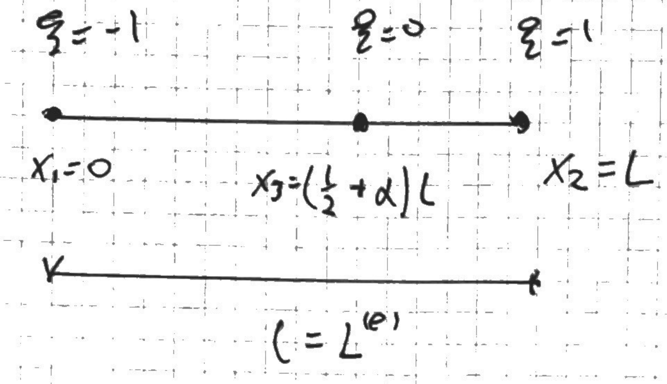
Assignment 4.1



$-\frac{1}{2} < \alpha < \frac{1}{2}$

1)

$$N_1(\xi) = \frac{1}{2}\xi(\xi-1)$$

$$N_3(\xi) = 1-\xi^2$$

$$N_2(\xi) = \frac{1}{2}\xi(1+\xi)$$

$$\Rightarrow \quad x = \sum_{i=1}^{3} x_i N_i = \frac{1}{2}\xi(1+\xi)\cdot L + \left(\frac{1}{2}+\alpha\right)\cdot L\,(1-\xi^2)$$

$$= L\left[\frac{\xi}{2} + \frac{\xi^2}{2} + \frac{1}{2} - \frac{\xi^2}{2} + \alpha - \alpha\xi^2\right]$$

$$\frac{dx}{d\xi} = J = L\left[\frac{1}{2} + \xi - \xi - 2\alpha\xi\right]$$

$$= L\left[\frac{1}{2} - 2\alpha\xi\right]$$

if $-1/4 < \alpha < 1/4$:

$$\left.\begin{aligned} \alpha = -1/4 &\Rightarrow J = L[1/2 + \xi/2] \\ \alpha = 1/4 &\Rightarrow J = L[1/2 - 1/2\,\xi] \end{aligned}\right\} \begin{array}{l} J > 0 \text{ if} \\ \underline{\underline{-1/4 < \alpha < 1/4}} \end{array}$$

if $\alpha = 0 \Rightarrow \underline{\underline{J = L/2 = \text{const.}}}$

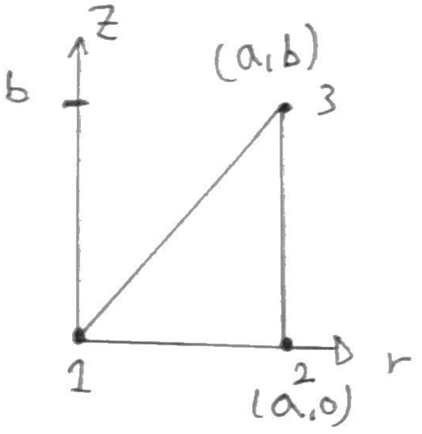
2)
$$e = \frac{du}{dx} = \frac{du}{d\xi} \cdot \frac{d\xi}{dx} = \frac{1}{J} \cdot \frac{du}{d\xi}$$

$$= \frac{1}{J} \frac{\partial N_i(\xi)}{d\xi} u$$

$$= \frac{1}{L[1/2 - 2\alpha\xi]} \begin{bmatrix} \xi - 1/2 & \xi + 1/2 & -2\xi \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{\mathbb{B}^e = \frac{1}{L[1/2 - 2\alpha\xi]} \begin{bmatrix} \xi - 1/2 & \xi + 1/2 & -2\xi \end{bmatrix}}}$$

Assignment 4.2



$$\nu = 0 \quad \mathbb{E} = E\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$A = \frac{ab}{2}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{zz} \\ \varepsilon_{\theta\theta} \\ \gamma_{rz} \end{bmatrix} = \begin{bmatrix} \partial/\partial r & 0 \\ 0 & \partial/\partial z \\ 1/r & 0 \\ \partial/\partial z & \partial/\partial r \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & 0 & & \\ & 0 & & N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Relation between area and axisymmetric coordinates:

$$N_1 = \frac{1}{2A}\left[r_2 z_3 - z_2 r_3 \quad z_{23} \cdot r \quad r_{32} \cdot z \right]$$

$$= \frac{1}{ab}\left[ab - 0 \quad (0-b)\, r \quad 0 \cdot z \right]$$

$$= \frac{1}{ab}\left(ab - br \right)$$

$$r_2 z_3 - z_2 r_3 = a \cdot b - 0 \cdot a = ab$$

$$z_{23} = 0 - b = -b$$

$$r_{32} = a - a = 0$$

$$N_2 = \frac{1}{2A}\left[r_3 z_1 - z_3 r_1 + z_{31} \cdot r + r_{13} z \right]$$

