



Nombre del estudiant: Rosa Eva González

Materia: Computational Structural Mechanics and Dynamics

Fecha de entrega: 26/02/2018

Descripción: Deber 3

In isotropic elastic materials (as well as in plasticity and viscoelasticity) it is convenient to use the so-called Lamé constants λ and μ instead of E and ν in the constitutive equations. Both λ and μ have the physical dimension of stress and are related to E and ν by

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad \mu = G = \frac{E}{2(1+\nu)}$$

1. Find the inverse relations for E, ν in terms of λ, μ

$$\text{Ecuación ①} \quad \lambda = \frac{E \cdot \nu}{(1 + \nu) \cdot (1 - 2 \cdot \nu)}$$

$$\text{Ecuación ②} \quad \mu = \frac{E}{2 \cdot (1 + \nu)}$$

$$\text{① despegado E} \quad E = \frac{\lambda \cdot (1 + \nu) \cdot (1 - 2 \cdot \nu)}{\nu}$$

$$\text{② despegado E} \quad E = 2 \cdot \mu \cdot (1 + \nu)$$

v en funcion de de λ, μ

$$E := E$$

$$\frac{\lambda \cdot (1 + \nu) \cdot (1 - 2\nu)}{\nu} := 2 \cdot \mu \cdot (1 + \nu)$$

$$\lambda \cdot (1 - 2\nu) := 2 \cdot \mu \cdot \nu$$

$$\lambda - 2\nu \cdot \lambda := 2 \cdot \mu \cdot \nu$$

$$\lambda := 2 \cdot \mu \cdot \nu + 2\nu \cdot \lambda$$

$$\lambda := \nu(2 \cdot \mu + 2 \cdot \lambda)$$

Ecuación④ $\nu := \frac{\lambda}{2 \cdot (\mu + \lambda)}$ //

E en funcion de de λ, μ

④en② $E := 2 \cdot \mu \cdot \left[1 + \frac{\lambda}{2 \cdot (\mu + \lambda)} \right]$

$$E := 2 \cdot \mu \cdot \frac{2 \cdot \mu + 2 \cdot \lambda + \lambda}{2 \cdot (\mu + \lambda)}$$

$$E := \mu \cdot \left(\frac{2 \cdot \mu + 3 \cdot \lambda}{\mu + \lambda} \right)$$
 //

2. Express the elastic matrix for plane stress and plane strain cases in terms of λ,μ.

plane stress: $\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix},$

plane strain: $\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1}{2}(1 - 2\nu) \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix}.$

Plane stress:

$$\nu := \frac{\lambda}{2 \cdot (\mu + \lambda)} \quad E := \mu \cdot \left(\frac{2 \cdot \mu + 3 \cdot \lambda}{\mu + \lambda} \right)$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E}{1 - \nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

① ②

$$\textcircled{1} \quad \frac{E}{1 - \nu^2} \rightarrow \frac{\mu \cdot (2 \cdot \mu + 3 \cdot \lambda)}{(\mu + \lambda) \cdot \left[\frac{\lambda^2}{(2 \cdot \mu + 2 \cdot \lambda)^2} - 1 \right]} \text{ simplify } \rightarrow \frac{4 \cdot \mu \cdot (\mu + \lambda)}{2 \cdot \mu + \lambda}$$

$$\textcircled{2} \quad \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{pmatrix} \rightarrow \begin{bmatrix} 1 & \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & 0 \\ \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & 1 & 0 \\ 0 & 0 & \frac{1}{2} - \frac{\lambda}{2 \cdot (2 \cdot \mu + 2 \cdot \lambda)} \end{bmatrix} \text{ simplify } \rightarrow \begin{bmatrix} 1 & \frac{\lambda}{2 \cdot (\mu + \lambda)} & 0 \\ \frac{\lambda}{2 \cdot (\mu + \lambda)} & 1 & 0 \\ 0 & 0 & \frac{2 \cdot \mu + \lambda}{4 \cdot (\mu + \lambda)} \end{bmatrix}$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{4 \cdot \mu \cdot (\mu + \lambda)}{2 \cdot \mu + \lambda} \begin{bmatrix} 1 & \frac{\lambda}{2 \cdot (\mu + \lambda)} & 0 \\ \frac{\lambda}{2 \cdot (\mu + \lambda)} & 1 & 0 \\ 0 & 0 & \frac{2 \cdot \mu + \lambda}{4 \cdot (\mu + \lambda)} \end{bmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

