Universitat Politècnica De Catalunya, Barcelona

MSc. Computational Mechanics Erasmus Mundus

Assignment 2: FEM Modelling and Variational Formulation

Computational Structural Mechanics & Dynamics

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Assignment 2.1

On "FEM Modelling: Introduction":

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete).

(b) the same disk under two diametrically opposite force pairs.

(c) a clamped semiannulus under a force pair oriented as shown.

(d) a stretched rectangular plate with a central circular hole.

(e) and (f) are half-planes under concentrated loads.

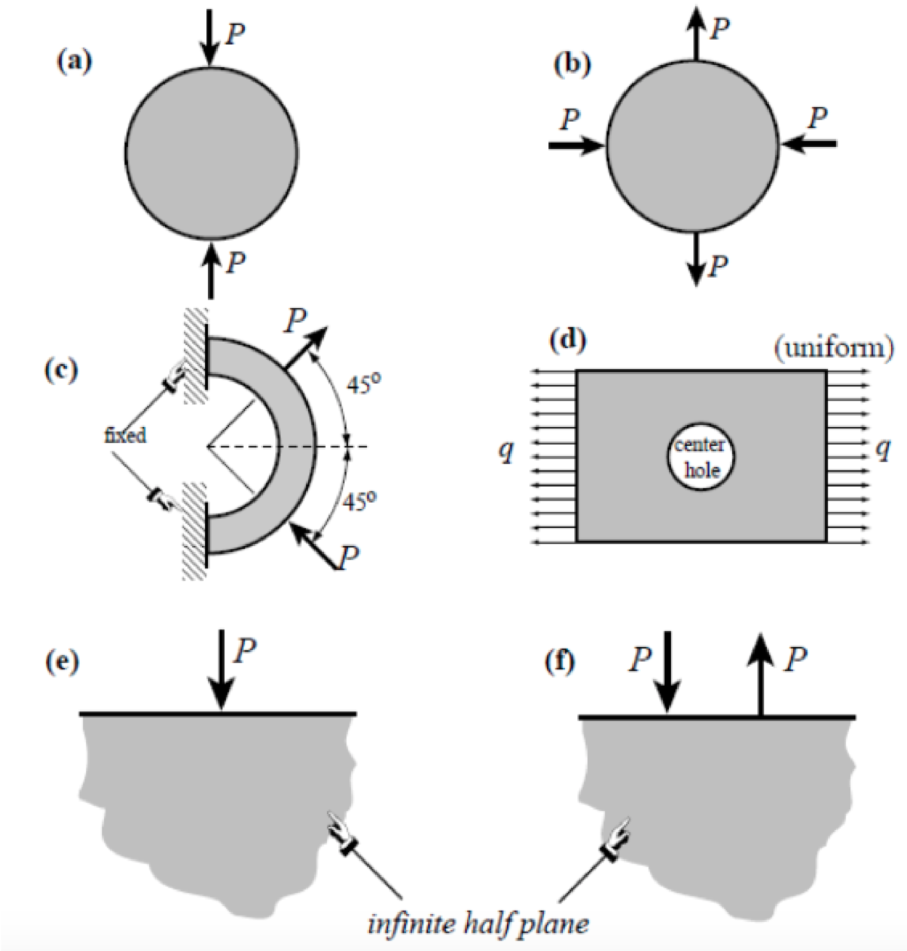

Figure 1: Problems for assignment 2.1

Solution: The symmetry and antisymmetry lines are identified for the given two-dimensional problems. Figure 2 shows the symmetry (dotted blue lines) and anti-symmetry (dotted green lines) for each given problem. We observe that symmetry exists in problems (a), (d) and (e) while the antisymmetry could be utilised in problems (c) and (f). Problem (b) gives us an option to exploit either the symmetry or antisymmetry property of the circular disk under two diametrically opposite force pairs.

