ASSIGNMENT 1

Computational Structural Mechanics and Dynamics

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Background:

Timoshenko beam theory accounts for the effect of transverse shear deformation. Timoshenko beam elements require C^0 continuity for the deflection and rotation fields and, therefore, are simpler than Euler-Bernoulli beam elements. Unfortunately, they suffer generally from the so-called shear locking defect which yields unrealistically stiffer solutions for slender beams. The effect of transverse shear deformation is negligible for a slender beam (i.e. for a large value of the slenderness ratio $\lambda = \frac{L}{h} = \frac{L}{a} = Length/Thichness$). Hence, Timoshenko solution should coincide for this case with that of conventional Euler-Bernoulli theory. But it has been observed that as the beam slenderness increases the numerical solution is progressively stiffer than the exact one. This means that the 2-noded Timoshenko beam element is unable to reproduce the conventional solution for slender beams. Many procedures to eliminate shear locking in Timoshenko beam elements have been proposed. A popular method is to reduce the influence of the transverse shear stiffness by under-integrating the terms in ${K_s}^e$using a quadrature of one order less than is needed for exact integration (the so-called reduced integration). The terms of ${K_b}^e$ are still integrated exactly. The reduced integration method is valid for both thick and slender beams.

(A) The basic goal is to implement the reduced integration algorithm in the provided code in order to avoid the shear locking effect associate with the Timoshenko beam theory. The algorithm has been successfully implemented and the code has been modified. The modified code is presented in the Annex. The correctness of the implementation have been verified for the case of the cantilever beam under end point load. It has been observed that the end deflection ratio of the reduced integration model and the Euler Bernoulli beam theory model converges rapidly to one for ($\lambda \rightarrow \infty$) as the mesh is refined. The exact quadrature leads to shear locking, whereas the reduced one point quadrature successfully overcomes the shear locking effect.

(B) **Goal: -** The goal is to solve the given problem with a 64 element mesh with the (a) 2 nodes Euler Bernoulli element, (b) 2 nodes Timoshenko Full Integrate element, (c) 2 nodes Timoshenko Reduce Integration element. We need to Compare maximum displacements, moments and shear for the 3 elements against a/L relationship.