//

Plane strain:

$$\nu := \frac{\lambda}{2 \cdot (\mu + \lambda)} \quad E := \mu \cdot \left(\frac{2 \cdot \mu + 3 \cdot \lambda}{\mu + \lambda} \right)$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E}{(1 + \nu) \cdot (1 - 2 \cdot \nu)} \cdot \begin{pmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1}{2} \cdot (1 - 2 \cdot \nu) \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

①
②

$$\textcircled{1} \quad \frac{E}{(1 + \nu) \cdot (1 - 2 \cdot \nu)} \rightarrow \frac{\mu \cdot (2 \cdot \mu + 3 \cdot \lambda)}{\left(\frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} + 1 \right) \cdot \left(\frac{2 \cdot \lambda}{2 \cdot \mu + 2 \cdot \lambda} - 1 \right) \cdot (\mu + \lambda)} \text{ simplify } \rightarrow 2 \cdot \mu + 2 \cdot \lambda$$

$$\textcircled{2} \quad \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1}{2} \cdot (1 - 2 \cdot \nu) \end{bmatrix} \rightarrow \begin{bmatrix} 1 - \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & 0 \\ \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & 1 - \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} & 0 \\ 0 & 0 & \frac{1}{2} - \frac{\lambda}{2 \cdot \mu + 2 \cdot \lambda} \end{bmatrix} \text{ simplify } \rightarrow \begin{bmatrix} \frac{2 \cdot \mu + \lambda}{2 \cdot (\mu + \lambda)} & \frac{\lambda}{2 \cdot (\mu + \lambda)} & 0 \\ \frac{\lambda}{2 \cdot (\mu + \lambda)} & \frac{2 \cdot \mu + \lambda}{2 \cdot (\mu + \lambda)} & 0 \\ 0 & 0 & \frac{\mu}{2 \cdot (\mu + \lambda)} \end{bmatrix}$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} 2 \cdot \mu + \lambda & \lambda & 0 \\ \lambda & 2 \cdot \mu + \lambda & 0 \\ 0 & 0 & \mu \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

- Split the stress-strain matrix E for plane strain as $E = E\lambda + E\mu$ in which $E\mu$ and $E\lambda$ contain only μ and λ , respectively. This is the Lamé $\{\lambda, \mu\}$ splitting of the plane strain constitutive equations, which leads to the so-called B-bar formulation of near-incompressible finite elements.

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} 2 \cdot \mu & 0 & 0 \\ 0 & 2 \cdot \mu & 0 \\ 0 & 0 & \mu \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix} + \begin{pmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

Ⓔ μ
Ⓔ λ

- Express $E\lambda$ and $E\mu$ also in terms of E and ν .

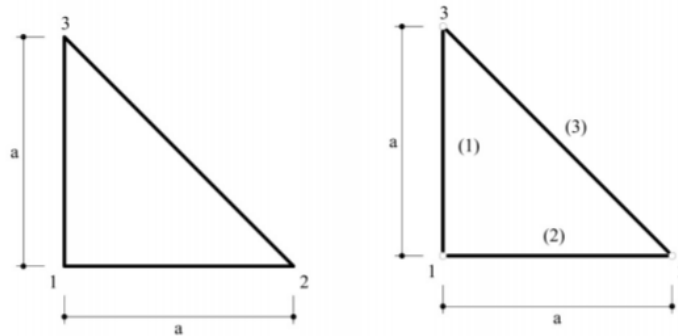
$$\lambda := \frac{E \cdot \nu}{(1 + \nu) \cdot (1 - 2 \cdot \nu)} \quad \mu := \frac{E}{2 \cdot (1 + \nu)}$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{E}{\nu + 1} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix} + \frac{E \cdot \nu}{(\nu + 1) \cdot (2 \cdot \nu - 1)} \cdot \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2 \cdot \epsilon_{xy} \end{pmatrix}$$

