \bar{K}^e T^e$$

$T^{(1)}$:



$$f_{x1} = -\sin\alpha\,\bar{f}_{x1} - \cos\alpha\,\bar{f}_{y1}$$
$$f_{y1} = \cos\alpha\,\bar{f}_{x1} - \sin\alpha\,\bar{f}_{y1}$$
$$f_{x2} = -\sin\alpha\,\bar{f}_{x2} - \cos\alpha\,\bar{f}_{y2}$$
$$f_{y2} = \cos\alpha\,\bar{f}_{x2} - \sin\alpha\,\bar{f}_{y2}$$

$$\begin{pmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x2} \\ \bar{u}_{y2} \end{pmatrix} = \overbrace{\begin{pmatrix} -S & C & 0 & 0 \\ -C & -S & 0 & 0 \\ 0 & 0 & -S & C \\ 0 & 0 & -C & -S \end{pmatrix}}^{T^{(1)}} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{pmatrix}$$

$$\begin{pmatrix} f_{x1} \\ f_{x2} \\ f_{y1} \\ f_{y2} \end{pmatrix} = \begin{pmatrix} -S & -C & 0 & 0 \\ C & -S & 0 & 0 \\ 0 & 0 & -S & -C \\ 0 & 0 & C & -S \end{pmatrix} \begin{pmatrix} \bar{f}_{x1} \\ \bar{f}_{y1} \\ \bar{f}_{x2} \\ \bar{f}_{y2} \end{pmatrix}$$

$\rightarrow (T^{(1)})^T$

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$$f^e = K^e \cdot u^e$$

$$K^e = (T^e)^T \cdot \bar{K}^e \cdot T^e$$

$$K^e = \begin{pmatrix} -S & -C & 0 & 0 \\ C & -S & 0 & 0 \\ 0 & 0 & -S & -C \\ 0 & 0 & C & -S \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -S & C & 0 & 0 \\ -C & -S & 0 & 0 \\ 0 & 0 & -S & C \\ 0 & 0 & -C & -S \end{pmatrix}$$

$$= \begin{pmatrix} -S & 0 & S & 0 \\ C & 0 & -C & 0 \\ S & 0 & -S & 0 \\ -C & 0 & C & 0 \end{pmatrix} \begin{pmatrix} -S & C & 0 & 0 \\ -C & -S & 0 & 0 \\ 0 & 0 & -S & C \\ 0 & 0 & -C & -S \end{pmatrix}$$

$$= \begin{pmatrix} S^2 & -SC & -S^2 & SC \\ -SC & C^2 & -SC & -C^2 \\ -S^2 & SC & +S^2 & -SC \\ SC & -C^2 & -SC & C^2 \end{pmatrix}$$ as I wanted to prove

BC ::

$f_{x_1} = H$

$f_{y_1} = -P$

The 5: row and column contains only zeros because no force is influenced by the u_{x3}, horizontal displacement of node 3, even the force in node 3, because u_{x3} is equal zero.

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2. BC:

$u_{x2} = u_{y2} = u_{x3} = u_{y3} = u_{x4} = u_{y4} = 0$

Remaining equations 3 – 8

$f_{x1} = H$
$f_{y1} = -P$

$$\frac{EA}{L}\begin{pmatrix} 2S^2 C & 0 \\ 0 & 2C^3+L \end{pmatrix} \cdot \begin{pmatrix} u_{x1} \\ u_{y1} \end{pmatrix} = \begin{pmatrix} H \\ -P \end{pmatrix}$$

3. $u_{x1} = \frac{H}{2S^2 \cdot C} \cdot \frac{L}{EA}$ $\quad u^T = \frac{L}{EA}\left[\frac{H}{2S^2C} \quad \frac{-P}{(2C^3+L)} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right]$

$u_{y1} = \frac{-P}{(2C^3+L)} \cdot \frac{L}{EA}$

$1 - \cos \quad (1-c^2)c$

$c - c^3 \quad c(1-c^2)$

If $\alpha \to 0$, $S \to 0$, $u_{x1} \to \infty$

If $\alpha \to \frac{\pi}{2}$, $c \to 0$, $u_{x1} \to \infty$

4. $u^{(1)T} = [u_{x1} \; u_{y1} \; u_{x2} \; u_{y2}]$

$$\begin{pmatrix} \bar{u}_{x1} \\ \bar{u}_{y1} \\ \bar{u}_{x2} \\ \bar{u}_{y2} \end{pmatrix} = \begin{pmatrix} -S & C & 0 & 0 \\ C & -S & 0 & 0 \\ 0 & 0 & -S & C \\ 0 & 0 & C & -S \end{pmatrix} \cdot \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{pmatrix} = \bar{u}^{(1)} = \begin{pmatrix} -\frac{H}{2SC} - \frac{P \cdot C}{(2C^3+L)} \\ \frac{H}{2S^2} + \frac{P \cdot S}{(2C^3+L)} \\ 0 \\ 0 \end{pmatrix} \cdot \frac{L}{EA}$$

