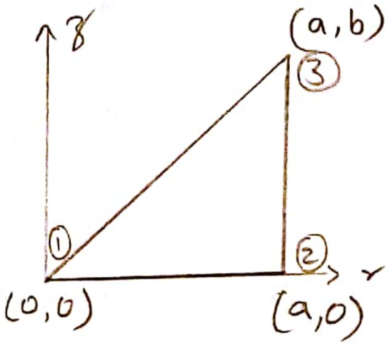
Assignment 4

4.1

Axisymmetric triangle

$r_1 = 0$ $\qquad$ $z_1 = z_2 = 0$

$r_2 = r_3 = a$ $\qquad$ $z_3 = b$.



$\nu = 0 \quad \therefore \quad E = E \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

$\Rightarrow$ Shape functions.

$$N_i = \frac{1}{2A}\left(f_i + r b_i + z c_i\right)$$

Where,

$$a_i = r_j z_k - r_k z_j$$

$$b_i = z_j - z_k$$

$$c_i = r_k - r_j$$

$f_1 = r_2 z_3 - r_3 z_2 = \quad ab - 0 = \underline{ab}$

$f_2 = r_3 z_1 - r_1 z_3 = \quad 0 - 0 = \underline{0}$

$f_3 = r_1 z_2 - r_2 z_1 = \quad 0 - 0 = \underline{0}$

$b_1 = z_2 - z_3 = -b$

$b_2 = z_3 - z_1 = b$

$b_3 = z_1 - z_2 = 0$

$c_1 = r_3 - r_2 = 0$

$c_2 = r_1 - r_3 = -a$

$c_3 = r_2 - r_1 = a$

$A = \frac{1}{2} ab$

$\therefore N_1 = 1 - r/a$

$N_2 = r/a - z/b$

$N_3 = z/b$

Stiffness matrix

$$K = 2\pi \bar{r} A \bar{B} E \bar{B}^T$$

$$B = (\bar{r}, \bar{z})$$

$$\bar{r} = \frac{r_1 + r_2 + r_3}{3}$$

$$\boxed{\bar{r} = \frac{2}{3} a}$$

$$\bar{z} = \frac{z_1 + z_2 + z_3}{3}$$

$$\boxed{\bar{z} = \frac{1}{3} b}$$

<u>Applying in shape functions</u>

$$\boxed{N_1 = \frac{1}{3}} \quad \boxed{N_2 = \frac{1}{3}} \quad \boxed{N_3 = \frac{1}{3}}$$

<u>Area of the triangle</u>

$$A = \frac{1}{2} \begin{bmatrix} r_1 & r_2 & r_3 \\ z_1 & z_2 & z_3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{2} ab$$

$$\bar{B}^T = \frac{1}{\cancel{2}\left(\frac{1}{\cancel{2}}ab\right)} \begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ b/2 & 0 & b/2 & 0 & b/2 & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix}$$

$$K = 2\pi \bar{r} A \bar{B} E \bar{B}^T$$

$$= \cancel{2}\pi \frac{1}{\cancel{2}}(ab) \times \frac{2}{3}a \times \bar{B} E \bar{B}^T$$

$$= \frac{2}{3} a^2 b \pi \bar{B} E B^T$$

$$B E B^T$$

$$= \frac{E}{4A^2} \begin{bmatrix} -b & 0 & b/2 & 0 \\ 0 & 0 & 0 & -b \\ b & 0 & b/2 & -a \\ 0 & -a & 0 & b \\ 0 & 0 & b/2 & a \\ 0 & a & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ b/2 & 0 & b/2 & 0 & b/2 & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix}$$

$$= \frac{E\pi 2}{3b} \begin{bmatrix} 5/4 & 0 & -3/4\, b^2 & 0 & \frac{1}{4}b^2 & 0 \\ & 1/2\, b^2 & 1/2\, ab & -1/2\, b^2 & -1/2\, ab & 0 \\ & & \left(\frac{5}{4}b^2 + \frac{1}{2}a^2\right) & -1/2\, ab & \left(\frac{1}{4}b^2 - 1/2\, a^2\right) & 0 \\ & & & \left(1/2\, b^2 + a^2\right) & 1/2\, ab & -a^2 \\ \text{symm} & & & & \frac{1}{4}b^2 + 1/2\, a^2 & 0 \\ & & & & & a^2 \end{bmatrix}$$

