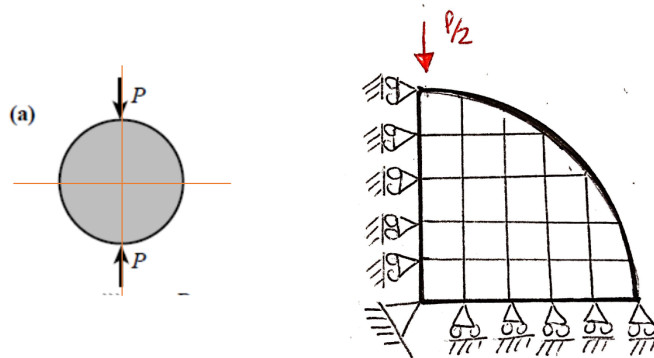


Assignment 2: FEM Modelling

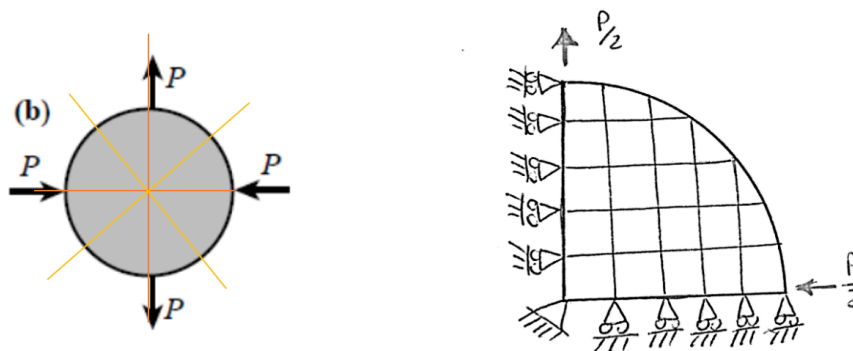
Assignment 2.1

In the 2D space we can define the symmetry line, the one which a 180° rotation of the body about it reproduces exactly the problem, and the anti-symmetry line, that is the one which a 180° rotation of the body about itself reproduce exactly the original problem except the loads (that are reversed). So after defined the symmetry or anti-symmetry lines we can cut the structure depending on the established lines and we must define for each case that appropriate boundary conditions and loads according to the system.

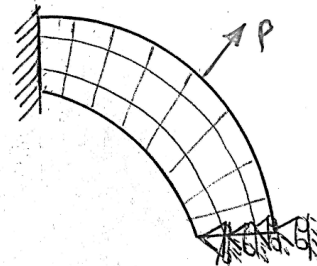
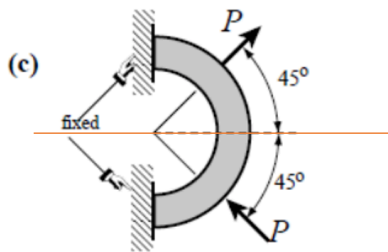
- (a) In this case we can work only with a quarter of the total structure because it is possible to define two symmetry lines: one matches with the diameter that is perpendicular to the forces direction and the another one matches the load directions. According to BCs there can be no displacement at the point corresponding to the center of the circle, and the nodes in symmetry lines will only have displacement in the same directions.



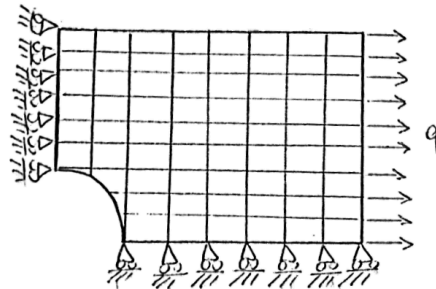
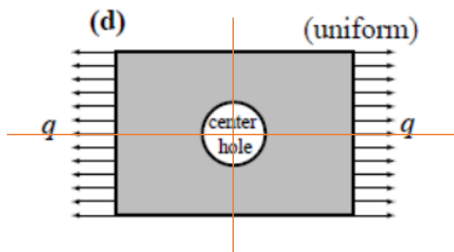
- (b) In this case we can work only with a quarter of the total structure but it is possible to define symmetry lines and also anti-symmetry lines, which have 45° grades. According to BC, for the symmetry case we have the same conditions as the previous case; for the anti-symmetry case the nodes corresponding with the center of the cylinder has no displacement, and the nodes on the anti-symmetry lines have no displacement in the lines directions.