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$$\bar{F}^{(1)} = \bar{K}_L \cdot \bar{u}$$

$$\bar{F}^{(1)} = \frac{EA}{L}\cdot\frac{L}{EA}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} \frac{-H}{2SC} + \frac{P.C}{(2C^3+L)} \\ \frac{H}{2S^2} + \frac{PS}{2C^3+L} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_{x1}^{(1)} \\ F_{y1}^{(1)} \\ F_{x2}^{(1)} \\ F_{y2}^{(1)} \end{pmatrix}$$

$$F_{x1}^{(1)} = \left(-\frac{H}{2S} + \frac{P.C^2}{2.C^3+L}\right)$$

$$F_{y1}^{(1)} = 0$$

$$F_{x2}^{(1)} = \left(\frac{H}{2S} - \frac{PC^2}{(2.C^3+L)}\right)$$

$$F_{y2}^{(1)} = 0$$

$$u^{(2)} = u_{1x} \quad u_{1y} \quad u_{3x} \quad u_{3y}$$

$$\bar{u}^{(2)} = \begin{pmatrix} \bar{u}_{1x} \\ \bar{u}_{1y} \\ \bar{u}_{3x} \\ \bar{u}_{3} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} \frac{H}{2S^2C} \\ \frac{-P}{2C^3+L} \\ 0 \\ 0 \end{pmatrix}\cdot\frac{L}{EA}$$

$$\bar{u}^{(2)} = \begin{pmatrix} -\frac{P}{2C^3+L} \\ \frac{-H}{2S^2C} \\ 0 \\ 0 \end{pmatrix}\cdot\frac{L}{EA}$$

$$F^{(2)} = \bar{K}\,\bar{u} \qquad F^{(2)} = \frac{EA}{L}\cdot\frac{L}{EA}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} -\frac{P}{2C^3+L} \\ \frac{-H}{2S^2C} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-P}{2C^3+L} \\ 0 \\ +\frac{P}{2C^3+L} \\ 0 \end{pmatrix}$$

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$$\bar{F}^{(2)T} = \left[\frac{-P}{2C^3+1} \quad 0 \quad \frac{P}{2C^3+1} \quad 0 \right]$$

$$F^{(3)} = ?$$

$\bar{M}_3 = \bar{R}\, M_3 \qquad M_3^T = [\, M_{x1}\ M_{y1}\ M_{x4}\ M_{y4} \,]$

$$\bar{M}_3 = \begin{pmatrix} S & C & 0 & 0 \\ -C & S & 0 & 0 \\ 0 & 0 & S & C \\ 0 & 0 & -C & S \end{pmatrix} \begin{pmatrix} \frac{H}{2S^2C} \\ \frac{-P}{2C^3+1} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{H}{2SC} - \frac{P.C}{2C^3+1} \\ -\frac{H}{S^2} - \frac{P.S}{2C^3+1} \\ 0 \\ 0 \end{pmatrix} \cdot \frac{L}{EA}$$

$$\bar{F}^{(3)} = \bar{K} \cdot \bar{u}^{(3)}$$

$$\bar{F}^3 = \frac{EAC}{L} \left(\frac{L}{EA}\right) \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{H}{2SC} - \frac{PC}{2C^3+1} \\ \frac{-H}{S^2} - \frac{PS}{2C^3+1} \\ 0 \\ 0 \end{pmatrix} =$$

$$\bar{F}_{xL}^{(3)} = C \left(\frac{H}{2SC} - \frac{PC}{2C^3+1} \right) = \left(\frac{H}{2S} - \frac{PC^2}{2C^3+L} \right)$$

$$F_{x_2}^{(3)} = C \left(\frac{-H}{2SC} + \frac{PC}{2C^3+1} \right) = \left(\frac{-H}{2S} + \frac{PC^2}{2C^3+L} \right)$$

