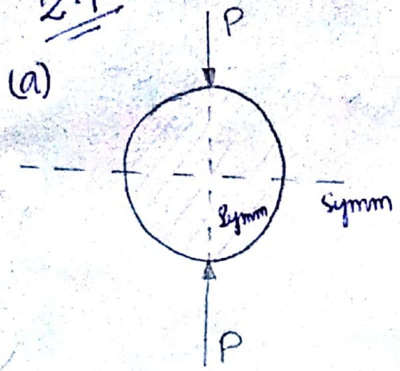
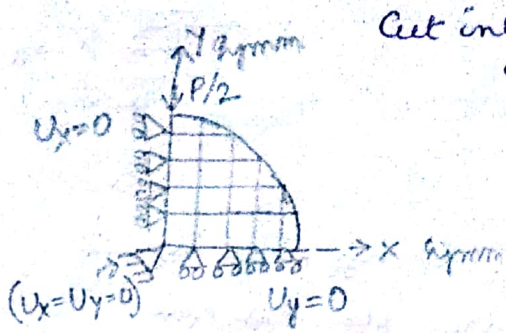
Assignment - 02 - Anurag Bhattacharjee

2.1

(a)

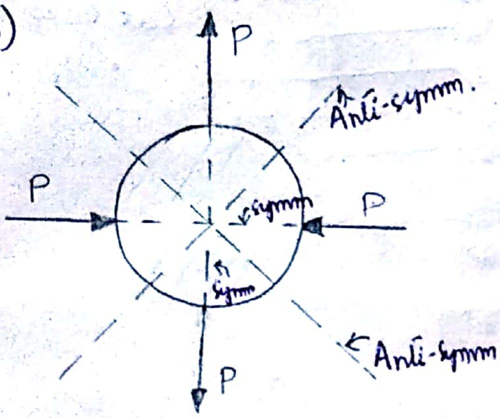


Circular disk with 2 diametrically opposite force

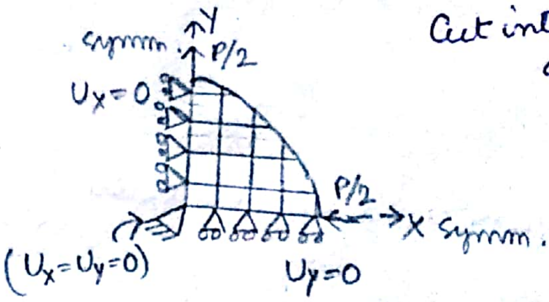


Cut into one-quarter

(b)



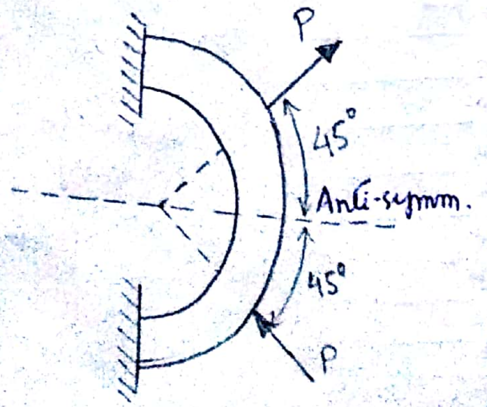
Circular disk with 2 diametrically opposite force pairs



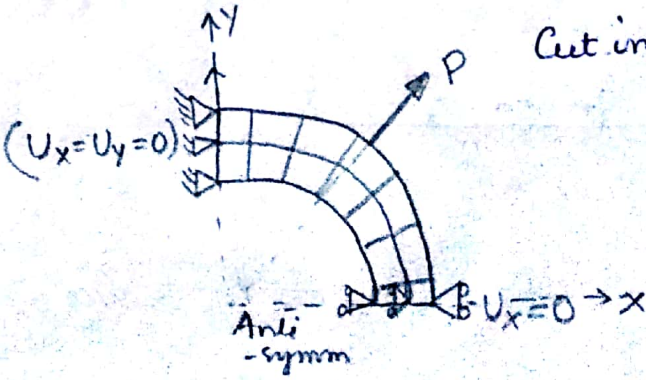
Cut into one-quarter

This can be further divided into a one-eighth part with zero displacement along anti-symmetric line.