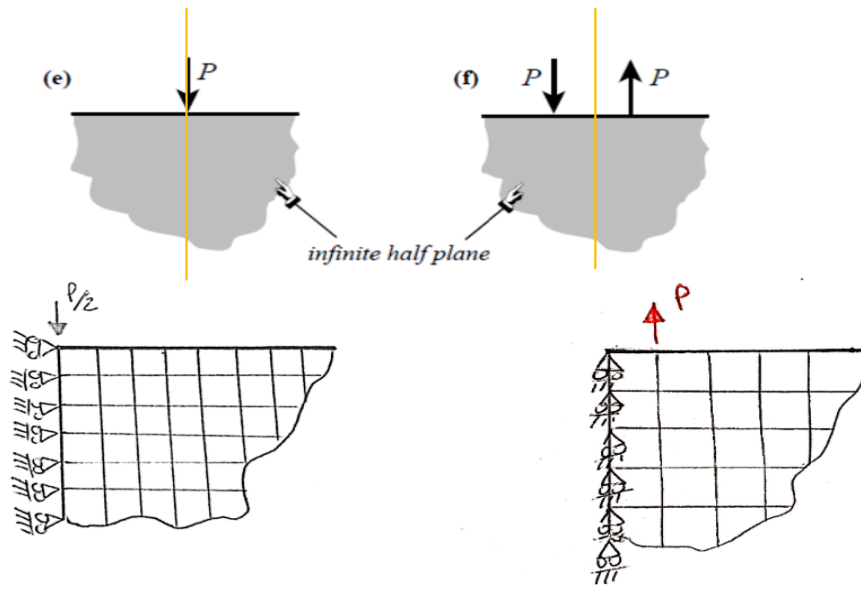
- (c) In this case it can be define the anti-symmetry line, which is a horizontal line that divide the structure in two parts. We are using only half of the structure. According to the BC we impose no displacement in the direction of the line for the nodes contained in the anti-symmetry line.



- (d) In this case we can define two symmetry lines, like the first case (a). According to the BC those nodes in the symmetry lines, will have displacement in the corresponding line direction.



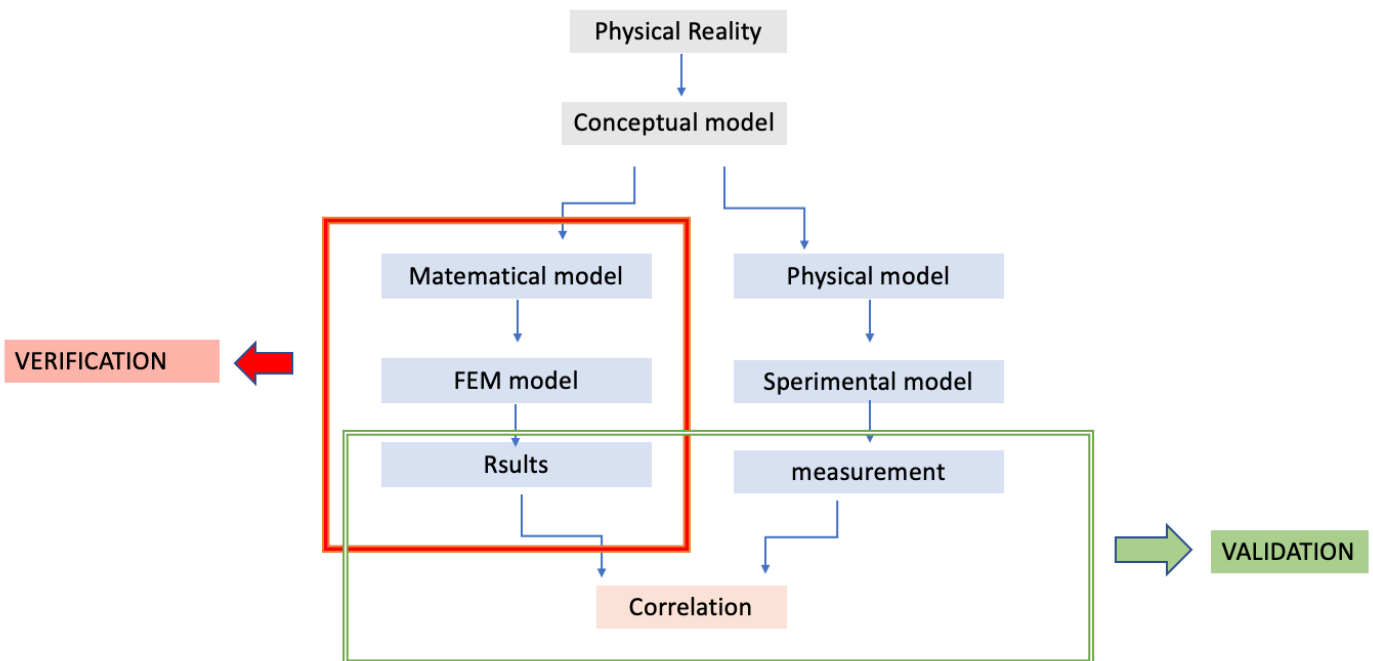
- (e) In this case is defined one symmetry line that is vertical which matches whit the load direction. According to the BC the nodes in the symmetry line must have no displacement in the horizontal direction. The loads in applied in the symmetry line, so we should only apply half $P/2$ of the total load P .
- (f) In this case is defined one anti-symmetry line that is vertical and fixed at the middle point between the two loads that are applied. According to the BC the nodes in the anti-symmetry line must have no displacement in the vertical direction.



