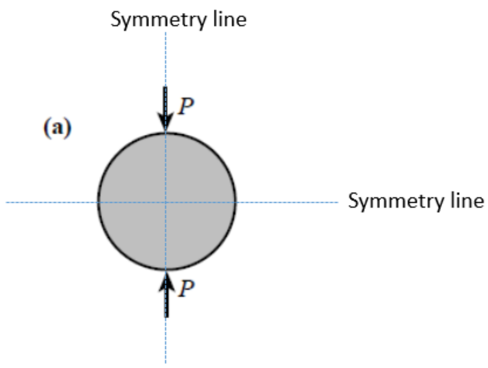
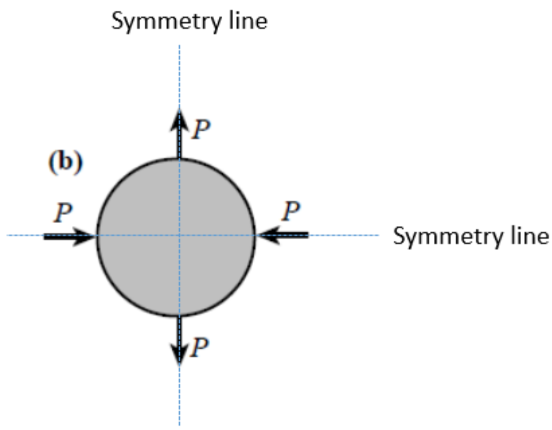
Assignement 2 – Computational Structural Mechanics and Dynamics

