Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports de Barcelona

UNIVERSITAT POLITÈCNICA DE CATALUNYA

Nombre del estudiante: Rosa Eva González

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Descripción: Deber 5

Problema 1

Consider a three-node bar element referred to the natural coordinate ξ. The two end nodes and the mid node are identified as 1, 2 and 3 respectively. The natural coordinates of nodes 1, 2 and 3 are $\xi = -1$, $\xi = 1$ and $\xi = 0$, respectively. The variation of the shape functions $N_1(\xi)$, $N_2(\xi)$ and $N_3(\xi)$ is sketched in the figure below. These functions must be quadratic polynomials in ξ:

$$N_1^e(\xi) = a_0 + a_1\xi + a_2\xi^2 \quad N_2^e(\xi) = b_0 + b_1\xi + b_2\xi^2 \quad N_3^e(\xi) = c_0 + c_1\xi + c_2\xi^2$$

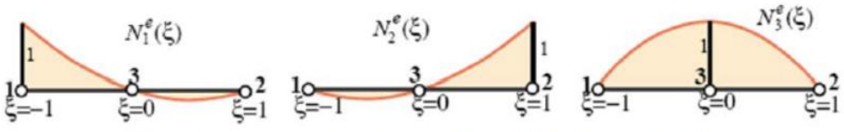


Figure.- Isoparametric shape functions for 3-node bar element (sketch). Node 3 has been drawn at the 1-2 midpoint but it may be moved away from it.

a) Determine the coefficients a_0, through, c_2 using the node value conditions depicted in figure. For example, N_{e1}=1 for ξ=1 and 0 for the rest of natural coordinates. The rest of the nodes follow the same scheme.

Condiciones de N1:

$$N1(-1) := -1$$

$$N1(0) := 0$$

$$N1(1) := 0$$

Ecuación:

$$N1 := a0 + a1 \cdot \xi + a2 \cdot \xi^2$$

Sistema de ecuaciones:

$$1 = a0 - a1 + a2$$

$$0 = a0$$

$$0 = a0 + a1 + a2$$

Resolución de incógnitas:

$$ans := find(a0, a1, a2) \rightarrow \begin{pmatrix} 0 \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$a0 := ans_0 \rightarrow 0$$

$$a1 := ans_1 \rightarrow -\frac{1}{2}$$

$$a2 := ans_2 \rightarrow \frac{1}{2}$$

Remplazando:

$$N1 := a0 + a1 \cdot \xi + a2 \cdot \xi^2 \rightarrow \text{ simplify } \rightarrow \frac{\xi \cdot (\xi - 1)}{2}$$

Condiciones de N2:

$$N2(-1) := 0$$

$$N2(0) := 0$$

$$N2(1) := 1$$

Ecuación:

$$N2 := b0 + b1 \cdot \xi + b2 \cdot \xi^2$$

Sistema de ecuaciones:

$$0 = b0 - b1 + b2$$

$$0 = b0$$

$$1 = b0 + b1 + b2$$

Resolución de incógnitas:

$$ans := find(b0, b1, b2) \rightarrow \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$b0 := ans_0 \rightarrow 0$$

$$b1 := ans_1 \rightarrow \frac{1}{2}$$

$$b2 := ans_2 \rightarrow \frac{1}{2}$$

Remplazando:

$$N2 := b0 + b1 \cdot \xi + b2 \cdot \xi^2 \rightarrow \text{ simplify } \rightarrow \frac{\xi \cdot (\xi + 1)}{2}$$

Condiciones de N3:

$$N3(-1) := 0$$

$$N3(0) := 1$$

$$N3(1) := 0$$

Ecuación:

$$N2 := c0 + c1\cdot\xi + c2\cdot\xi^2$$

Sistema de ecuaciones:

$$0 = c0 - c1 + c2$$

$$1 = c0$$

$$0 = c0 + c1 + c2$$

Resolución de incógnitas:

$$ans := find(c0,c1,c2) \rightarrow \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$c0 := ans_0 \rightarrow 1$$

$$c1 := ans_1 \rightarrow 0$$

$$c2 := ans_2 \rightarrow -1$$

Remplazando:

$$N3 := c0 + c1\cdot\xi + c2\cdot\xi^2 \rightarrow \text{ simplify } \rightarrow 1 - \xi^2$$

b) Verify that their sum is identically one.

$$N1 := \frac{\xi\cdot(\xi - 1)}{2}$$

$$N2 := \frac{\xi\cdot(\xi + 1)}{2}$$

$$N3 := 1 - \xi^2$$

$$N1 + N2 + N3 \rightarrow \frac{\xi\cdot(\xi - 1)}{2} + \frac{\xi\cdot(\xi + 1)}{2} - \xi^2 + 1 \text{ simplify } \rightarrow 1$$

c) Calculate their derivatives respect to the natural coordinates.