Assignment 3.2

On “The 3-node Plane Stress Triangle”: Consider a plane triangular domain of thickness h , with horizontal and vertical edges of length a . Let us consider for simplicity $a = 1$, $h = 1$. The material parameters are E , ν . Initially ν is set to zero. Two discrete structural models are considered as depicted in the figure:

- a) A plane linear Turner triangle with the same dimensions.
- b) A set of three bar elements placed over the edges of the triangular domain. The cross sections for the bars are $A_1 = A_2$ and A_3 .



1. Calculate the stiffness matrices K_{tri} and K_{bar} for both discrete models.

Longitud de las barras

$$L_1 = a \quad L_2 = a \quad L_3 = a\sqrt{2}$$

Matriz de rigidez de Barra 1, 2 y 3

$$K_1 := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \frac{E \cdot A_1}{L_1}$$

$$K_2 := \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{E \cdot A_1}{L_2}$$

$$K_3 := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \frac{E \cdot A_3}{L_3}$$

Matriz de rigidez K_{bar}

$$K_1 + K_2 + K_3 \rightarrow \begin{pmatrix} A_1 \cdot E & 0 & -A_1 \cdot E & 0 & 0 & 0 \\ 0 & A_1 \cdot E & 0 & 0 & 0 & -A_1 \cdot E \\ -A_1 \cdot E & 0 & A_1 \cdot E + \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & 0 & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & 0 & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & -A_1 \cdot E & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & A_1 \cdot E + \frac{\sqrt{2} \cdot A_3 \cdot E}{4} \end{pmatrix}$$

Coordenadas del triangulo

$$\begin{aligned} x1 &:= 0 & y1 &:= 0 \\ x2 &:= a & y2 &:= 0 \\ x3 &:= 0 & y3 &:= a \end{aligned}$$

Vectores

$$\begin{aligned} y23 &:= -a & x32 &:= -a \\ y31 &:= a & x13 &:= 0 \\ y12 &:= 0 & x21 &:= a \end{aligned}$$

Área

$$2 \cdot A := \begin{vmatrix} 1 & 1 & 1 \\ x1 & x2 & x3 \\ y1 & y2 & y3 \end{vmatrix} \rightarrow a^2 \quad a := 1$$

$$A := \frac{a^2}{2} \rightarrow \frac{1}{2}$$

Si:

$$E := \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \rightarrow \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \frac{E}{2} \end{pmatrix}$$

Matriz de rigidez Ktri

$$K := \frac{h}{4 \cdot A} \begin{pmatrix} y23 & 0 & x32 \\ 0 & x32 & y23 \\ y31 & 0 & x13 \\ 0 & x13 & y31 \\ y12 & 0 & x21 \\ 0 & x21 & y12 \end{pmatrix} \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \frac{E}{2} \end{pmatrix} \begin{pmatrix} y23 & 0 & y31 & 0 & y12 & 0 \\ 0 & x32 & 0 & x13 & 0 & x21 \\ x32 & y23 & x13 & y31 & x21 & y12 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{3 \cdot E}{4} & \frac{E}{4} & \frac{E}{2} & \frac{E}{4} & \frac{E}{4} & 0 \\ \frac{E}{4} & \frac{3 \cdot E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & \frac{E}{2} \\ \frac{E}{2} & 0 & \frac{E}{2} & 0 & 0 & 0 \\ \frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ \frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ 0 & \frac{E}{2} & 0 & 0 & 0 & \frac{E}{2} \end{pmatrix}$$

2. Is there any set of values for the cross sections $A_1=A_2$ and A_3 to make both stiffness matrix equivalent: $K_{bar} = K_{tri}$? If not, which are the values that make them more similar?