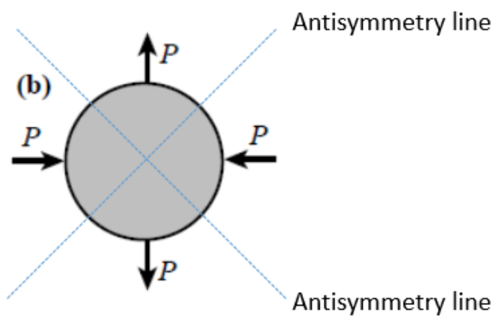
Trond Jørgen Opheim

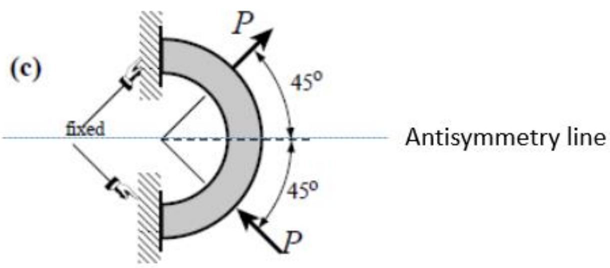
Problem 1

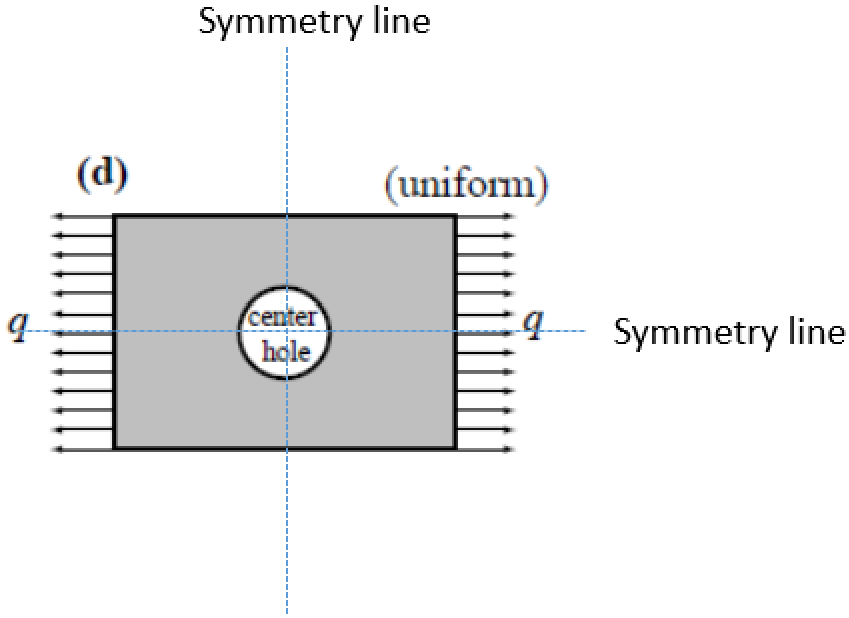
1. Symmetry and antisymmetry lines

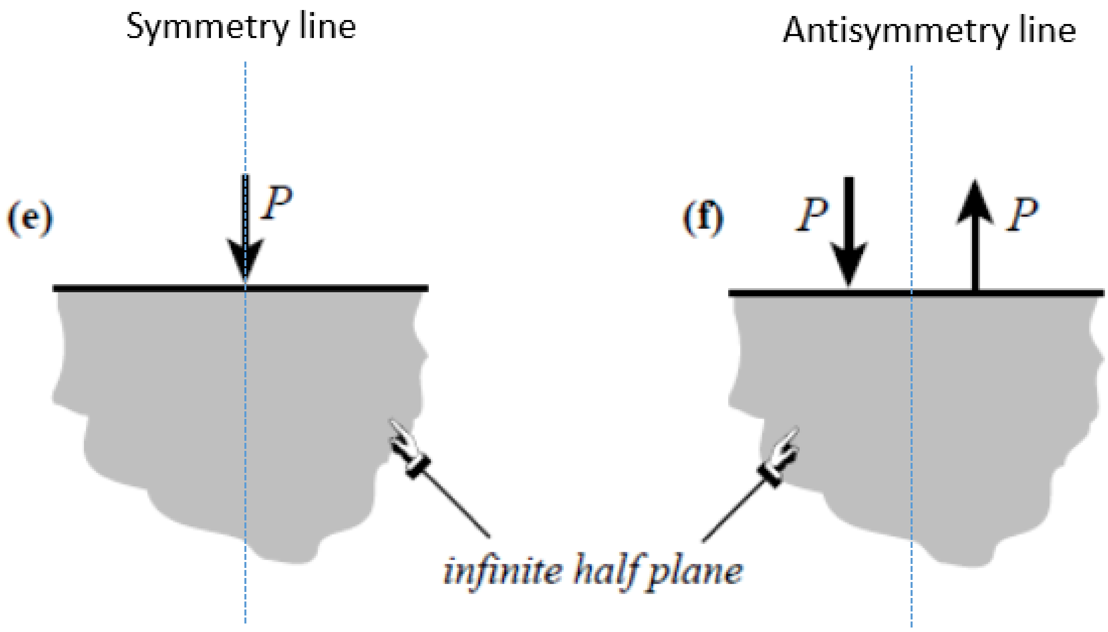









Symmetry line
(d)
(uniform)
center
hole
q
q
Symmetry line

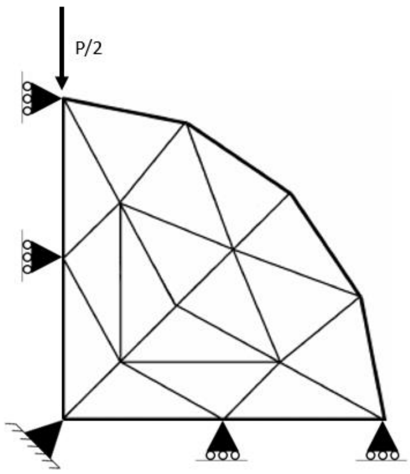

Symmetry line
Antisymmetry line
(e)
P
(f)
P
P
infinite half plane

2. Using the symmetry and antisymmetry lines to simplify

The advantages by using the symmetry and antisymmetry lines is that you can reduce the size of the model and use the boundary conditions that the symm. and antisymm. lines gives you. These are:

- No rotation along the symmetry line in the plane
- No displacement along the antisymmetry line in the plane

