Computational Structural Mechanics and Dynamics

Master of Science in Computational Mechanics
Spring Semester 2018

Homework 2

Juan Diego Iberico Leonardo

1987 - 2017 CIMNE 30 years International Centre for Numerical Methods in Engineering

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Assignment 2.1

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure.

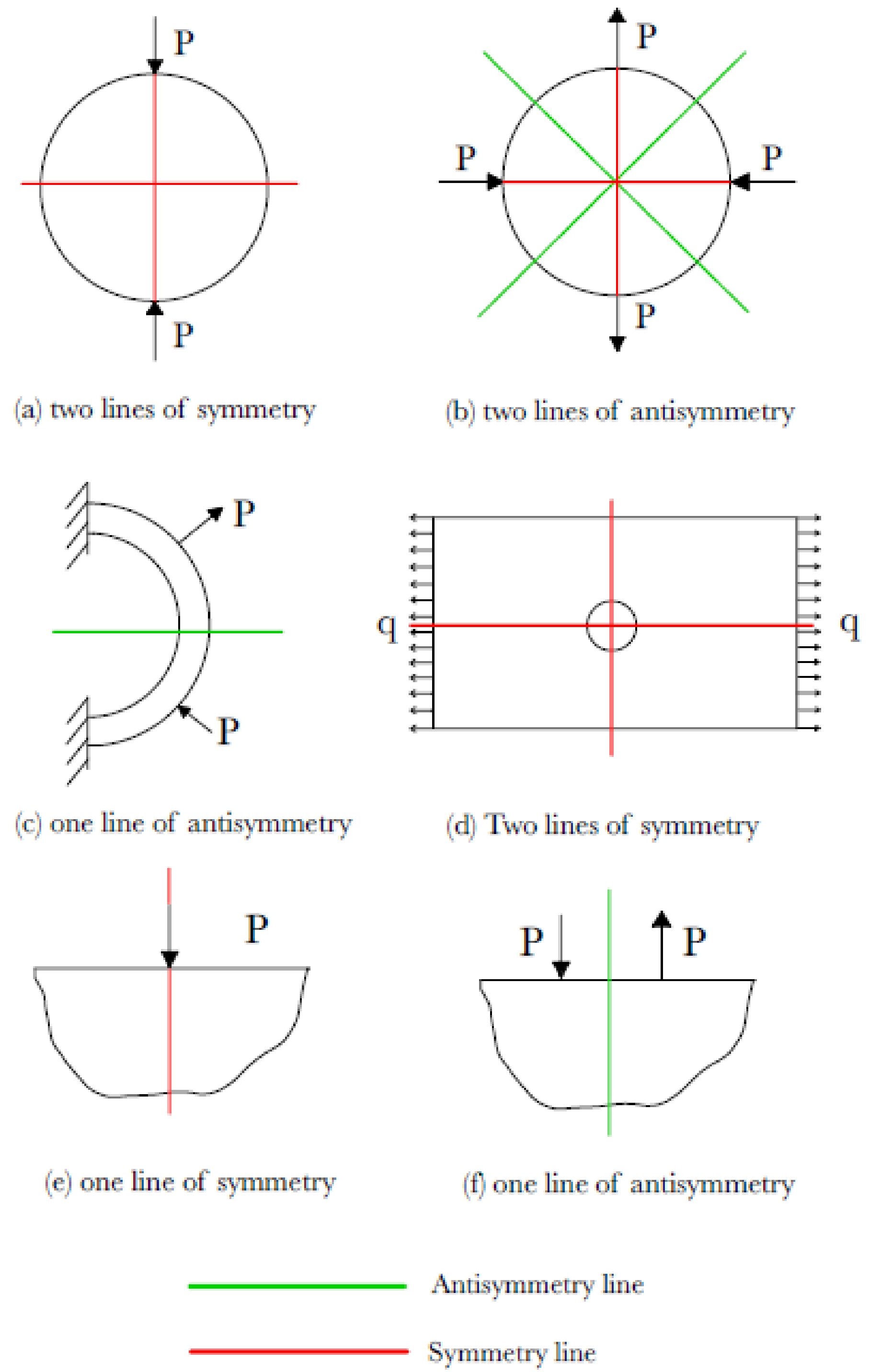

Figure 1: Problems for assignment 2.1

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

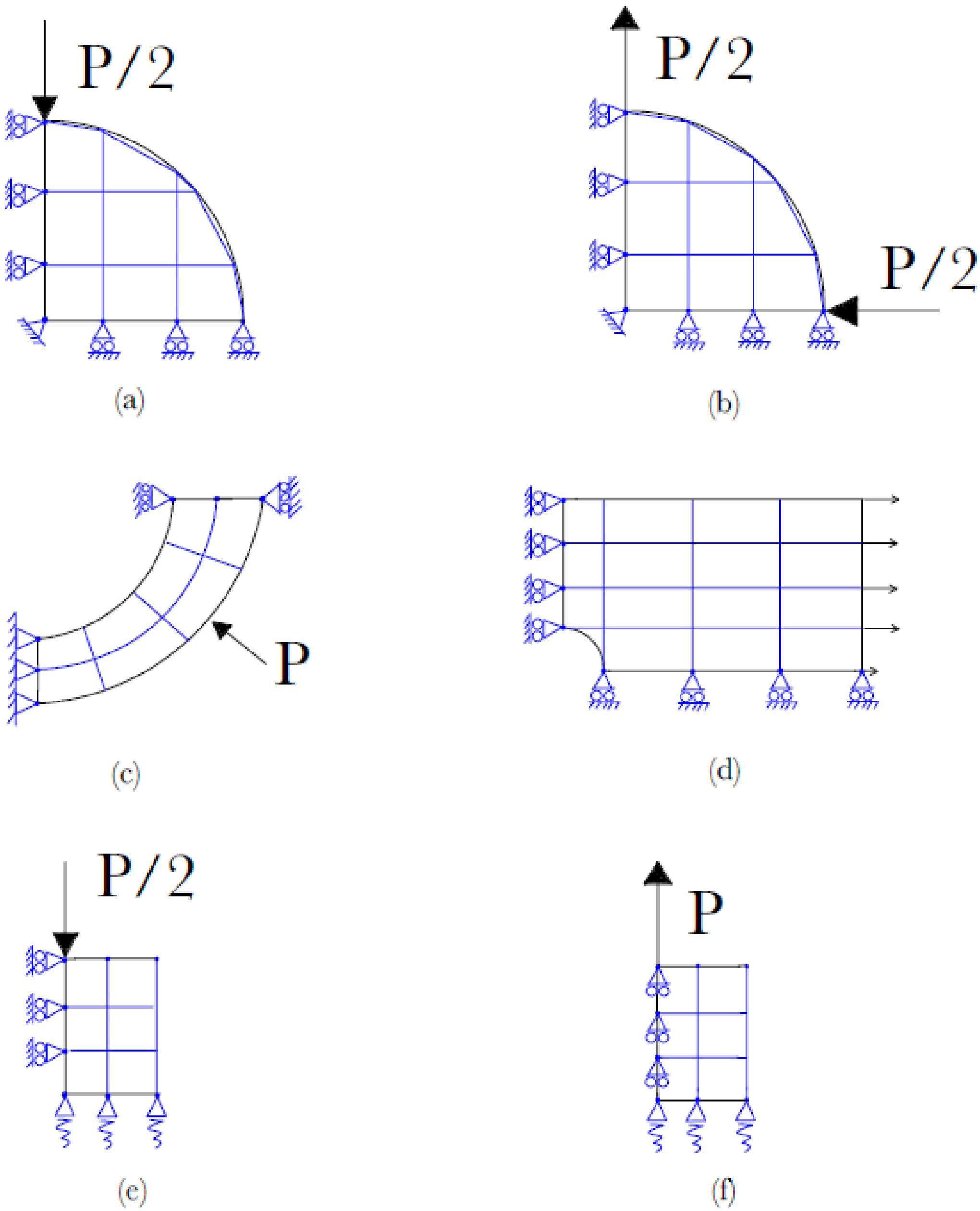

Figure 2

Assignment 2.2

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

   Answer: The points that requiere a finer mesh are B, D, F, I, J, M and N.
   For points D, I and N are under hight strees gradient due to the direct load and reaction forces. On the other side, the remaining points are under hight stress gradient due to its location of transition of different width of the beam.

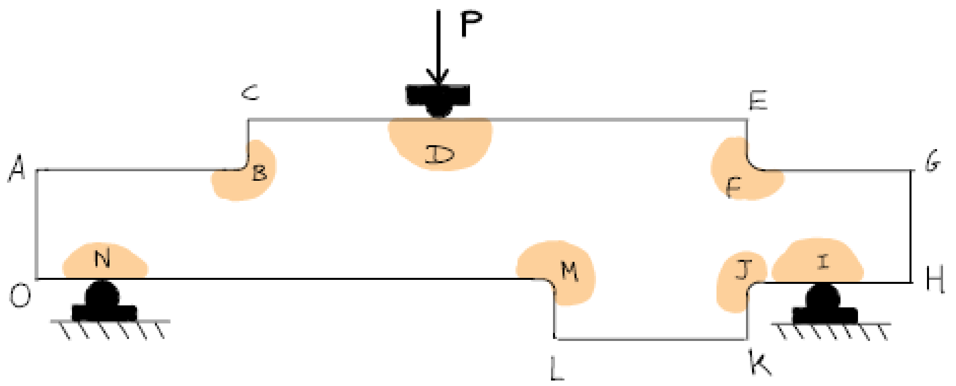

Figure 3: Problem for assignment 2.2

Assignment 2.3

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as:

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

The consistency nodal force vector is definded as:

$$f_{ext} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l \, d\xi$$

$$q = \rho A\omega^2 x = \rho A\omega^2 \xi l = \rho\omega^2\xi l(A_i(1-\xi) + A_j\xi)$$

After normalize the axial centrifugal force:

$$\begin{aligned} f_{ext} &= \int_0^1 \rho l^2\omega^2\xi(A_i(1-\xi) + A_j\xi) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi \\ &= \rho l^2\omega^2 \int_0^1 \begin{bmatrix} A_i(\xi^3 - 2\xi^2 + \xi) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j\xi^3 \end{bmatrix} d\xi \\ &= \rho l^2\omega^2 \begin{bmatrix} A_i(\xi^4/4 - 2/3\,\xi^3 + \xi^2/2) + A_j(\xi^3/3 - \xi^4/4) \\ A_i(\xi^3/3 - \xi^4/4) + A_j\xi^4/4 \end{bmatrix} \\ &= \rho l^2\omega^2 \begin{bmatrix} A_i(1/4 - 2/3 + 1/2) + A_j(1/3 - 1/4) \\ A_i(1/3 - 1/4) + A_j\,1/4 \end{bmatrix} \end{aligned}$$

Achieving as result for tapered beam and prismating beam:

$$f_{ext} = \rho l^2\omega^2 \begin{bmatrix} A_i/12 + A_j/12 \\ A_i/12 + A_j/4 \end{bmatrix} \longrightarrow f_{ext} = \rho l^2\omega^2 \begin{bmatrix} A/6 \\ A/3 \end{bmatrix}$$