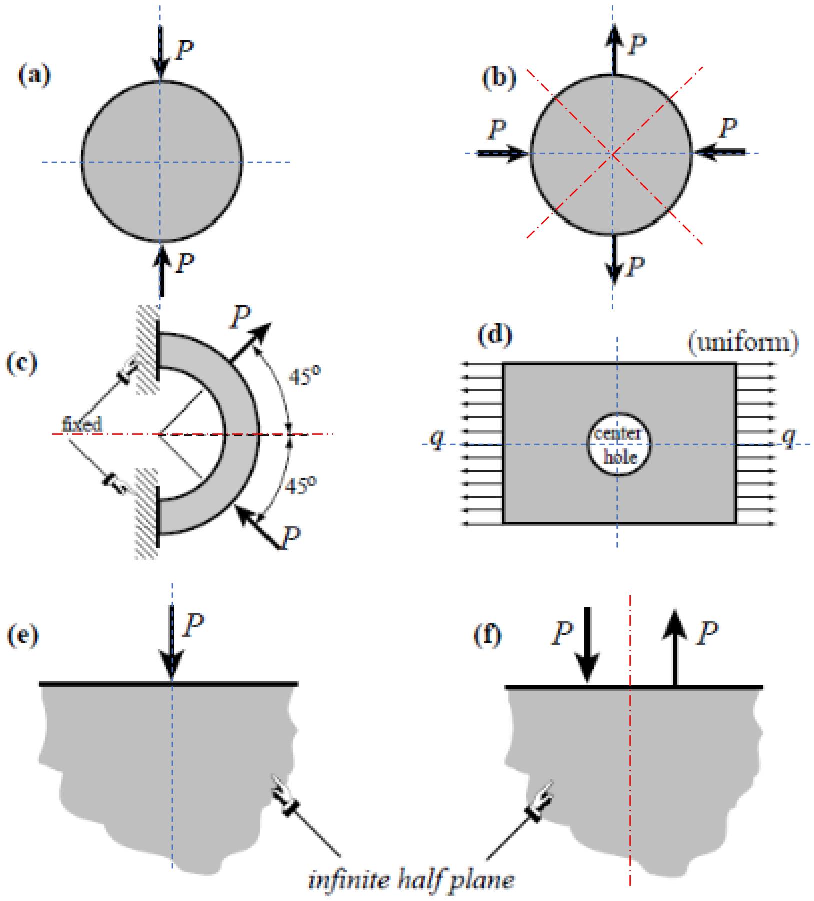
Assignment 2.1

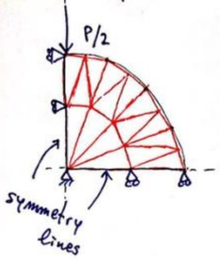
1. **Identification of the symmetry and antisymmetry lines in the two-dimensional problems illustrated.**



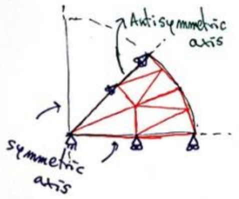
Symmetry axis
Antisymmetry axis

2. Displacement BCs in a coarse mesh for every two-dimensional problem stated above

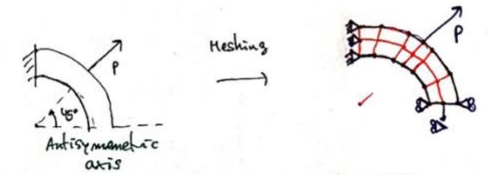
(a) We only need 1/4 of the circular disk to solve the problem



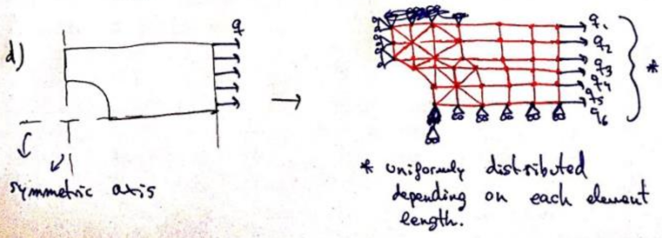
(b) In this problem only 1/8 of the disk is needed.