Matriz de rigidez K_{tri}	Matriz de rigidez K_{bar}
$\begin{pmatrix} \frac{3-E}{4} & \frac{E}{4} & -\frac{E}{2} & \frac{E}{4} & \frac{E}{4} & 0 \\ \frac{E}{4} & \frac{3-E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & -\frac{E}{2} \\ -\frac{E}{2} & 0 & \frac{E}{2} & 0 & 0 & 0 \\ -\frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ -\frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ 0 & -\frac{E}{2} & 0 & 0 & 0 & \frac{E}{2} \end{pmatrix}$	$\begin{pmatrix} A_1 E & 0 & -A_1 E & 0 & 0 & 0 \\ 0 & A_1 E & 0 & 0 & 0 & -A_1 E \\ -A_1 E & 0 & A_1 E + \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & 0 & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & 0 & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} \\ 0 & -A_1 E & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & -\frac{\sqrt{2} \cdot A_3 \cdot E}{4} & \frac{\sqrt{2} \cdot A_3 \cdot E}{4} & A_1 E + \frac{\sqrt{2} \cdot A_3 \cdot E}{4} \end{pmatrix}$

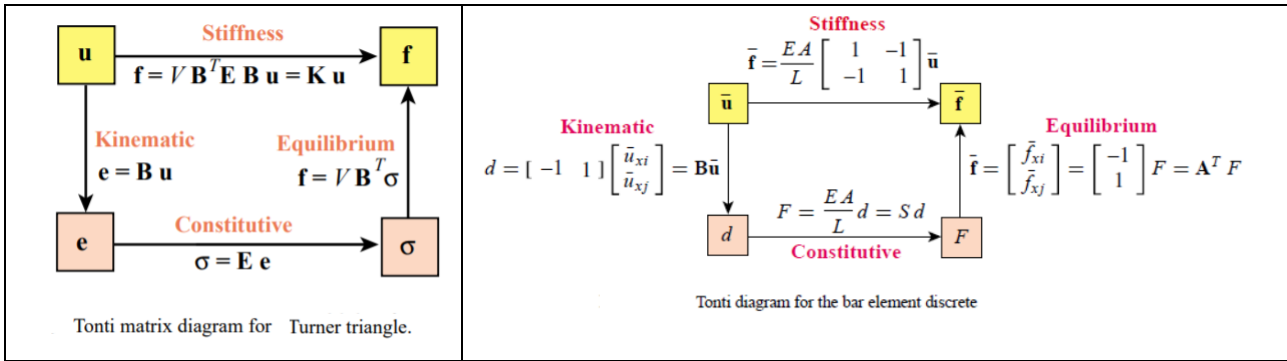
Si:

$$A_1 = \frac{3}{4} \quad A_3 = \frac{1}{\sqrt{2}}$$

Matriz de rigidez K_{tri}	Matriz de rigidez K_{bar}
$\begin{pmatrix} \frac{3-E}{4} & \frac{E}{4} & -\frac{E}{2} & \frac{E}{4} & \frac{E}{4} & 0 \\ \frac{E}{4} & \frac{3-E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & -\frac{E}{2} \\ -\frac{E}{2} & 0 & \frac{E}{2} & 0 & 0 & 0 \\ -\frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ -\frac{E}{4} & \frac{E}{4} & 0 & \frac{E}{4} & \frac{E}{4} & 0 \\ 0 & -\frac{E}{2} & 0 & 0 & 0 & \frac{E}{2} \end{pmatrix}$	$\begin{pmatrix} \frac{3-E}{4} & 0 & -\frac{3-E}{4} & 0 & 0 & 0 \\ 0 & \frac{3-E}{4} & 0 & 0 & 0 & -\frac{3-E}{4} \\ -\frac{3-E}{4} & 0 & E & \frac{E}{4} & -\frac{E}{4} & -\frac{E}{4} \\ 0 & 0 & \frac{E}{4} & \frac{E}{4} & -\frac{E}{4} & -\frac{E}{4} \\ 0 & 0 & -\frac{E}{4} & -\frac{E}{4} & \frac{E}{4} & \frac{E}{4} \\ 0 & -\frac{3-E}{4} & -\frac{E}{4} & -\frac{E}{4} & \frac{E}{4} & E \end{pmatrix}$