Assignment 2.2

Explain the difference between “Verification” and “Validation” in the context of the FEM-Modelling procedure.

The Verification and Validation phases of a Finite Element model through the FEM-Test Correlation tools constitute an essential step in simulation-based design processes as they allow you to objectively compare the simulation product with the response of the real product.



Validation is to ensures reliability, and verification is to ensure the accuracy of the numerical analysis of a structure, meaning the definitions with reliability and accuracy.

- **Verification** is the set of activities that determine that a computational model accurately represents the aspects of mathematical modeling and the mathematical solution it produces. The questions that we can ask to ourselves to satisfy the verification is “did I model correctly?”
- **Validation** is the procedure for quantifying the degree of accurate representation of physical reality, in the areas of interest, by the model. The questions that we can ask to ourselves to satisfy the validation is "Did I make the right model?"

Assignment 3.

We have tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A_i = A_i(1 - \xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i . Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the length in which x is the longitudinal coordinate $x = xe$.

Find the consistent node forces as functions of ρ , A_i , A_j , ω and l , and specialize the result to the prismatic bar $A = A_i = A_j$.

We can write the nodal element force of the element through the following relationship

$$f^e = \int_{x_i^e}^{x_j^e} N^T q dx$$

It is decided to work with the natural coordinates ξ , so therefore we must have to make a variables change:

$$\xi = \frac{x-x_i}{l^e} = \frac{x}{l^e} \quad d\xi = \frac{dx}{l^e}$$

Where $x_i = 0$.

Is possible to write and solve the previous expression of the nodal element force as function of those parameters:

$$\begin{aligned} f^e &= \int_0^1 p\omega^2(A_i(1-\xi) + A_j\xi)\xi l^e d\xi \\ &= p\omega^2 l^e \int_0^1 \left[\begin{array}{c} (A_i\xi - A_i\xi^2 + A_j\xi^2)(1-\xi) \\ (A_i\xi - A_i\xi^2 + A_j\xi^2)\xi \end{array} \right] d\xi \\ &= p\omega^2 l^e \left[\begin{array}{c} A_i \left(\frac{\xi^2}{2} - \frac{\xi^3}{3} - \frac{\xi^3}{3} + \frac{\xi^4}{4} \right) + A_j \left(\frac{\xi^3}{3} - \frac{\xi^4}{4} \right) \\ A_i \left(\frac{\xi^3}{3} - \frac{\xi^4}{4} \right) + A_j \frac{\xi^4}{4} \end{array} \right] = \omega^2 l^e \left[\begin{array}{c} A_i \frac{1}{12} + A_j \frac{1}{12} \\ A_i \frac{1}{12} + A_j \frac{1}{4} \end{array} \right] \end{aligned}$$

If the solution is generalized for the prismatic bar in which $A = A_i = A_j$, the previous expression became :

$$f^e = p\omega^2 l^e \left[\begin{array}{c} A \frac{1}{6} \\ A \frac{1}{3} \end{array} \right]$$