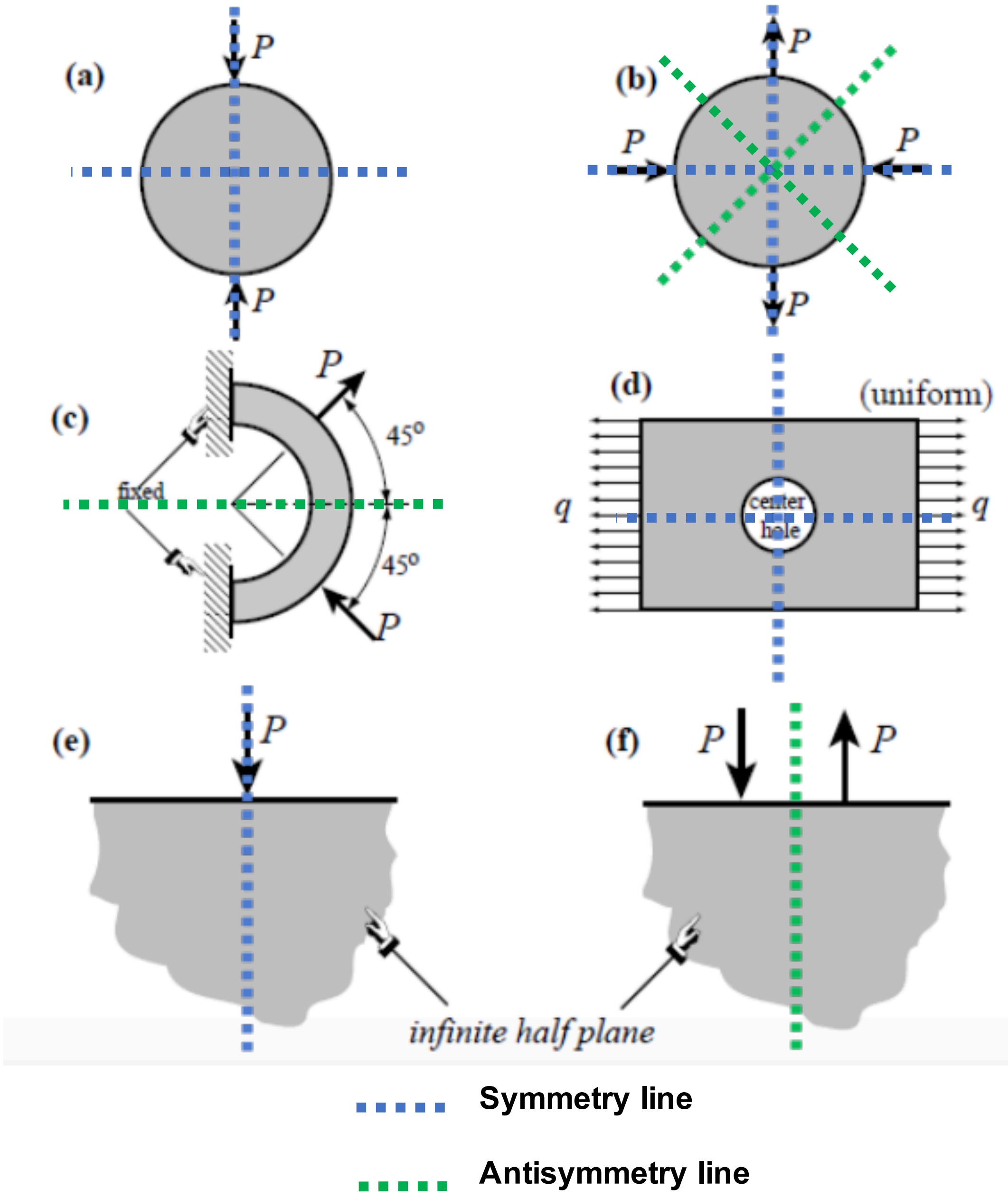

Figure 2: Symmetry and antisymmetry lines in the two-dimensional problems

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

Solution: It is very useful to exploit the symmetry or antisymmetry of a problem for analysing the structural system. With proper specification of the boundary conditions and loading, the structures given in each problem could be cut along the symmetry and antisymmetry lines and a finite element mesh can be generated to solve half or quarter model as shown in Figures 3-8. The meshes shown in blue and green are laid out using the symmetry and antisymmetry of the problem respectively. The orientation of the roller or fixed supports along the symmetry and antisymmetry lines are recognised by the displacement patterns of the structure. This approach reduces the size of the model leading to reduction in computational cost.

