Computational Structural mechanics and Dynamics

Assignment 3.1

(1)

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad , \quad \mu = G = \frac{E}{2(1+\nu)}$$

$$E = \frac{(1+\nu)(1-2\nu)\lambda}{\nu} \qquad \nu = \frac{E}{2\mu} - 1$$

After some basic operations we get:

$$\boxed{E = \frac{\mu(2\lambda + 3\mu)}{\lambda + \mu}} \qquad \boxed{\nu = \frac{\lambda}{2(\lambda+\mu)}}$$

(2)

Plane stress elastic matrix:

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\frac{\nu}{2} \end{bmatrix} \xrightarrow{\text{substituting}} \frac{8\mu^3 + 20\mu^2\lambda + 12\mu\lambda^2}{3\lambda^2 + 4\mu^2}\begin{bmatrix} 1 & \frac{\lambda}{2(\lambda+\mu)} & 0 \\ \frac{\lambda}{2(\lambda+\mu)} & 1 & 0 \\ 0 & 0 & \frac{\lambda+2\mu}{4(\lambda+\mu)} \end{bmatrix}$$

Planestrain elastic matrix:

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \xrightarrow{\text{substituting}} \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

(3)

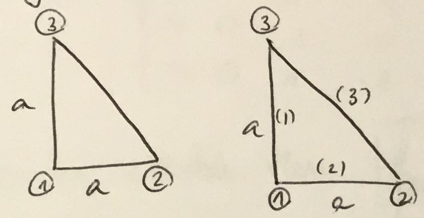
$$\boxed{E = E_\lambda + E_\mu}$$

$$E = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} = \text{Elastic matrix for plane strain.}$$

④ $E_\lambda = \begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_\mu = \begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$

$$E = \frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{E}{2(1+\upsilon)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Assignment 3.2



①

$$K_{tri} = \int_{\Omega^e} h B^T E B \, d\Omega = h B^T E B \int_{\Omega^e} d\Omega$$

Taking into account B by definition, $J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \left[J^{-1} \right] \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}$

Taking $N_1 = 1 - \xi - \eta$, $N_2 = \xi$, $N_3 = \eta$

$$B = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix} \Rightarrow \frac{h}{2} \underbrace{\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{B^T} \underbrace{\begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \frac{E}{2} \end{bmatrix}}_{E} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

So at the end we have:

$$K_{tri} = E \begin{bmatrix} 3/4 & 1/4 & -1/2 & -1/4 & -1/4 & 0 \\ 1/4 & 3/4 & 0 & -1/4 & -1/4 & -1/2 \\ -1/2 & 0 & 1/2 & 0 & 0 & 0 \\ -1/4 & -1/4 & 0 & 1/4 & 1/4 & 0 \\ -1/4 & -1/4 & 0 & 1/4 & 1/4 & 0 \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

For K_{bar}, we have 3 bars that are equal: (1=2) 3 is different.

$$k^{(1)} = EA\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad k^{(2)} = EA\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k^{(3)} = \frac{EA_3}{2\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

When we assemble them $\boxed{K_{bar} = K^{(1)} + K^{(2)} + K^{(3)}}$

(2) No, there is no way possible that could happen. Trying to chg the cross section values of K_{bar} may be difficult to make it look like K_{ni}.

But we could use: $\boxed{A = \frac{E}{2} \qquad A_3 = -2E\sqrt{2}}$

(3) These matrices aren't equal due the fact they are completely different even if they look alike. Finite element method base its solution on degrees of freedom of systems. In this case, those are completely different from each other.

(4)