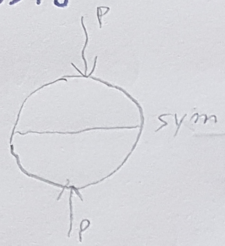
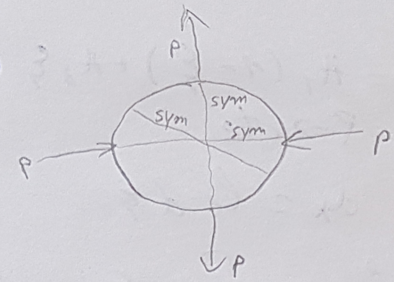
Assigment 2

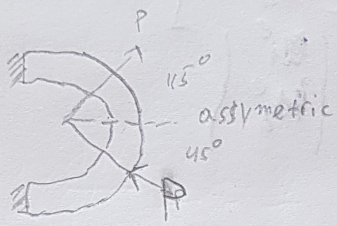
2-1 a)



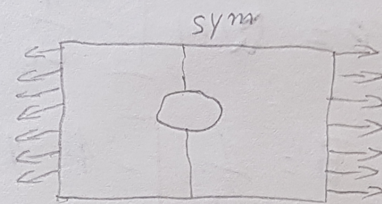
b)



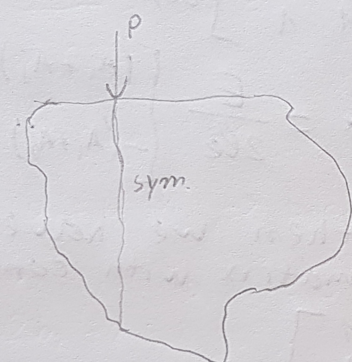
c)



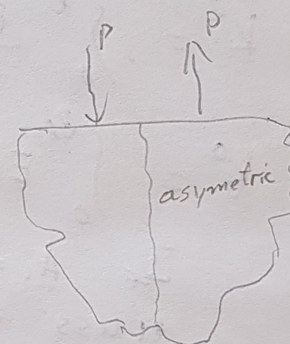
d)



e)



f)



2-2

1- N, D, I -> concentrated loads
B, M ->

2.3

$$A = A_1(1-\xi) + A_j\,\xi \qquad \xi = \frac{x - x_1}{l_e}$$

$\rho \to cte$

$$dx = l_e\, d\xi$$

$$u_e^e(x) = \begin{bmatrix} 1-\xi & \xi \end{bmatrix} \begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}$$

$$e = u'(x) \begin{bmatrix} -\frac{1}{L_e} & \frac{1}{L_e} \end{bmatrix} \begin{vmatrix} u_1^e \\ u_2^e \end{vmatrix}$$

$$B = \frac{1}{l_e}\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$K^e = \int_0^l \frac{E_e A_e}{l_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx$$

$$K^e = \int_0^{l_e} EA(x) . B^T . B\, dx = \frac{E}{2l_e} \begin{bmatrix} (A_i + A_j) & -(A_j + A_j) \\ -(A_i + A_j) & (A_i + A_j) \end{bmatrix}$$

If $A_i = A_j = A$, then we have the truss stiffness matrix with constant ones.

$$K^e = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The nodal forces are given by:

$$W = \int_0^{L_e} q\, M^T dx, \text{ where } q(x) = \rho A(x) w^2 x$$

Then:

$$W = \int_0^{l_e} \rho A(x) w^2 x \,. \begin{bmatrix} 1-\xi & \xi \end{bmatrix}^T \begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix} dx$$

$$W = \rho w^2 \int_0^{l_e} x \begin{bmatrix} 1-\xi & \xi \end{bmatrix} \begin{Bmatrix} A_i \\ A_j \end{Bmatrix} \begin{bmatrix} 1-\xi & \xi \end{bmatrix} \begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix}^T dx$$

$$W = \rho w^2 \begin{Bmatrix} u^e \end{Bmatrix}^T \int_0^{l_e} x \begin{bmatrix} 1-\xi & \xi \end{bmatrix} \begin{Bmatrix} A_i \\ A_j \end{Bmatrix} \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} dx$$

$$f_e = \frac{\rho w^2 l_e^2}{12} \begin{Bmatrix} A_i + A_j \\ A_i + 3A_j \end{Bmatrix} \quad \text{when } A = A_i = A_j$$

$$f_e = \rho w^2 l_e^2 A \begin{Bmatrix} 1/2 \\ 1/2 \end{Bmatrix}$$