(c)

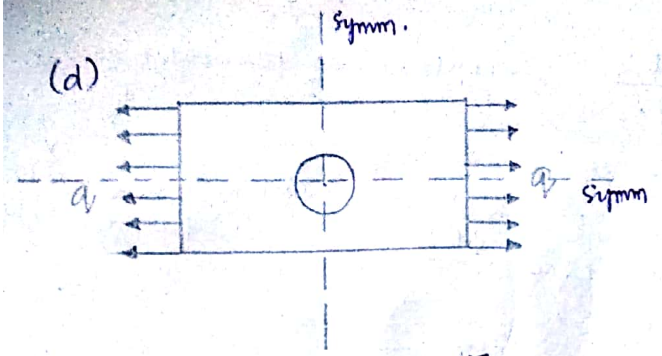


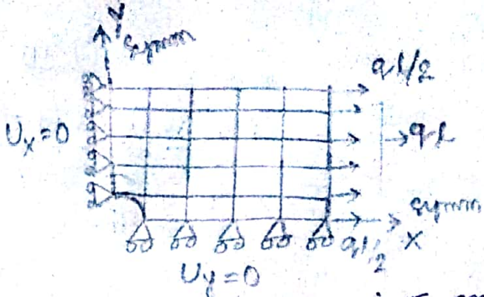
Clamped semiannulus



Cut into half

(d)

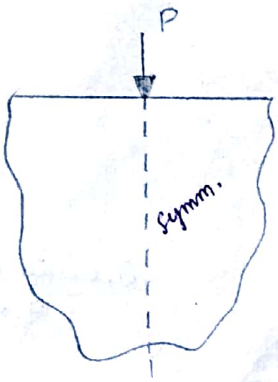


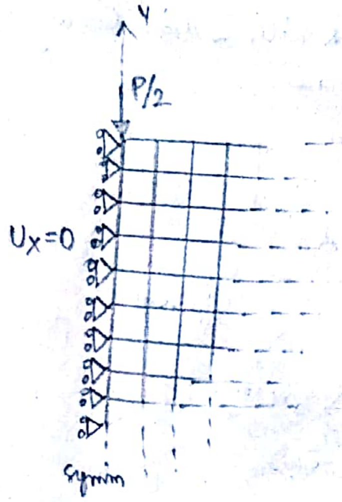


Cut into one-quarter

Stretched rectangular plate with central circular hole

(e)

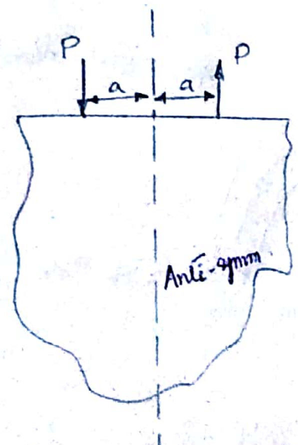


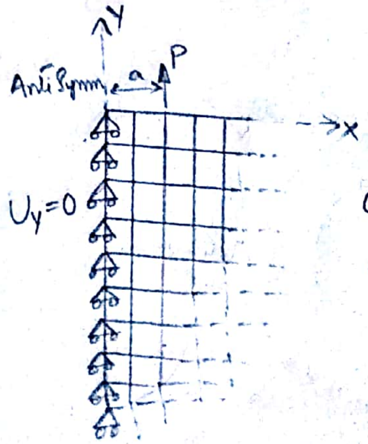


Cut into half

Half plane under concentrated load

(f)