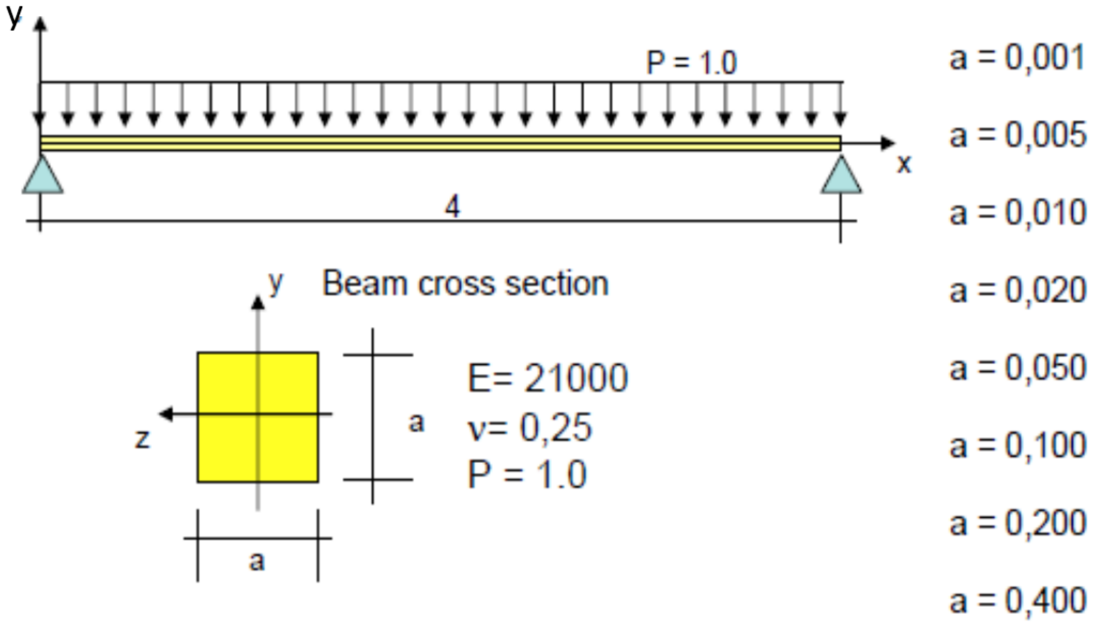

Figure 1: The given beam element under uniformly distributed loading

Case	a	a/L	$\lambda = L/a$	Inertia (I)=$\frac{a^4}{12}$
1	0.001	0.00025	4000	8.333 e-14
2	0.005	0.00125	800	52.08333 e-12
3	0.01	0.0025	400	8.3333 e-10
4	0.02	0.005	200	1.3333 e-8
5	0.05	0.0125	80	52.08333 e-8
6	0.1	0.025	40	8.3333 e-6
7	0.2	0.05	20	1.3333 e-4
8	0.4	0.1	10	21.3333 e-4
9	4	1	1	21.33333

P=1; $\gamma = 0.25$; E= 21000; L= 4m

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(a) **Maximum Displacement Comparison:-**

a/L	Euler Bernoulli	Timoshenko	Reduced Integration
0.00025	-1.90E+09	-1.46E+06	-1.90E+09
0.00125	-3.05E+06	-57401	-3.05E+06
0.0025	-1.90E+05	-13583	-1.90E+05
0.005	-8658	-2570.8	-8655.3
0.0125	-304.76	-200.43	-304.76
0.025	-19.048	-16.875	-19.069
0.05	-0.8658	-0.85259	-0.87261
0.1	-0.074405	-0.075561	-0.076161
1	-7.44E-06	-2.53E-05	-2.53E-05

Normalized maximum displacement with respect to the Euler Bernoulli Theory:

a/L	$\frac{U_{Max,EB}}{U_{Max,EB}}$	$\frac{U_{Max,Timoshenko}}{U_{Max,EB}}$	$\frac{U_{Max,Reduced\ Int.}}{U_{Max,EB}}$
0.00025	1	7.69E-04	1.00E+00
0.00125	1	1.88E-02	1.00E+00
0.0025	1	7.14E-02	1.00E+00
0.005	1	2.97E-01	1.00E+00
0.0125	1	6.58E-01	1.00E+00
0.025	1	8.85E-01	1.00E+00
0.05	1	9.80E-01	1.01E+00
0.1	1	1.02E+00	1.02E+00
1	1	3.40E+00	3.40E+00

The obtained normalized maximum displacement values have been plotted versus a/L values in the **Figure 2**. It can be clearly observed that for slender beams Timoshenko solution does not coincide with that of the conventional Euler-Bernoulli theory. As the effect of transverse shear deformation is negligible for a slender beam (i.e. for a large value of the slenderness ratio λ=L/h=L/a=Length/Thickness), the Timoshenko solution is not able to depict this characteristics which displays the shear locking effect. On the other hand it can be lucidly observed that for slender beams the reduced integration method is able to overcome the shear locking effect. It exactly produces the results as obtained using the Euler-Bernoulli thin beam theory. For thick/stocky beams, Timoshenko beam is physically more realistic because it includes the shear deformations. It can be investigated from the attached figure that for thick beams, the solutions obtained using the reduced integration method and the Timoshenko beam theory are similar. The application of the EB theory is usually restricted to situations where dimensions along the axis of the beam are at least ten times those of the transverse (cross-section) dimensions: a/ L < 1/10. In contrast to the EB theory, the Timoshenko theory includes transverse shear deformations and is applicable when the length to thickness ratio is lower. However for 0.05< a/L< 0.1, all the theories give similar results for the maximum displacement.

