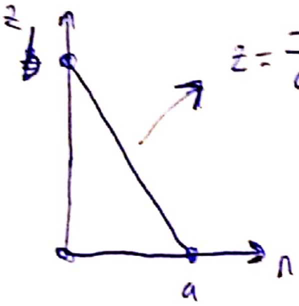
Oriol Falip

Assignment 4.1

1)




$$z = \frac{-b}{a} r + b$$

To compute $K^{(e)}$

$$\boxed{K^{(e)} = \iint_{\Omega} B^T E B \, d\Omega}$$

where

$$B = DN = \begin{pmatrix} \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial r} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \frac{\partial N_3}{\partial z} \\ \frac{N_1}{r} & \frac{N_2}{r} & \frac{N_3}{r} & 0 & 0 & 0 \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \frac{\partial N_3}{\partial z} & \frac{\partial N_1}{\partial r} & \frac{\partial N_2}{\partial r} & \frac{\partial N_3}{\partial r} \end{pmatrix}$$

2)

The sum of the rows and columns 2, 4 and 6 vanishes because it corresponds to the translation in the z direction of the element. then it is a solid rigid translation so no force is produced.

columns 1, 3, 5 doesn't vanishes because if joints are moved in the "r" direction, then it produces a force!!

3)

$$\vec{f}_{ext} = \int_A N^T b \, r \, dr \, dz = \int_A \begin{bmatrix} N_1 & 0 \\ N_2 & 0 \\ N_3 & 0 \\ 0 & N_1 \\ 0 & N_2 \\ 0 & N_3 \end{bmatrix} \begin{bmatrix} 0 \\ -g \end{bmatrix} r \, dr \, dz =$$

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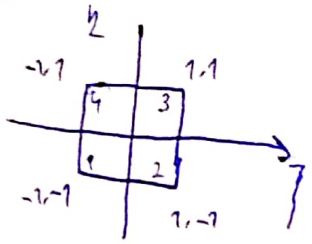
Assignment 4.2




Using line-product method:

$$\bar{N}_1 = A(\xi-1)(\eta-1)$$

$$1 = \bar{N}_1(-1,-1) = A(-1-1)(-1-1) = 4A \rightarrow \boxed{A = \frac{1}{4}}$$

then $\boxed{\bar{N}_1 = \frac{1}{4}(\xi-1)(\eta-1)}$

Doing the same procedure:

$$\boxed{\begin{aligned} \bar{N}_2 &= -\frac{1}{4}(\xi+1)(\eta-1) = \frac{1}{4}(\xi+1)(1-\eta) \\ \bar{N}_3 &= \frac{1}{4}(\xi+1)(\eta+1) \\ \bar{N}_4 &= -\frac{1}{4}(\xi-1)(\eta+1) = \frac{1}{4}(1-\xi)(\eta+1) \end{aligned}}$$

To find $N_5$ we also impose the SF to be equal to zero in the other nodes and equal to one in node 5:

$$\boxed{N_5 = (\xi-1)(\eta+1)(\xi+1)(\eta-1)}$$

Now if $N_i = \bar{N}_i + \alpha N_5$ $\quad i=1,2,3,4$

for $i=1$:

$$N_1 = \bar{N}_1 + \alpha N_5 = \frac{1}{4}(\xi-1)(\eta-1) + \alpha(\xi-1)(\eta+1)(\xi+1)(\eta-1)$$

$$N_1(0,0) = 0 = \frac{1}{4}(-1)(-1) + \alpha(-1)(1)(1)(-1) = +\frac{1}{4} + \alpha = 0 \Longrightarrow \boxed{\alpha = -\frac{1}{4}}$$

Then $\boxed{N_i = \bar{N}_i + -\frac{1}{4}N_5}$

$$\sum_{i=1}^{5} N_i = \bar{N}_1 - \frac{1}{4}N_5 + \bar{N}_2 - \frac{1}{4}N_5 + \bar{N}_3 - \frac{1}{4}N_5 + \bar{N}_4 - \frac{1}{4}N_5 + N_5 = \boxed{\bar{N}_1 + \bar{N}_2 + \bar{N}_3 + \bar{N}_4} = 1$$

which we know that it is consistent!!!