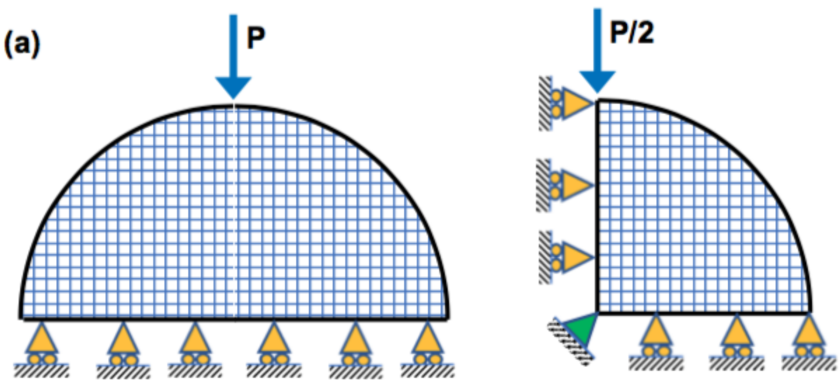

Figure 3: Half and quarter model for problem (a) using symmetry

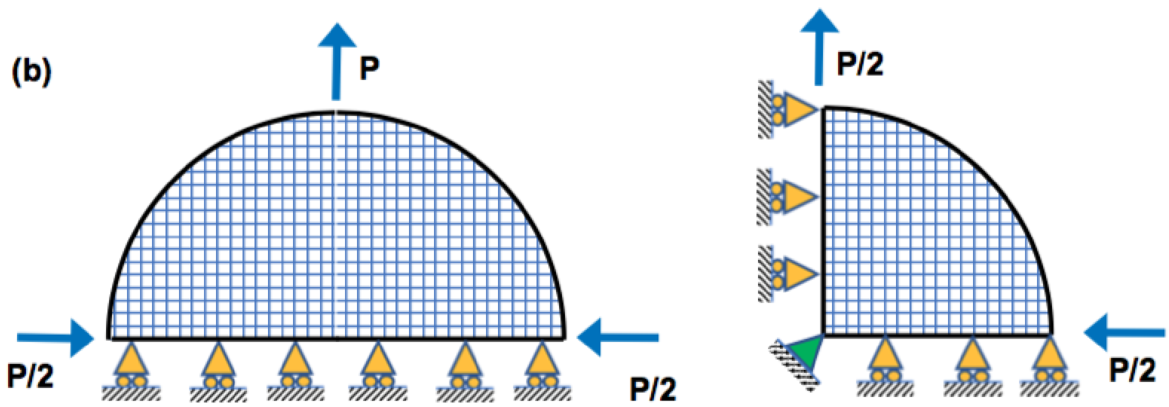

Figure 4: Half and quarter model for problem (b) using symmetry

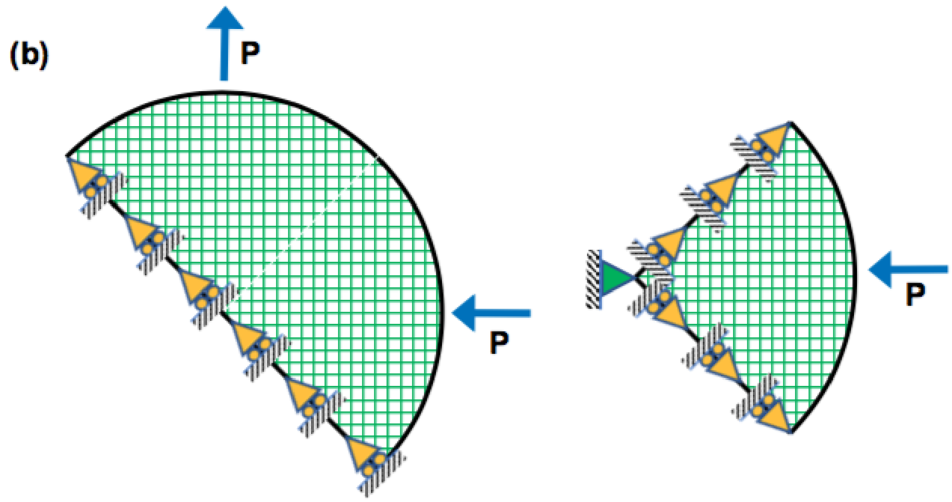

Figure 5: Half and quarter model for problem (b) using antisymmetry

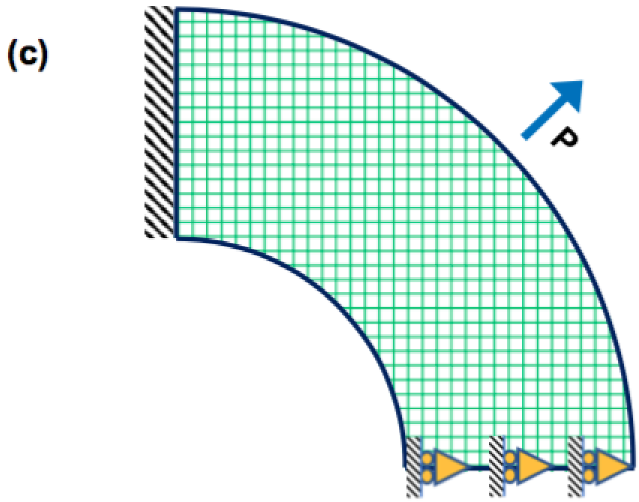

Figure 6: Half model for problem (c) using antisymmetry

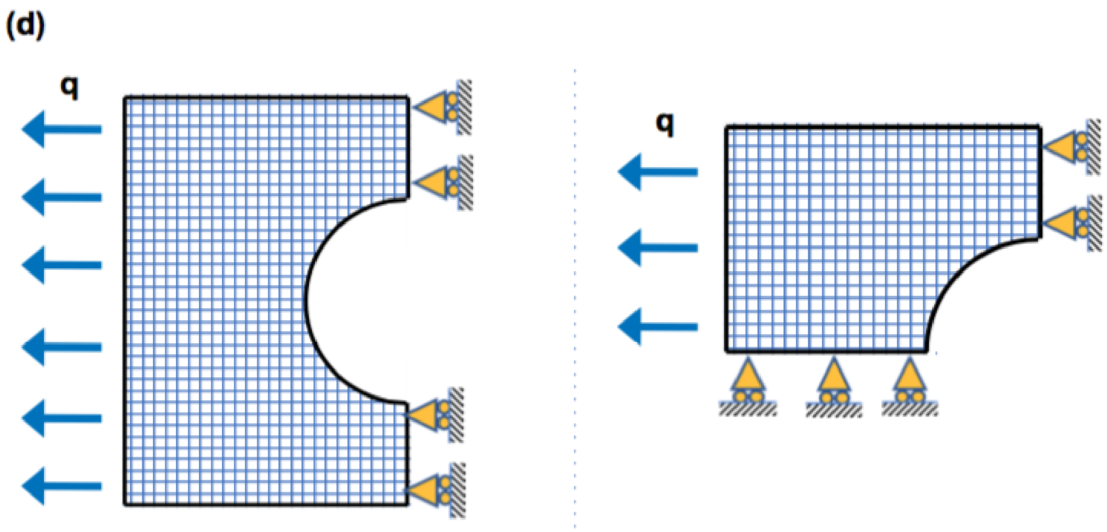

Figure 7: Half and quarter model for problem (d) using symmetry