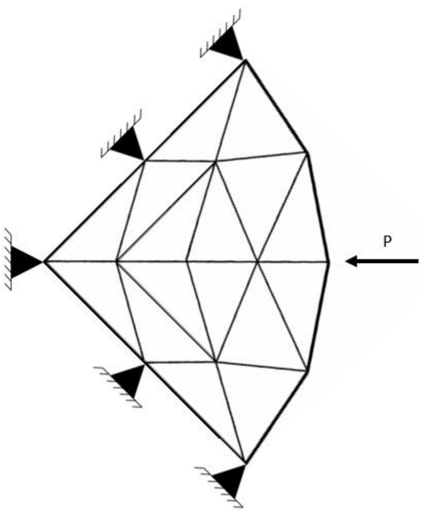
a



A long the symmetry lines there will be no rotation, and therefore it could be more convenient to use the symbol given in figure 1. But since this is not used in the presentation slides shown in class, I continue to use the same symbols (rollers) when the figures are allowed to move freely.

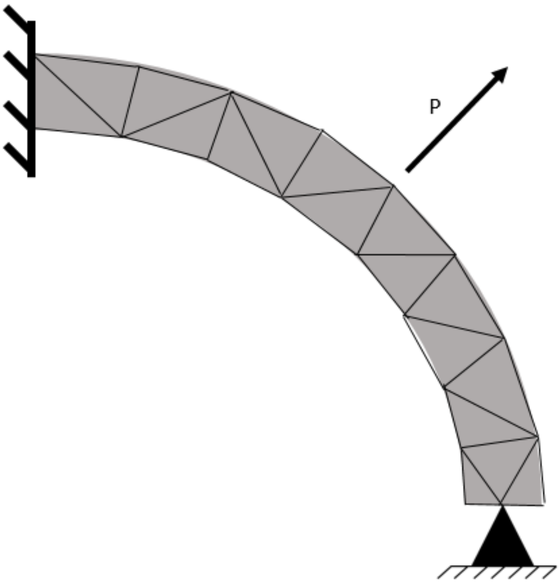
Figure 1 - Support without rotation, but with displacement in one direction

b



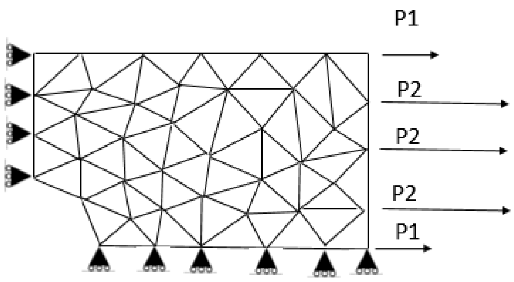
With the antisymmetry lines defined as in problem 1, we know that there can be no displacement along these lines, which is presented with the fixed supports at the nodes along the antisymmetry line.

c



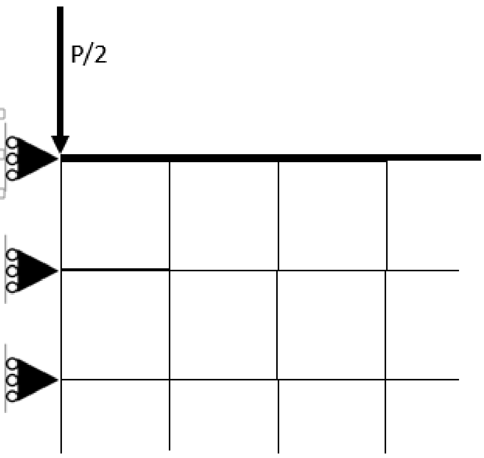
With the antisymmetry line cutting the figure in two, we knot that the displacement at this point is zero, but with an allowed rotation.

d



The two symmetry lines defined in problem 1 gives us no rotation but with an allowed displacement along these lines. In this case I would also use the symbol for support condition shown in Figure 1, but since this is not used in the presentation slides, I continue to use the “rollers”.

e



The point load is splitted in half and with the symmetry condition we use that there is no rotation a long the line, but with an allowed vertical displacement. In my opinion, this should be represented with the symbol of support condition shown in Figure 1.