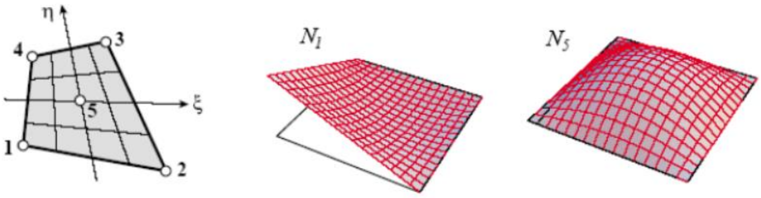
$$\frac{d}{d\xi}N1 \rightarrow \xi - \frac{1}{2}$$

$$\frac{d}{d\xi}N2 \rightarrow \xi + \frac{1}{2}$$

$$\frac{d}{d\xi}N3 \rightarrow -2\cdot\xi$$

Problema 2

A five node quadrilateral element has the nodal configuration shown if the figure with two perspective views of N^e_1 and N^e_5. Find five shape functions N^e_i, i=1,…,5 that satisfy compatibility and also verify that their sum is unity.



Hint: develop $N_5(\xi,\eta)$ first for the 5-node quad using the line-product method. Then the corner shape functions $\underline{N}_i(\xi,\eta)$, i=1,2,3,4, for the 4-node quad (already given in the notes). Finally combine $N_i = \underline{N}_i + \alpha N_5$ determining α so that all N_i vanish node 5. Check that $N_1 + N_2 + N_3 + N_4 + N_5 = 1$ identically.

N5:

$$N_i^e = c_i\, L_1\, L_2\, \ldots\, L_m,$$

$$N5 := c5\cdot L12\cdot L23\cdot L34\cdot L14$$

L12	L34
$n := -1$	$n := 1$
$n + 1 := 0$	$n - 1 := 0$

L23	L14
$\xi := 1$	$\xi := -1$
$\xi - 1 := 0$	$\xi + 1 := 0$

$L12 := n + 1$

$L23 := \xi - 1$

$L34 := n - 1$

$L14 := \xi + 1$

$$N5 := c5\cdot L12\cdot L23\cdot L34\cdot L14 \rightarrow c5\cdot(\xi - 1)\cdot(\xi + 1)\cdot(n - 1)\cdot(n + 1)$$

$$N5 := 1$$

$$\xi := 0$$

$$n := 0$$

$$c5\cdot(\xi - 1)\cdot(\xi + 1)\cdot(n - 1)\cdot(n + 1) \rightarrow c5$$

$$c5 := 1$$

Valor de α:

$$N_i = \underline{N_i} + \alpha N_5$$

$$N_i^e = \tfrac{1}{4}(1 + \xi_i\, \xi)(1 + \eta_i\, \eta).$$

$\underline{N_i}$ ·

$\xi1 := -1$ $\eta1 := -1$

$\xi2 := 1$ $\eta2 := -1$

$\xi3 := 1$ $\eta3 := 1$

$\xi4 := -1$ $\eta4 := 1$

$$N1 := \frac{1}{4}\cdot(1 + \xi1\cdot\xi)\cdot(1 + \eta1\cdot\eta) \text{ explicit, ALL } \rightarrow \frac{1}{4}\cdot[1 + (-1)\cdot\xi]\cdot[1 + (-1)\cdot\eta]$$

$$N2 := \frac{1}{4}\cdot(1 + \xi2\cdot\xi)\cdot(1 + \eta2\cdot\eta) \text{ explicit, ALL } \rightarrow \frac{1}{4}\cdot(1 + 1\cdot\xi)\cdot[1 + (-1)\cdot\eta]$$

$$N3 := \frac{1}{4}\cdot(1 + \xi3\cdot\xi)\cdot(1 + \eta3\cdot\eta) \text{ explicit, ALL } \rightarrow \frac{1}{4}\cdot(1 + 1\cdot\xi)\cdot(1 + 1\cdot\eta)$$

$$N4 := \frac{1}{4}\cdot(1 + \xi4\cdot\xi)\cdot(1 + \eta4\cdot\eta) \text{ explicit, ALL } \rightarrow \frac{1}{4}\cdot[1 + (-1)\cdot\xi]\cdot(1 + 1\cdot\eta)$$