3. Why these two stiffness matrices are not equal?. Find a physical explanation.

Las consideraciones son diferentes, ya que en el caso triangular se considera que las tracciones son constantes a lo largo de todo el elemento, y se considera que las funciones de forma son lineales, con los grados de libertad en las esquinas del triángulo, considerando el área del elemento, y sus propiedades. En el caso de K_{bar} cada barra tendrá su esfuerzo, en función de la longitud, modulo de elasticidad, y su área transversal en el caso de barras se considera como un elemento constante a lo largo del mismo, se puede considerar como un resorte de rigidez constante, y con una deformación axial relativa entre el nudo de inicio y el final.



4. Consider nowidering $\nu \neq 0$ and extract some conclusions.

$$E = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \rightarrow \begin{bmatrix} \frac{E}{\nu^2-1} & \frac{E\nu}{\nu^2-1} & 0 \\ \frac{E\nu}{\nu^2-1} & \frac{E}{\nu^2-1} & 0 \\ 0 & 0 & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{\nu^2-1} \end{bmatrix}$$

$$K = \frac{h}{4A} \begin{pmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{pmatrix} \begin{bmatrix} \frac{E}{\nu^2-1} & \frac{E\nu}{\nu^2-1} & 0 \\ \frac{E\nu}{\nu^2-1} & \frac{E}{\nu^2-1} & 0 \\ 0 & 0 & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{\nu^2-1} \end{bmatrix} \begin{pmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{pmatrix} \rightarrow$$

$$\begin{bmatrix} \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & -\frac{E}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & -\frac{E\nu}{2(\nu^2-1)} & \frac{E}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E\nu}{2(\nu^2-1)} \\ \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E\nu}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & -\frac{E}{2(\nu^2-1)} & \frac{E\nu}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E}{2(\nu^2-1)} \\ \frac{E}{2(\nu^2-1)} & -\frac{E\nu}{2(\nu^2-1)} & -\frac{E}{2(\nu^2-1)} & 0 & 0 & -\frac{E}{2(\nu^2-1)} & -\frac{E\nu}{2(\nu^2-1)} & \frac{E\nu}{2(\nu^2-1)} \\ \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & 0 & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & 0 & 0 & 0 \\ \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & 0 & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & \frac{E(\frac{\nu}{2}-\frac{1}{2})}{2(\nu^2-1)} & 0 & 0 & 0 \\ \frac{E\nu}{2(\nu^2-1)} & \frac{E}{2(\nu^2-1)} & -\frac{E\nu}{2(\nu^2-1)} & 0 & 0 & -\frac{E}{2(\nu^2-1)} & -\frac{E\nu}{2(\nu^2-1)} & \frac{E}{2(\nu^2-1)} \end{bmatrix}$$

Al ser el módulo de poisson diferente de 0, la matriz de elasticidad aumenta en E_{12} , esto quiere decir que se considera como una placa el elemento triangular, y las deformaciones por fuerzas axiales, producirán deformaciones laterales.

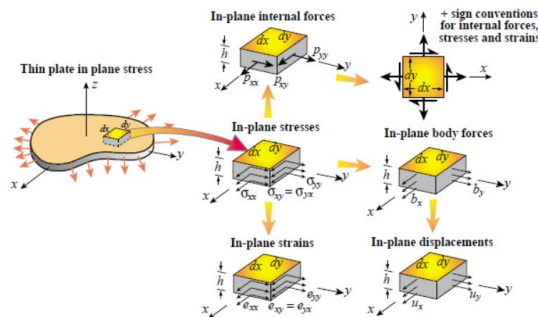


FIGURE 14.2. Notational conventions for in-plane stresses, strains, displacements and internal forces of a thin plate in plane stress.