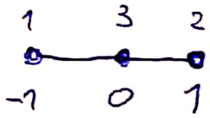
Oriol Falip

Assignment 5.1

$$\begin{bmatrix}1\\x\\u\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ x_1 & x_2 & x_3\\ u_1 & u_2 & u_3\end{bmatrix}\begin{bmatrix}N_1\\N_2\\N_3\end{bmatrix}$$



$x_1 = 0 \quad x_2 = L \quad x_3 = \frac{1}{2}L + \alpha L$

$$\begin{cases} N_1(\xi) = \frac{1}{2}(\xi-1)\xi \\ N_2 = \frac{1}{2}(\xi+1)\xi \\ N_3 = -(\xi+1)(\xi-1) = -(\xi^2-1) \end{cases}$$

$$x = 0\cdot N_1 + L N_2 + \left(\tfrac{1}{2}L+\alpha L\right)N_3 = \tfrac{1}{2}L\left(\xi^2+\xi\right) + -\left(\tfrac{1}{2}L+\alpha L\right)\left(\xi^2-1\right)$$

$$J = \frac{\partial x}{\partial \xi} = \frac{2L\xi}{2} + \frac{1}{2}L - 2\left(\tfrac{1}{2}L+\alpha L\right)\xi = L\xi + \tfrac{1}{2}L - 2\left(\tfrac{1}{2}L+\alpha L\right)\xi$$

$$J = 0 = \xi + \tfrac{1}{2} - (1+\alpha\cdot 2)\xi = \tfrac{1}{2} - 2\alpha\xi \rightarrow \boxed{\xi = \frac{1}{4}\frac{1}{\alpha}}$$

$\boxed{\xi = \frac{1}{4\alpha}}$ this gives us the point of the element bar for which the jacobian vanishes. Remember that $\xi \in [-1, 1]$ and $\alpha \in [-\frac{1}{2}, \frac{1}{2}]$

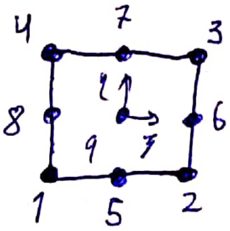
Then, $\boxed{\text{if } \alpha = -\frac{1}{4} \rightarrow \xi = -1}$

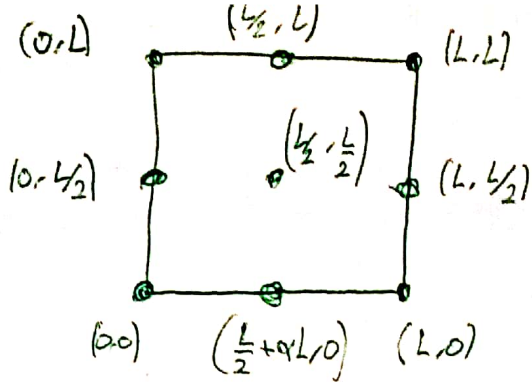
if $\alpha < -\frac{1}{4}$ for example $\left(-\frac{1}{5}\right)$ then $\xi < -1$ for example $-1{,}0001\ldots$ which is not possible

If $\alpha = \frac{1}{4}$ then $\xi = 1$. if $\alpha > \frac{1}{4}$ then $\xi > 1$ which is not possible!

Then $\boxed{\alpha \in [-\frac{1}{4}, \frac{1}{4}]}$

Assignment 5.2





Shape functions are written as:

$$\begin{cases} N_1 = \frac{1}{4}(\xi^2-\xi)(\eta^2-\eta) \\ N_2 = \frac{1}{4}(\xi^2+\xi)(\eta^2-\eta) \\ N_3 = \frac{1}{4}(\xi^2+\xi)(\eta^2+\eta) \\ N_4 = \frac{1}{4}(\xi^2-\xi)(\eta^2+\eta) \end{cases}
\qquad
\begin{cases} N_5 = \frac{1}{2}(\eta^2-\eta)(1-\xi^2) \\ N_6 = \frac{1}{2}(\xi^2+\xi)(1-\eta^2) \\ N_7 = \frac{1}{2}(\eta^2+\eta)(1-\xi^2) \\ N_8 = \frac{1}{2}(\xi^2+\xi)(1+\eta^2) \end{cases}
\qquad
\left[N_9 = (1-\xi^2)(1-\eta^2) \right.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & L & L & 0 & \left(\frac{L}{2}+\alpha L\right) & L & L/2 & 0 & L/2 \\ 0 & 0 & L & L & 0 & L/2 & L & L/2 & L/2 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \end{bmatrix}$$