Reemplazando N1, en $N_1 = \underline{N_1} + \alpha * N_5$

$$N1 = \frac{1}{4}\cdot(1 + 1\cdot\xi)\cdot[1 + (-1)\cdot\eta] + \alpha\cdot\left(\eta^2 - 1\right)\cdot\left(\xi^2 - 1\right)$$

$$N1 := 0$$

$$\xi := 0$$

$$\eta := 0$$

$$\frac{1}{4}\cdot(1 + 1\cdot\xi)\cdot[1 + (-1)\cdot\eta] + \alpha\cdot\left(\eta^2 - 1\right)\cdot\left(\xi^2 - 1\right) \text{ explicit, ALL } \rightarrow \frac{1}{4}\cdot(1 + 1\cdot 0)\cdot[1 + (-1)\cdot 0] + \alpha\cdot\left(0^2 - 1\right)\cdot\left(0^2 - 1\right)$$

$$0 = \frac{1}{4}\cdot(1 + 1\cdot 0)\cdot[1 + (-1)\cdot 0] + \alpha\cdot\left(0^2 - 1\right)\cdot\left(0^2 - 1\right) \text{ solve}, \alpha \rightarrow -\frac{1}{4}$$

El valor de α, es igual para $N_2, N_3,\ y\ N_4$

Sumatorias de funciones de forma:

$$N1 := \frac{1}{4}\cdot\left[(1-\xi)\cdot(1-\eta) - \left(n^2 - 1\right)\cdot\left(\xi^2 - 1\right)\right]$$

$$N2 := \frac{1}{4}\cdot\left[(1+\xi)\cdot(1-\eta) - \left(n^2 - 1\right)\cdot\left(\xi^2 - 1\right)\right]$$

$$N3 := \frac{1}{4}\cdot\left[(1+\xi)\cdot(1+\eta) - \left(n^2 - 1\right)\cdot\left(\xi^2 - 1\right)\right]$$

$$N4 := \frac{1}{4}\cdot\left[(1-\xi)\cdot(1+\eta) - \left(n^2 - 1\right)\cdot\left(\xi^2 - 1\right)\right]$$

$$N5 := \left(n^2 - 1\right)\cdot\left(\xi^2 - 1\right)$$

$$N1 + N2 + N3 + N4 + N5 \rightarrow \frac{(\xi-1)\cdot(\eta-1)}{4} - \frac{(\xi-1)\cdot(\eta+1)}{4} - \frac{(\xi+1)\cdot(\eta-1)}{4} + \frac{(\xi+1)\cdot(\eta+1)}{4} \text{ simplify } \rightarrow 1$$

Problema 3

Which minimum integration rules of Gauss-product type gives a rank sufficient stiffness matrix for these elements:

1. the 8-node hexahedron
2. the 20-node hexahedron
3. the 27-node hexahedron
4. the 64-node hexahedron

	n (nodos)	nf (Grados de libertad) n*3	nf-nr	ng min =(nf-nr)/ne	Mínimo puntos de integración Gauss
8-node hexahedron	8	24	18	3	2x2x2
20-node hexahedron	20	60	54	9	3x3x3
27-node hexahedron	27	81	75	13	3x3x3
64-node hexahedron	64	192	186	31	4x4x4

nr (modos de cuerpo rigido)	6
ne(matriz de rigidez 6x6)	6

$$ne * ng \geq nF - nr$$

$$ng \geq \frac{nf - nr}{ne}$$

Puntos de intregración Gauss				
	x	y	z	ng
1x1x1	1	1	1	1
2x2x2	2	2	2	8
3x3x3	3	3	3	27
4x4x4	4	4	4	64
5x5x5	5	5	5	125
6x6x6	6	6	6	216