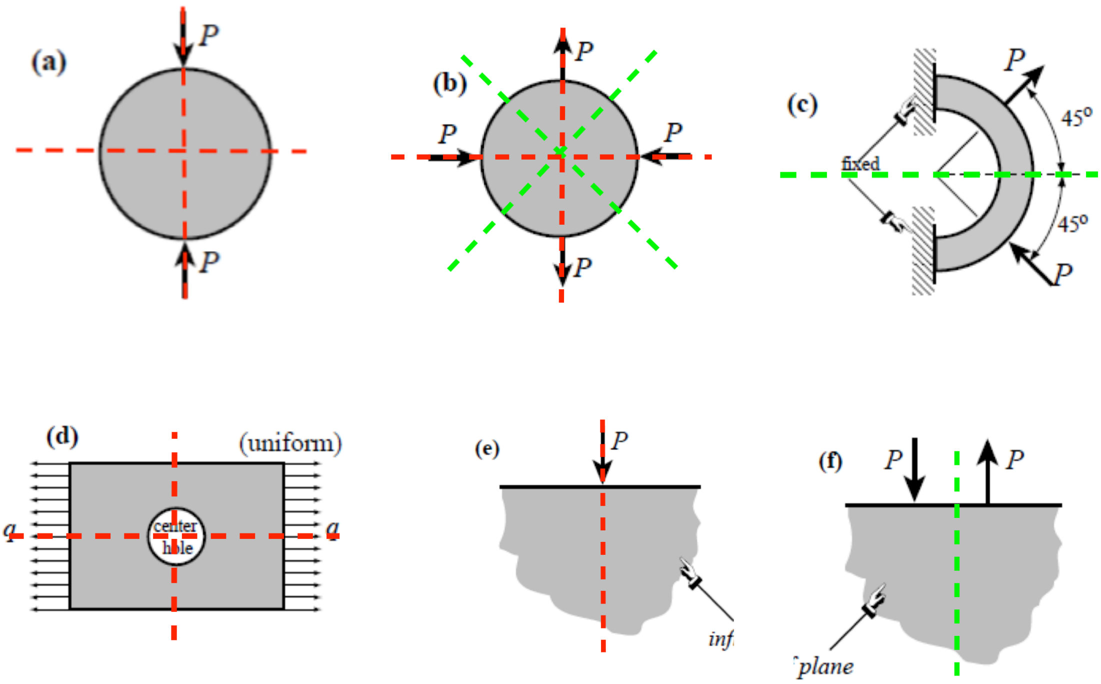
As2-Benjaminsson

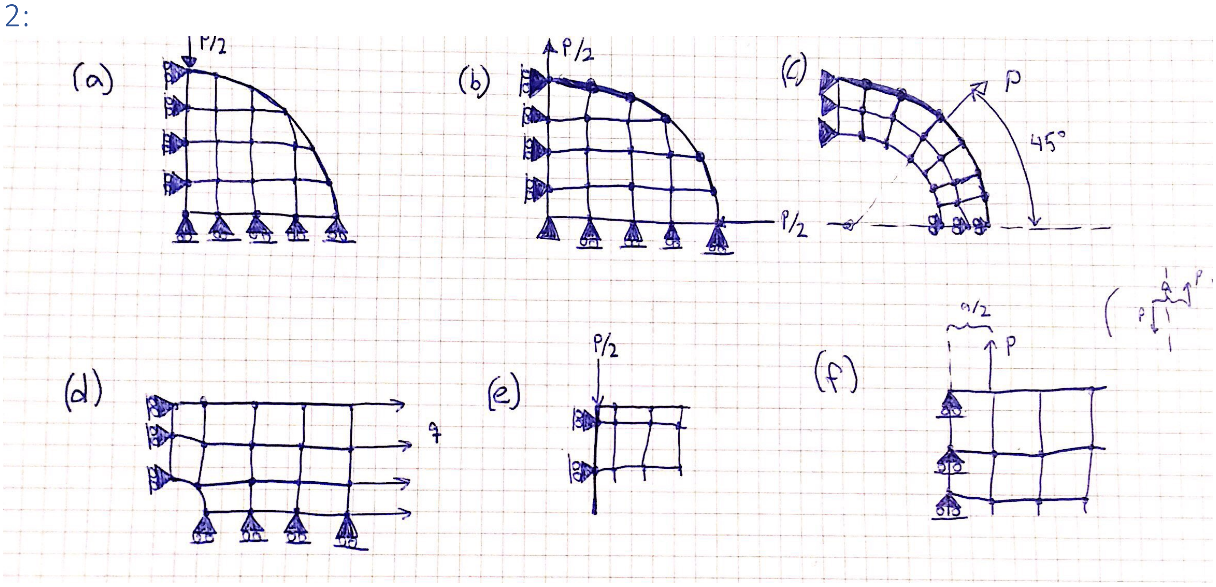
2.1

1:

Below the symmetry and anti symmetri lines are shown. Antisymmetry lines are presented with green color and the symmetry lines with red color.



2:



2.2

There are two groups of trouble spots in the example that would need a more fine mesh. At point D there is a concentrated load with sharp contact area and at N, I there are concentrated reaction points. At points B,M,J,F there are entrant corners and abrupt thickness changes.

2.3

Given:

l, ρ, ω

$A = A_i(1-\xi) + A_j\xi$

$q(x) = \rho A \omega^2 x$

Prismatic case: $A = A_i = A_j$

Solution:

With the nodal force in the x-direction we have

$$f_{ext} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi = \int_0^1 \rho[A_i(1-\xi) + A_j\xi]\omega^2 x \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi = \left\{ x_1 = 0 \rightarrow \xi = \frac{x}{l} \right\} =$$

$$= \int_0^1 \rho[A_i(1-\xi) + A_j\xi]\omega^2 \xi \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l^2 d\xi =$$

$$= \rho l^2 \omega^2 \int_0^1 A_i \left(\begin{bmatrix} \xi - \xi^2 \\ \xi^2 \end{bmatrix} - \begin{bmatrix} \xi^2 - \xi^3 \\ \xi^3 \end{bmatrix} \right) + A_j \begin{bmatrix} \xi^2 - \xi^3 \\ \xi^3 \end{bmatrix} d\xi =$$

$$= \rho l^2 \omega^2 \left[A_i \left(\begin{bmatrix} \frac{\xi^2}{2} - \frac{\xi^3}{3} \\ \frac{\xi^3}{3} \end{bmatrix} - \begin{bmatrix} \frac{\xi^3}{3} - \frac{\xi^4}{4} \\ \frac{\xi^4}{4} \end{bmatrix} \right) + A_j \begin{bmatrix} \frac{\xi^3}{3} - \frac{\xi^4}{4} \\ \frac{\xi^4}{4} \end{bmatrix} \right]_0^1 =$$

$$= \rho l^2 \omega^2 \left[A_i \left(\begin{bmatrix} \frac{1}{2} - \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{3} - \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} \right) + A_j \begin{bmatrix} \frac{1}{3} - \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} \right] =$$

$$= \rho l^2 \omega^2 \left[A_i \begin{bmatrix} \frac{1}{12} \\ \frac{1}{12} \end{bmatrix} + A_j \begin{bmatrix} \frac{1}{12} \\ \frac{1}{4} \end{bmatrix} \right].$$

For a prismatic bar with $A = A_i = A_j$ the consistent nodal force vector becomes

$$f_{ext} = \rho l^2 \omega^2 A \begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix}.$$