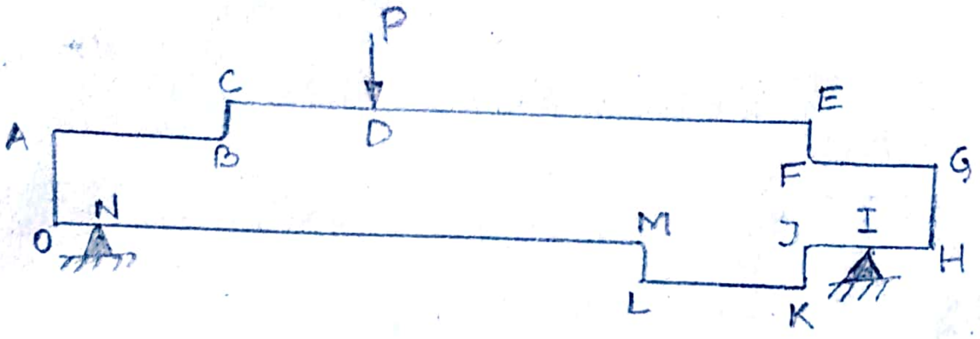


Cut into half

Half plane under concentrated load

* We need to put a constraint in the infinite plates in (e) & (f) as finite element analysis favors finite length models & not infinite models.

2.2



The spots in the above figure that would require a locally finer mesh are as follows –

(i) Point - D (Concentrated load at D)

(ii) Points - N & I (Points where reaction forces will act)

(iii) Points - B, F, J & M (Re-entrant points - Although they are not sharp edged, still they are likely to have a high stress concentration)

2.3

$A = A_i(1-\xi_s) + A_j \xi_s$

$q(x) = \rho A \omega^2 x$

$\therefore$ Nodal force, $f^e = q \int_0^l \begin{bmatrix} 1-\xi_s \\ \xi_s \end{bmatrix} dx$

$$= \int_0^l \rho A \omega^2 x \begin{bmatrix} 1-\xi_s \\ \xi_s \end{bmatrix} dx \quad \text{— (1)}$$

Now, $\xi_s = \frac{x - x_i}{l}$

$\Rightarrow x = \xi_s l + x_i$

$\Rightarrow dx = l \, d\xi_s$

$\therefore$ From (1),

$$f^e = \int_0^1 \rho A \omega^2 (\xi_s l + x_i) \begin{bmatrix} 1-\xi_s \\ \xi_s \end{bmatrix} l \, d\xi_s$$

For $x_i = 0$,

$$f^e = \int_0^1 \rho A \omega^2 (\xi_s l) \begin{bmatrix} 1-\xi_s \\ \xi_s \end{bmatrix} l \, d\xi_s$$

Substituting the value of A, we get

$$f^e = \int_0^1 \rho \omega^2 l^2 \left[A_i(1-\xi_s) + A_j \xi_s\right] \xi_s \begin{bmatrix} 1-\xi_s \\ \xi_s \end{bmatrix} d\xi_s$$

$$= \int_0^1 \rho \omega^2 l^2 \left[A_i(1-\xi_s) + A_j \xi_s\right] \begin{bmatrix} \xi_s - \xi_s^2 \\ \xi_s^2 \end{bmatrix} d\xi_s$$

$$= \int_0^1 \rho \omega^2 l^2 \begin{bmatrix} A_i \xi_s - \xi_s^2(2A_i - A_j) + \xi_s^3(A_i - A_j) \\ A_i(\xi_s^2 - \xi_s^3) + A_j \xi_s^3 \end{bmatrix} d\xi_s$$

$$= \rho \omega^2 l^2 \begin{bmatrix} A_i \frac{\xi_s^2}{2} - \frac{\xi_s^3}{3}(2A_i - A_j) + \frac{\xi_s^4}{4}(A_i - A_j) \\ A_i\left(\xi_s^3/3 - \xi_s^4/4\right) + A_j \xi_s^4/4 \end{bmatrix}_0^1$$

$$= \rho\omega^2 l^2 \begin{bmatrix} \frac{A_i}{2} - \left(\frac{2A_i - A_j}{3}\right) + \frac{(A_i - A_j)}{4} \\ A_i\left(\frac{1}{3} - \frac{1}{4}\right) + A_j/4 \end{bmatrix}$$

$$\boxed{f^e = \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}(A_i + A_j) \\ \frac{A_i}{12} + A_j/4 \end{bmatrix}}$$

This is the required expression of consistent nodal forces.

For a prismatic bar,

$A = A_i = A_j$

$$\therefore f^e = \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}(A + A) \\ A\left(\frac{1}{12} + \frac{1}{4}\right) \end{bmatrix}$$

$$\boxed{f^e = \rho\omega^2 l^2 \begin{bmatrix} A/6 \\ A/3 \end{bmatrix}}$$