$$x = L N_2 + L N_3 + (L/2 + \alpha L) N_5 + L N_6 + \frac{L}{2} N_7 + L/2\, N_9$$

$$y = L N_3 + L N_4 + \frac{L}{2} N_6 + L N_7 + \frac{L}{2} N_8 + \frac{L}{2} N_9$$

$$J(\xi,\eta) = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

let's compute the derivatives:

$$\frac{\partial x}{\partial z} = \frac{L}{4}(2z+1)(y^2-y) + \frac{L}{4}(2z+1)(y^2+y) + \left(\frac{L}{2}+\alpha L\right)\frac{1}{2}(y^2-y)(-2z) + \frac{L}{2}(2z+1)(1-y^2) +$$

$$+ \frac{L}{4}(y^2+y)(-2z) + \frac{L}{2}(1-y^2)(-2z)$$

evaluating at $(1,-1)$:

$$\left.\frac{\partial x}{\partial z}\right|_{(1,-1)} = \frac{6L}{4} + \left(\frac{L}{2}+\alpha L\right)(-2) = \frac{6L}{4} - L - 2\alpha L = \frac{2}{4}L - 2\alpha L = \boxed{\frac{L}{2} - 2\alpha L}$$

$$\frac{\partial y}{\partial z} = \frac{L}{4}(2z+1)(y^2+y) + \frac{L}{4}(2z-1)(y^2+y) + \frac{L}{4}(2z+1)(1-y^2) + \frac{L}{2}(y^2+y)(-2z) +$$

$$+ \frac{L}{4}(2z+1)(1-y^2) + \frac{L}{2}(-2z)(1-y^2)$$

$$\left.\frac{\partial y}{\partial z}\right|_{(1,-1)} = \boxed{0}$$

$$\frac{\partial x}{\partial y} = \frac{L}{4}(z^2+z)(2y-1) + \frac{L}{4}(z^2+z)(2y+1) + \left(\frac{L}{2}+\alpha L\right)\frac{1}{2}(2y-1)(1-z^2) +$$

$$+ \frac{L}{2}(z^2+z)(-2y) + \frac{L}{4}(2y+1)(1-z^2) + \frac{L}{2}(1-z^2)(-2y)$$

$$\left.\frac{\partial x}{\partial y}\right|_{(1,-1)} = \frac{L}{4}2(-3) + \frac{L}{4}2(-1) + \frac{L}{2}2\cdot 2 = -\frac{3L}{2} - \frac{L}{2} + 2L = \boxed{0}$$

$$\frac{\partial y}{\partial \eta} = \frac{L}{4}(\xi^2+\xi)(2\eta+1) + \frac{L}{4}(\xi^2-\xi)(2\eta+1) + \frac{L}{4}(\xi^2+\xi)(-2\eta) + \frac{L}{2}(2\eta+1)(1-\xi^2) +$$

$$+ \frac{L}{2}\frac{1}{2}(\xi^2+\xi)(-2\eta) + \frac{L}{2}(1-\xi^2)(-2\eta)$$

$$\left.\frac{\partial y}{\partial \eta}\right|_{(1,-1)} = \frac{L}{4}\,2\cdot(-1) + \frac{L}{4}\,2\cdot 2 + \frac{L}{4}\,2\cdot 2 = -\frac{L}{2} + L + L = \boxed{\frac{3L}{4}}$$

$$J = \begin{bmatrix} \frac{L}{2} - 2\alpha L & 0 \\ 0 & \frac{3L}{4} \end{bmatrix} = \frac{3L}{4}\left(\frac{L}{2} - 2\alpha L\right) = 0 \Rightarrow \frac{L}{2} - 2\alpha L = 0$$

$$\boxed{\alpha = \frac{1}{4}}$$