$$= \frac{1}{ab}\left[0 - 0 + (b-0) r + (0-a) z \right]$$

$$= \frac{1}{ab}\left(br - az \right)$$

$$N_3 = \frac{1}{ab}\left[r_1 z_2 - r_2 z_1 + z_{12} r + r_{21} z \right]$$

$$= \frac{1}{ab}\left[(0 - 0) + 0 \cdot r + a z \right]$$

$$= \frac{1}{ab}\left(az \right)$$

$$\Rightarrow \mathbb{B} = \frac{1}{ab}\begin{bmatrix} -b & b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a & a \\ \frac{b}{r}(a-r) & b-\frac{az}{r} & \frac{az}{r} & 0 & 0 & 0 \\ 0 & -a & -a & -b & b & 0 \end{bmatrix}$$

Now $\mathbb{K}^e = \int_V \mathbb{B}^T \mathbb{C}\, \mathbb{B}\; dV$

$$= \int_V \mathbb{B}^T \mathbb{C}\, \mathbb{B}\; r\,dr\,d\theta\,dz$$

$$= 2\pi \iint \mathbb{B}^T \mathbb{E}\, \mathbb{B} \cdot r\,dr\,dz$$

$0 \le r \le a$ and $0 \le z \le \frac{b}{a} r$

$\Rightarrow$ Analytically solved using Matlab:

$$\mathbb{K}^e = 2\pi \int_0^a \int_0^{br/a} \mathbb{B}^T \mathbb{E}\, \mathbb{B}\; r\,dz\,dr$$

$$= 2\pi \cdot E \begin{bmatrix} 2b/3 & -b/4 & b/12 & 0 & 0 & 0 \\ -b/4 & a^2/6b + 4b/9 & -\frac{a^2}{6b} + \frac{b}{18} & a/6 & -a/6 & 0 \\ b/12 & -a^2/6b + \frac{b}{18} & a^2/6b + b/9 & -a/6 & a/6 & 0 \\ 0 & a/6 & -a/6 & b/6 & -b/6 & 0 \\ 0 & -a/6 & a/6 & -b/6 & \frac{a^2}{3b} + b/6 & \frac{-a^2}{3b} \\ 0 & 0 & 0 & 0 & \frac{-a^2}{3b} & \frac{a^2}{3b} \end{bmatrix}$$

2)

Since I have chosen to arrange the $\mathbb{B}$ matrix according to the presentation on Cimne "CSMD-07-Revolution" page 14:

$$\mathbb{B} = \begin{bmatrix} \partial/\partial r & 0 \\ 0 & \partial/\partial z \\ 1/r & 0 \\ \partial/\partial z & \partial/\partial r \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_1 & N_2 & N_3 \end{bmatrix}$$

my arrengemet of the dofs are different and thus for me it's column (rows) 4,5,6 that cancle each other and column (rows) 1,2,3 that dont cancle.

$$4+5+6 \Rightarrow [0, 0, 0, 0, 0, 0] \Rightarrow 0$$

$$1+2+3 \Rightarrow [6/2, 6/4, 6/4, 0, 0, 0] = 6$$

The reason for this is that column 4,5,6 are related to the dof's z_1, z_2 and z_3, and we have not imposed BC's to prevent rigid body motion in this direction.

On the other hand we have prevention of rigid body motion in r-direction and thus column 1,2,3 related to r_1, r_2, r_3 do not vanish.

3) $\mathbb{b} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$

$$\mathbb{f}^e = \int_V \mathbb{N}^T \mathbb{b}\, dV = 2\pi \int_A \mathbb{N}^T \mathbb{b}\, r\, dr\, d\theta$$

$$= 2\pi \int_A \begin{bmatrix} N_1 & 0 \\ N_2 & 0 \\ N_3 & 0 \\ 0 & N_1 \\ 0 & N_2 \\ 0 & N_3 \end{bmatrix} \begin{bmatrix} 0 \\ -g \end{bmatrix} r\, dr\, d\theta$$

$$= 2\pi \int_0^a \int_0^{br/a} \frac{1}{ab} \begin{bmatrix} ab-br & 0 \\ br-az & 0 \\ az & 0 \\ 0 & ab-br \\ 0 & br-az \\ 0 & az \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} g r\, dz\, dr$$

Analytically solved in matlab

$$= 2\pi \cdot g \begin{bmatrix} 0 \\ 0 \\ 0 \\ a^2/12 \\ a^2 \cdot (b+1/2))/4b \\ -a^2/8b \end{bmatrix}$$