CSMD HW 5

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1 1st Question

According to the isoparametric representation, the geometry is interpolated using the same shape functions as the unknowns. For 1d quadratic elements, the geometric representation is as follows ($x_1 = 0, x_2 = L, x_3 = 0.5L + \alpha L$):

$$
\begin{aligned}
x &= N_1 x_1 + N_2 x_2 + N_3 x_3 \\
&= \frac{1}{2}\zeta(1-\zeta)x_1 + \frac{1}{2}\zeta(1+\zeta)x_2 + (1-\zeta^2)x_3 \\
&= \frac{1}{2}\zeta L + \frac{1}{2}\zeta^2 L + \frac{1}{2}L + \alpha L - \frac{1}{2}\zeta^2 L - \alpha\zeta^2 L \\
&= \frac{1}{2}\zeta L + \frac{1}{2}L + \alpha L - \alpha\zeta^2 L
\end{aligned}
$$

The Jacobian is defined as:

$$
\begin{aligned}
J = \frac{\partial x}{\partial \zeta} &= \frac{1}{2}L - 2\alpha\zeta L \\
0 &= \frac{1}{2}L - 2\alpha\zeta L \\
\alpha &= \frac{1}{4\zeta}
\end{aligned}
$$

The minimum α which makes the Jacobian vanishes, occurs at $\zeta = \pm 1$

$$\alpha = \pm\frac{1}{4}$$

The strain is calculated as follows

$$\epsilon = \frac{\partial \mathbf{N}}{\partial \zeta}\mathbf{u}J^{-1}$$

$J = 0$ at the end points, therefore, the strain tends to infinity which represents a singularity.

2 2nd Question

Same process is used for the 9-node quadrilateral element. A MATLAB code is implemented to show this result.

$$
\begin{aligned}
x &= \sum_{i=1}^{9} N_i x_i \\
y &= \sum_{i=1}^{9} N_i y_i
\end{aligned}
$$

The Jacobian in 2d is a matrix calculated as follows:

$$
J = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}
$$

For a 2d element with x from [0,1], y from [0,1], the Jacobian determinant is then calculated at node 2 ($\zeta = 1, \eta = -1$), we find that when node 5 is moved towards node 2 till it's halfway between the original location and node 2 (x=0.75), the Jacobian vanishes

3 MATLAB code

```
syms z i

N1= 0.25*(z-1)*(i-1)*z*i;
N2= -0.25*z*(1+z)*i*(1-i);
N3= 0.25*z*(1+z)*(1+i)*i;
N4=-0.250*1*(1-z)*(1+i)*i;
N5= -0.5*(1+z)*(1-z)*(1-i)*i;
N6= 0.5*z*(z+1)*(1+i)*(1-i);
N7= -0.5*(1-z^2)*(1+i)*i;
N8= -0.5*z*(1-z)*(1-i)*i;
N9= (1-z^2)*(1-i^2);

x = 0*N1+1*N2+1*N3+0*N4+0.75*N5+1*N6+0.5*N7+0*N8+0.5*N9;
y = 0*N1+0*N2+1*N3+1*N4+0*N5+0.5*N6+1*N7+0.5*N8+0.5*N9;

J=[diff(x,z),diff(y,z);diff(x,i),diff(y,i)];
subs(det(J), [z, i], [1, -1])
```

```
>> p2CSMD

ans =

0
```