f

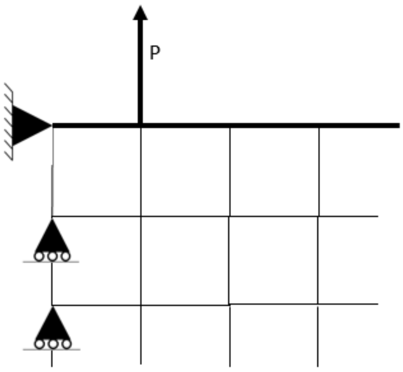
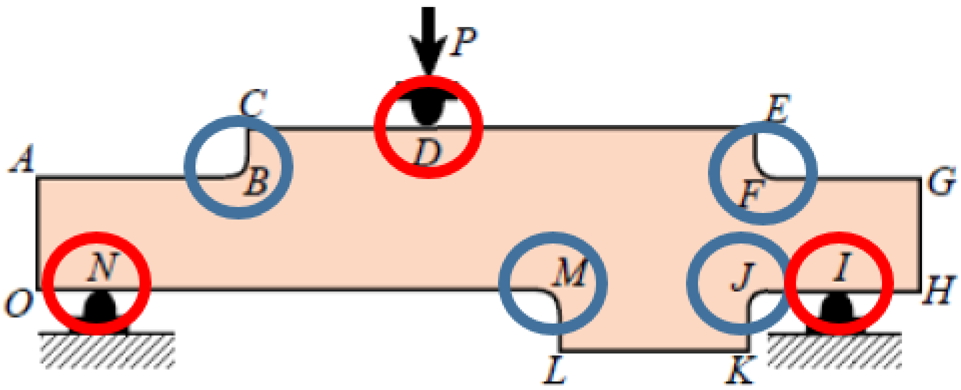


Figure f can be splitted in two with the use of the antisymmetry line, but here I am more insecure about the support conditions. Since this is a continuous 2D figure (instead of a beam for example) I would say it is allowed with the horizontal displacement at the lower support conditions. At the upper support I set it to be fixed because there could be no displacement, but with an allowed rotation.

Problem 2



For the figure I have marked out the areas that need a finer mesh. The entrant corners are marked out in blue and points with applied concentrated loads in red. These areas need a finer mesh because the gradient stress is higher.

Problem 3

Area: $A = A_i(1-\zeta) + A_j\zeta$

Centrifugal axial force: $q(x) = \rho A \omega^2 x$

Consistent node force vector: $\boldsymbol{f}_{ext} = \int_0^1 q \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} l \, d\zeta$

Relationship x-ζ: $x = \zeta * l$

Calculation of the consistent node force vector

$$\boldsymbol{f}_{ext} = \rho\omega^2 \int_0^1 \left(A_i(1-\zeta) + A_j\zeta\right)\zeta l^2 \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} d\zeta$$

$$\boldsymbol{f}_{ext} = \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} \zeta\left(A_i(1-\zeta) + A_j\zeta\right)(1-\zeta) \\ \zeta^2\left(A_i(1-\zeta) + A_j\zeta\right) \end{bmatrix} d\zeta$$

$$f_{ext} = \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i\zeta^3 - 2A_i\zeta^2 + A_i\zeta - A_j\zeta^3 + A_j\zeta^2 \\ -A_i\zeta^3 + A_i\zeta^2 + A_j\zeta^3 \end{bmatrix} d\zeta$$

$$f_{ext} = \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{4}A_i\zeta^4 - \frac{2}{3}A_i\zeta^3 + \frac{1}{2}A_i\zeta^2 - \frac{1}{4}A_j\zeta^4 + \frac{1}{3}A_j\zeta^3 \\ -\frac{1}{4}A_i\zeta^4 + \frac{1}{3}A_i\zeta^3 + \frac{1}{4}A_j\zeta^4 \end{bmatrix}_0^1$$

$$f_{ext} = \rho\omega^2 l^2 \begin{bmatrix} \frac{A_i + A_j}{12} \\ \frac{A_i + 3A_j}{12} \end{bmatrix}$$

For the prismatic bar with $A = A_i = A_j$:

$\boldsymbol{f}_{ext} = \frac{\boldsymbol{\rho\omega^2 l^2 A}}{\boldsymbol{6}} \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{2} \end{bmatrix}$ **consistant node forces**