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$$K \quad U \quad = F$$

$$\begin{bmatrix} \frac{5}{4}b^2 & 0 & -\frac{3}{4}b^2 & 0 & \frac{1}{4}b^2 & 0 \\ & \frac{1}{2}b^2 & \frac{1}{2}ab^2 & -\frac{1}{2}b^2 & -\frac{1}{2}ab & 0 \\ & & \left(\frac{5}{4}b^2+\frac{1}{2}a^2\right) & -\frac{1}{2}ab & \left(\frac{1}{4}b^2-\frac{1}{2}a^2\right) & 0 \\ & & & \left(\frac{1}{2}b^2+a^2\right) & \left(\frac{1}{2}ab\right) & -a^2 \\ & \text{symm} & & & \left(\frac{1}{4}b^2+\frac{1}{2}a^2\right) & 0 \\ & & & & & a^2 \end{bmatrix} \begin{bmatrix} u_{r_1} \\ u_{z_1} \\ u_{r_2} \\ u_{z_2} \\ u_{r_3} \\ u_{z_3} \end{bmatrix} = \begin{bmatrix} f_{r_1} \\ f_{z_1} \\ f_{r_2} \\ f_{z_2} \\ f_{r_3} \\ f_{z_3} \end{bmatrix}$$

(2)

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \to 0 \qquad \varepsilon_{\theta\theta} = \frac{u_r}{r} \neq 0.$$

There is displacement in radial direction
∴ there is no stresses in radial direction.

On the other hand while rotating the stresses get formed.
∴ row 2, 4, 6 gets vanished on the other hand 1, 3, 5 remains in the matrix.

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3)

$$f = \int_A N^T b r \, dA \quad \cdots \text{(4)}$$

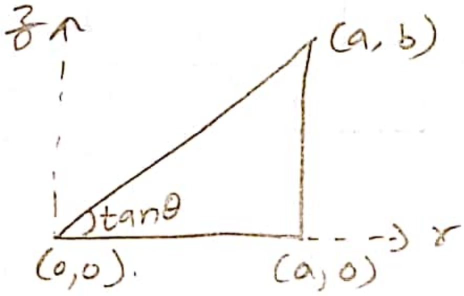
$$r = \sum_{i=1}^{3} r_i N_i$$

$$= r_1 N_1 + r_2 N_2 + r_3 N_3$$

$$= r$$

$$f = \int_r \int_z F \, dz \, dr$$

$$F = N^T b r$$



$$\frac{z}{r} = \tan\theta$$

$$= \frac{b}{a}$$

$$\boxed{z = b\,\frac{r}{a}}$$

$$F = N^T b r$$

$$= r \begin{bmatrix} (1 - r/a) & 0 \\ (r/a - z/b) & 0 \\ z/b & 0 \\ 0 & 1 - r/a \\ 0 & r/a - z/b \\ 0 & z/b \end{bmatrix} \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

$$= \left[0 \quad 0 \quad 0 \quad -gr(1-r/a) \quad -gr\left(\frac{r}{a}-\frac{z}{b}\right) \quad -gr\frac{z}{b} \right]$$

$$f = \int_0^a \int_0^{br/a} \left[0 \quad 0 \quad 0 \quad -gr(1-r/a) \quad -gr\left(\frac{r}{a}-\frac{z}{b}\right) \quad -gr\frac{z}{b} \right] dy\, dr$$

$$f_5 \; -\frac{gba^2}{8} = \int_0^a \int_0^{br/a} -gr\left(r/a - z/b\right) dz\, dr$$

$$-\frac{gba^2}{8} = \int_0^a \int_0^{br/a} -g\, r\, z/b \; dz\, dr$$

$$\boxed{\int_0^a \int_0^{br/a} 0 \; dz\, dr = 0}$$

$$\therefore \boxed{f = \frac{-ga^2b}{12} \left[0 \quad 0 \quad 0 \quad 1 \quad 3/2 \quad 3/2 \right]}$$