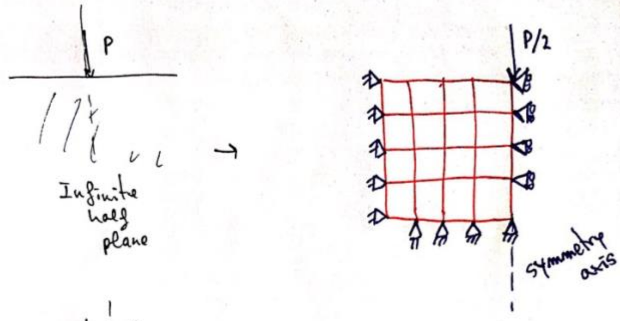
(c)



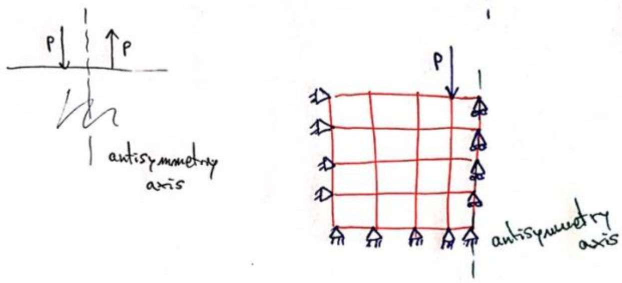
d)



(e)



(f)



Assignment 2.2

Difference between "Verification" and "Validation" in the context of the FEM-Modelling procedure

Validation is the evaluation of the error that appears when the 'real problem' is simplified in a mathematical model. During the 'Validation' process it is critical to asses some factors such as the constitutive equations (material behaviour), the boundary conditions, as well as the physical equations selected to represent the physics of the problem. For example, through experimental results, it is possible to check if the mathematical model is describing properly the 'real problem'.

On the other hand, 'Verification' is the process of evaluating the error of the Finite-Element results in comparison with analytical results or numerical results of a finer mesh. Here, the error estimates are the "tool" used to assess the error, and several techniques (a priori, a posteriori estimates…) are used to refine the mesh optimally to reduce this numerical error.

Assignment 2.3

1. **Tapered bar rotation**

   Problem data

   Bar section area: $A = A_i(1-\xi) + A_j\xi$

   Centrifugal axial force: $q(x)=\rho A\omega^2 x$ being x the longitudinal coordinate

   The objective is to obtain the node forces, and then, specialize the result to the prismatic bar

   Solution

   It is possible to write the nodal force of an element through the following expression

$$f^{(e)} = \int_{l^{(e)}} N^T q dx$$

   Being *N* the linear shape function used to approximate the solution.

   It is convenient to use the natural coordinates ξ to simplify the resolution of the integral. Therefore, a variable change must be considered.

$$\xi = \frac{x - x_i}{l^e} \qquad d\xi = \frac{dx}{l^e}$$

As the origin is at node *i*, $x_i = 0$

The shape functions N_i and N_j in natural coordinates results as

$$N^T = \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix}$$

Thus the integral can be solved as follows

$$f^e = \int_0^1 \rho\omega^2 (A_i(1-\xi) + A_j\xi)\xi l^{e2} \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$
$$= \rho\omega^2 l^{e2} \begin{bmatrix} A_i\left(\frac{\xi^2}{2} - \frac{2\xi^3}{3} + \frac{\xi^4}{4}\right) + A_j\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) \\ A_i\left(\frac{\xi^3}{3} - \frac{\xi^4}{4}\right) + A_j\frac{\xi^4}{4} \end{bmatrix}_0^1$$
$$= \omega^2 l^{e2} \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

A particular solution of this problem is the prismatic bar $A_i = A_j$

$$f^e = \rho\omega^2 l^{e2} \begin{bmatrix} \frac{A}{6} \\ \frac{A}{3} \end{bmatrix}$$