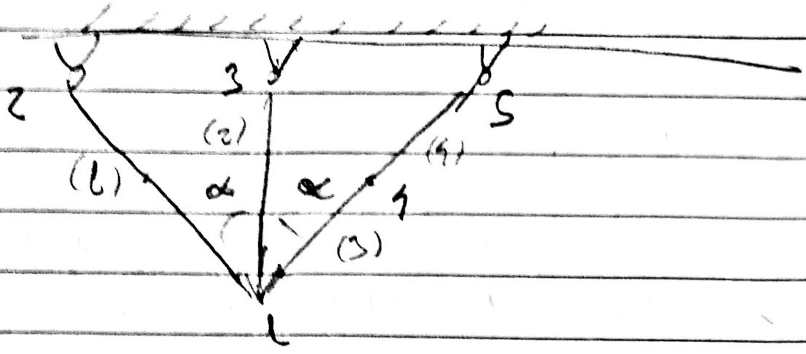
If $\alpha \to 0$, and $H \neq 0$, $F^{(1)} \left(= \frac{H}{2S} - \frac{PC^2}{2C^3+1} \right) \to \infty$

If $\alpha \to 0$, and $H \neq 0$, $F^{(3)} \left(= -\frac{H}{SC} + \frac{PC^2}{2C^3+L} \right) \to -\infty$

$F^{(1)}$ and $F^{(3)}$ "blows up" because the truss won't "hold" the force H wich is perpendicular to all the members of the truss when $\alpha \to 0$.

$c(\alpha)$:

5.



$f^{(1)}$ and $f^{(2)}$ are already known

$$K_3 = K_4 = \begin{pmatrix} S^2 & SC & -S^2 & -SC \\ SC & C^2 & -SC & -C^2 \\ -S^2 & -SC & S^2 & SC \\ -SC & -C^2 & SC & C^2 \end{pmatrix} \cdot \frac{2E \cdot A \cdot C}{L}$$

$$f^{(3)} = \begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x4} \\ f_{y4} \end{pmatrix} = \begin{pmatrix} 2S^2C & 2SC^2 & -2S^2C & -2SC^2 \\ 2SC^2 & 2C^3 & -2SC^2 & -2C^3 \\ -2S^2C & -2SC^2 & 2S^2C & 2SC^2 \\ -2SC^2 & -2C^3 & 2SC^2 & 2C^3 \end{pmatrix} \frac{EA}{L} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x4} \\ u_{y4} \end{pmatrix}$$

$$f^{4} = \begin{pmatrix} f_{x4} \\ f_{y4} \\ f_{x5} \\ f_{y5} \end{pmatrix} \begin{pmatrix} 2S^2C & 2SC^2 & -2SC^2 & -2SC^2 \\ 2SC^2 & 2C^3 & -2SC^2 & -2C^3 \\ -2S^2C & -2SC^2 & 2S^2C & 2SC^2 \\ -2SC^2 & -2C^3 & 2SC^2 & 2C^3 \end{pmatrix} \frac{EA}{L} \begin{pmatrix} u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ f_{y4} \\ f_{x5} \\ f_{y5} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 3S^2C & SC^2 & -S^2C & -SC^2 & 0 & 0 & -2S^2C & -2SC^2 & 0 & 0 \\ SC^2 & 3C^3+1 & SC^2 & -C^3 & 0 & -1 & -2SC^2 & -2C^3 & 0 & 0 \\ -S^2C & SC^2 & S^2C & -SC^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ SC^2 & -C^3 & -SC^2 & C^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -2SC^2 & -2SC^2 & 0 & 0 & 0 & 0 & 4S^2C & 4SC^2 & -2S^2C & -2SC^2 \\ -2SC^2 & -2C^3 & 0 & 0 & 0 & 0 & 4SC^2 & 4C^3 & -2SC^2 & -2C^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2S^2C & -2SC^2 & -2S^2C & 2SC \\ 0 & 0 & 0 & 0 & 0 & 0 & -2SC^2 & -2C^3 & 2SC^2 & 2C^3 \end{pmatrix} \begin{pmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \\ u_{x5} \\ u_{y5} \end{pmatrix}$$

(column groups: -1- -2- -3- -4- -5-)

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$M_{x2} = M_{y2} = M_{x3} = M_{y3} = M_{x5} = M_{y5} = 0$

$$\begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{x4} \\ f_{y4} \end{pmatrix} = \frac{EA}{L} \begin{pmatrix} 3S^2C & SC^2 & -2SC^2 & -2SC^2 \\ SC^2 & 3C^3+1 & -2SC^2 & -2C^3 \\ -2S^2C & -2SC^2 & 4S^2C & 4SC^2 \\ -2SC^2 & -2C^3 & 4SC^2 & 4C^3 \end{pmatrix} \begin{pmatrix} M_{x1} \\ M_{x2} \\ M_{x4} \\ M_{y4} \end{pmatrix} = \begin{pmatrix} H \\ -P \\ f_{x4} \\ f_{y4} \end{pmatrix}$$