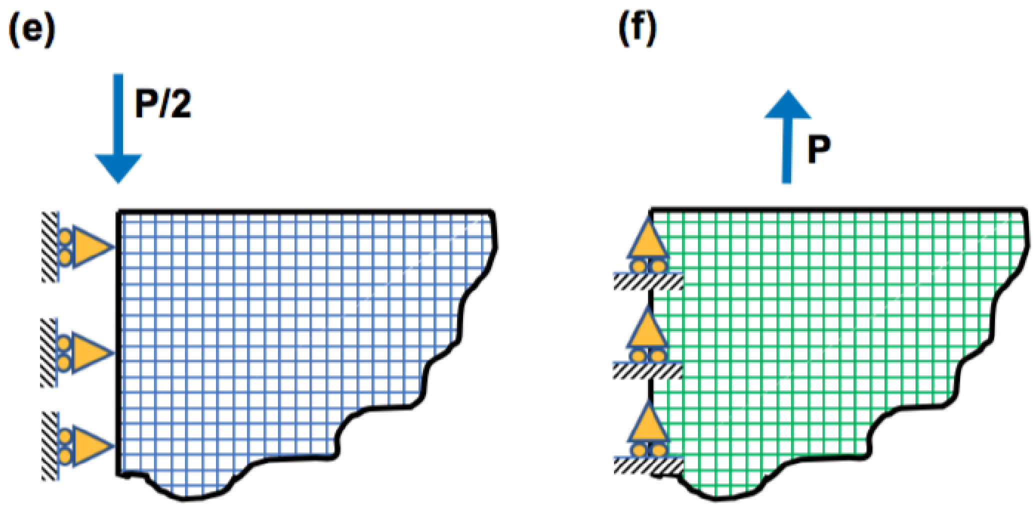

Figure 8: Half models for problem (e) using symmetry and for problem (f) using anti-symmetry

Assignment 2.2

On "FEM Modelling: Introduction":

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports at *I* and *N* extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

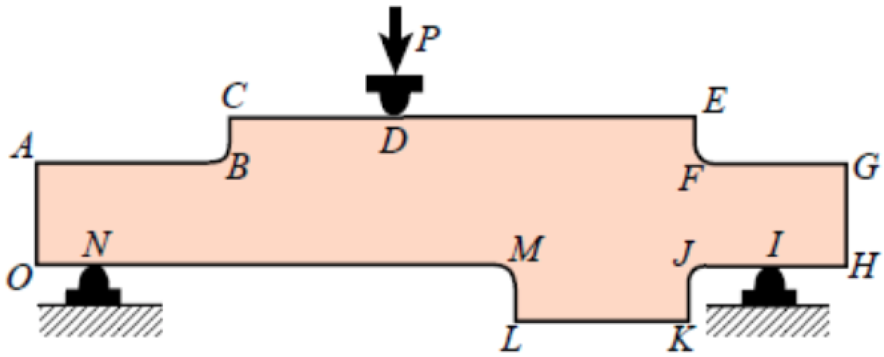

Figure 9: Inplane bent plate

Solution: The 'trouble spots' in the plate structure shown in figure 9 are expected in the region of high stress or strain gradients i.e. where a sudden variation of these quantities could be anticipated. In order to capture these changes reasonably well, we need to refine the mesh locally at these regions. A few situations which require local mesh refinement are:

- Around cracks and cutouts.
- Near entrant corners and sharp curved edges.
- In the neighbourhood of concentrated point loads and supports/reaction forces.
- At sharp contact areas, joints or welds.
- Surrounding the region where the thickness, cross-section area or material properties changes within the structure.

