Computational Structural Mechanics and Dynamics:

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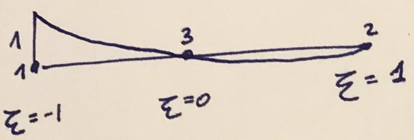
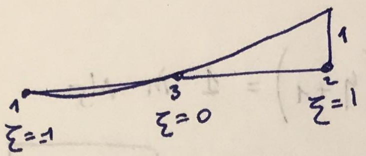
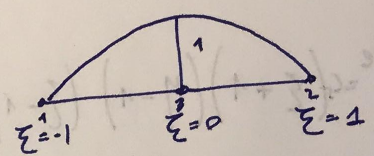
Assignment 5:

Problem 1:

$N_1^{(e)}(\xi) = a_0 + a_1\xi + a_2\xi^2$ $\quad N_2^{(e)}(\xi) = b_0 + b_1\xi + b_2\xi^2$ $\quad N_3^{(e)}(\xi) = c_0 + c_1\xi + c_2\xi^2$







a) For $\xi = -1$:

$N_1^e(\xi=-1) = a_0 - a_1 + a_2 = 1$

$N_2^e(\xi=-1) = b_0 - b_1 + b_2 = 0$

$N_3^e(\xi=-1) = c_0 - c_1 + c_2 = 0$

$\begin{cases} -a_1 + a_2 = 1 \\ a_1 + a_2 = 0 \end{cases} \Rightarrow \boxed{a_1 = -1/2}\ \boxed{a_2 = 1/2}$

$\begin{cases} b_1 + b_2 = 1 \\ -b_1 + b_2 = 0 \end{cases} \Rightarrow \boxed{b_1 = 1/2}\ \boxed{b_2 = 1/2}$

$\begin{cases} c_1 + c_2 + 1 = 0 \\ -c_1 + c_2 + 1 = 0 \end{cases} \Rightarrow \boxed{c_1 = 0}\ \boxed{c_2 = -1}$

For $\xi = 0$:

$N_1^e(\xi=0) = \boxed{a_0 = 0}$

$N_2^e(\xi=0) = \boxed{b_0 = 0}$ $\Rightarrow$

$N_3^e(\xi=0) = \boxed{c_0 = 1}$

For $\xi = 1$:

$N_1^e(\xi=1) = a_1 + a_2 = 0$

$N_2^e(\xi=1) = b_1 + b_2 = 1$

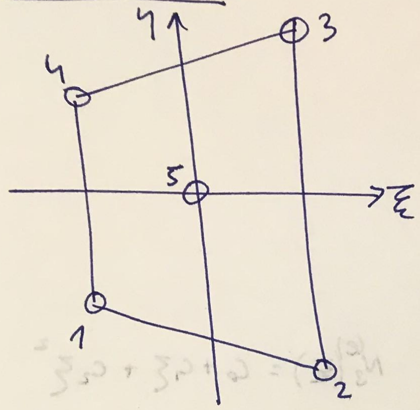
$N_3^e(\xi=1) = \underbrace{c_0}_{1} + c_1 + c_2 = 0$

c) $\dfrac{\partial N_1}{\partial \xi} = \xi - 1/2$ $\quad \dfrac{dN_2}{d\xi} = \dfrac{1}{2} + \xi$ $\quad \dfrac{dN_3}{d\xi} = -2\xi$, derivatives respect the natural coordinates.

b) Verification of the sum:

$N_1 + N_2 + N_3 = -\frac{1}{2}\xi + \frac{1}{2}\xi^2 + \frac{1}{2}\xi + \frac{1}{2}\xi^2 + 1 - \frac{2}{2}\xi^2 = \boxed{1}$ The sum is verified.

Problem 5.2:



$$\boxed{N_5^e = c_5 L_{1-2} L_{23} L_{3-4} L_{4-1}}$$

$$\begin{cases} L_{1-2} \Rightarrow \eta = -1 \\ L_{2-3} \Rightarrow \xi = 1 \\ L_{3-4} \Rightarrow \eta = 1 \\ L_{4-1} \Rightarrow \xi = -1 \end{cases}$$

$N_5^e = c_5 (\xi+1)(\eta-1)(\xi-1)(\eta+1) = 1$ in N_5

$N_5^e(\xi=0, \eta=0) = (-1)(-1)(1)(1) c_5 = 1 \Rightarrow \boxed{c_5 = 1}$

$N_5 = (\eta+1)(\xi-1)(\eta-1)(\xi+1)$

For 4 noded elents:

$$N_i^e = \frac{1}{4}(1+\xi_i \xi)(1+\eta_i \eta) \Rightarrow \begin{cases} N_1^e = \frac{1}{4}(1-\xi)(1-\eta) \\ N_2^e = \frac{1}{4}(1+\xi)(1-\eta) \\ N_3 = \frac{1}{4}(1+\xi)(1+\eta) \\ N_4 = \frac{1}{4}(1-\xi)(1+\eta) \end{cases} \quad \boxed{N_i = 1}$$

$N_i + \alpha N_5 = 0$

$N = N_1 + N_2 + N_3 + N_4 + \alpha N_5 = 1$

$N_i^e = \frac{1}{4}(1+\xi_i \xi)(1+\eta_i \eta) + \alpha(\eta+1)(\xi-1)(\eta-1)(\xi-1) = 0 \quad \boxed{\alpha = \frac{3}{4}}$

$N_i = \frac{1}{4}(1+\xi_1 \xi)(1+\eta_1 \eta) + \frac{3}{4}(\eta+1)(\xi-1)(\eta-1)(\xi-1) = \boxed{1}$

Problem 5.3 :

Following the convergence requirements (Rank sufficiency parameter)

$$\boxed{\begin{array}{ll} \text{rank } K : & r = \min(n_F - n_r, n_E \cdot n_G) \\ \text{rank deficiency} & d = (n_F - n_r) - r \end{array}}$$

n_F = number of element DoF

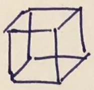
n_R = number of independent rigid body modes

n_E = number of ε - stress - strain matrix

n_G = number of Gauss points in integration rule for K.

r = actual rank of stiffness matrix

we want to determine.

a) 8 node hexahedron =  typical cube. 3Dimensions: 6 degrees of freedom

$$r = \min\left(48 - 12, n_G \cdot 12\right) \Rightarrow r = \min\left(36, 12 \cdot n_G\right) \quad \boxed{n_G = 3}$$

$$\hookrightarrow \boxed{n_G \cdot n_E \geq n_F - n_r}$$

b) 20-node hexahedron

$$r = \min\left(\overset{120}{\cancel{48}}, n_G \cdot 12\right) \Rightarrow \boxed{n_G = 10}$$

c) 27 node hexahedron

$$r = \min\left(152, n_G \cdot 12\right) \Rightarrow \boxed{n_G = 13} \longrightarrow \boxed{n_G \cdot n_E \geq n_F - n_r}$$

d) 64-node hexahedron

$$r = \min\left(372, n_G \cdot 12\right) = \boxed{n_G = 31}$$