Keeping the above situations in mind, Table 1 presents the trouble spots in the given plate structure.

Trouble spots	Situation
D	Concentrated point load
N, I	Supports/reaction forces
B, F, J, M	Entrant corners

Table 1: Trouble spots and the related situation in the given plate structure

In the regions away from these spots, we could use a coarser mesh which can justifiably represent the problem i.e. geometry, loads and the support conditions. By doing this, we could save the computational cost without compromising the solution of the problem.

Assignment 2.3

On "Variational Formulation":

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialise the result to the prismatic bar $A = A_i = A_j$.

Solution: The consistent node force vector $\boldsymbol{f}^e$ is given as,

$$\boldsymbol{f}^e = \int_{x_1}^{x_2} q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} dx = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi$$

where, $\xi = (x - x_1)\,/\,l$ and considering only one bar element $x_1 = 0 \longrightarrow x = \xi\, l$

Using the given centrifugal axial force $q(x)$, we get,

$$\boldsymbol{f}^e = \int_0^1 \rho A\omega^2 x \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi$$

Now, utilising the area interpolation for the tapered bar element, we have,

$$\boldsymbol{f}^e = \int_0^1 \rho(A_i(1-\xi) + A_j\xi)\omega^2 x \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi$$

We know $x = \xi\ l$, therefore we get the consistent node force vector as,

$$\boxed{\boldsymbol{f}^e = \rho\omega^2 l^2 \int_0^1 (A_i(1-\xi) + A_j\xi)\ \xi\ \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi}$$

Hence, we have the consistent node force component as,

$$f_1^e = \rho\omega^2 l^2 \int_0^1 A_i\xi(1-\xi)^2 + A_j\xi^2(1-\xi)\ d\xi$$

$$f_1^e = \rho\omega^2 l^2 \int_0^1 A_i(\xi + \xi^3 - 2\xi^2) + A_j(\xi^2 - \xi^3)\ d\xi$$

$$f_1^e = \rho\omega^2 l^2 \left[A_i(\frac{\xi^2}{2} + \frac{\xi^4}{4} - \frac{2\xi^3}{3}) + A_j(\frac{\xi^3}{3} - \frac{\xi^4}{4}) \right]_0^1$$

$$f_1^e = \rho\omega^2 l^2 \left[A_i(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}) + A_j(\frac{1}{3} - \frac{1}{4}) \right]$$

$$\boxed{f_1^e = \rho\omega^2 l^2 \left[\frac{A_i}{12} + \frac{A_j}{12} \right]}$$

Similarly, the second component is given as,

$$f_2^e = \rho\omega^2 l^2 \int_0^1 A_i\xi^2(1-\xi) + A_j\xi^3\ d\xi$$

$$f_2^e = \rho\omega^2 l^2 \int_0^1 A_i(\xi^2 - \xi^3) + A_j\xi^3\ d\xi$$

$$f_2^e = \rho\omega^2 l^2 \left[A_i(\frac{\xi^3}{3} - \frac{\xi^4}{4}) + A_j\frac{\xi^4}{4} \right]_0^1$$

$$f_2^e = \rho\omega^2 l^2 \left[A_i(\frac{1}{3} - \frac{1}{4}) + A_j(\frac{1}{4}) \right]$$

$$\boxed{f_2^e = \rho\omega^2 l^2 \left[\frac{A_i}{12} + \frac{A_j}{4} \right]}$$

Therefore,

$$\boxed{\boldsymbol{f}^e = \frac{\rho\omega^2 l^2}{12} \begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}}$$

For the special case of a prismatic bar, $A = A_i = A_j$, we get,

$$\boxed{\boldsymbol{f}^e = \frac{\rho A\omega^2 l^2}{6} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

It is interesting to note that due to the specification of centrifugal axial force $q(x)$ as a linear function of x, the force on node 2 is double the force on node 1.