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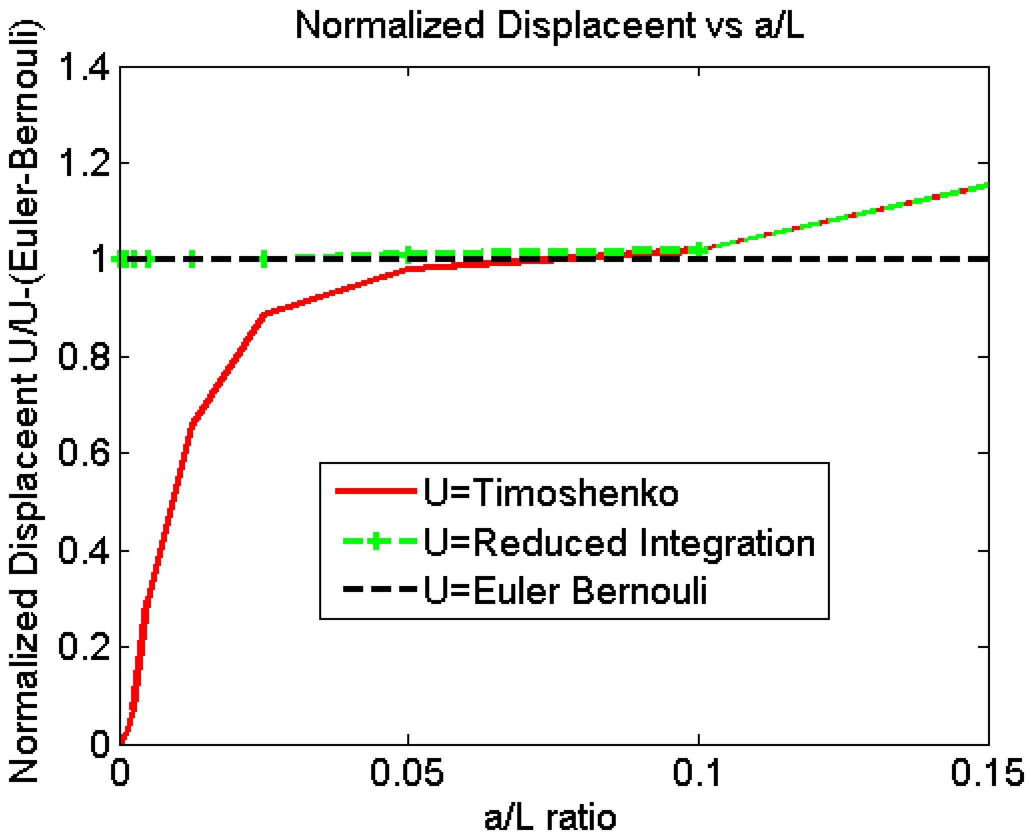

Figure 2: Normalized displacement vs. a/L ratio

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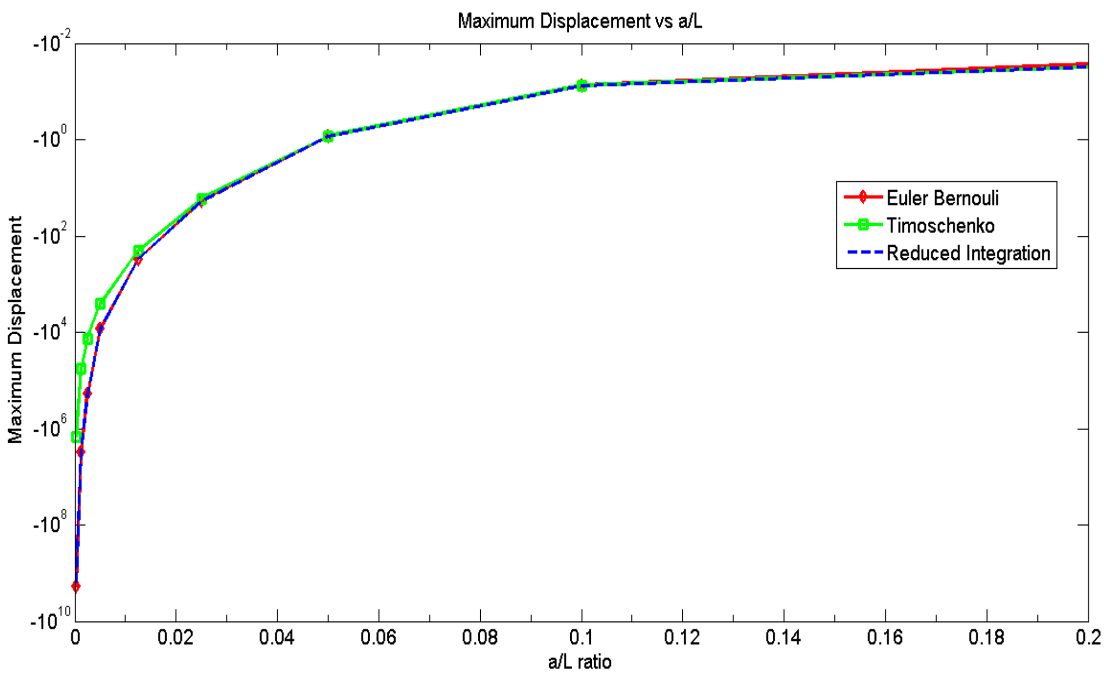

Figure 3: Maximum Displacement vs. a/L ratio

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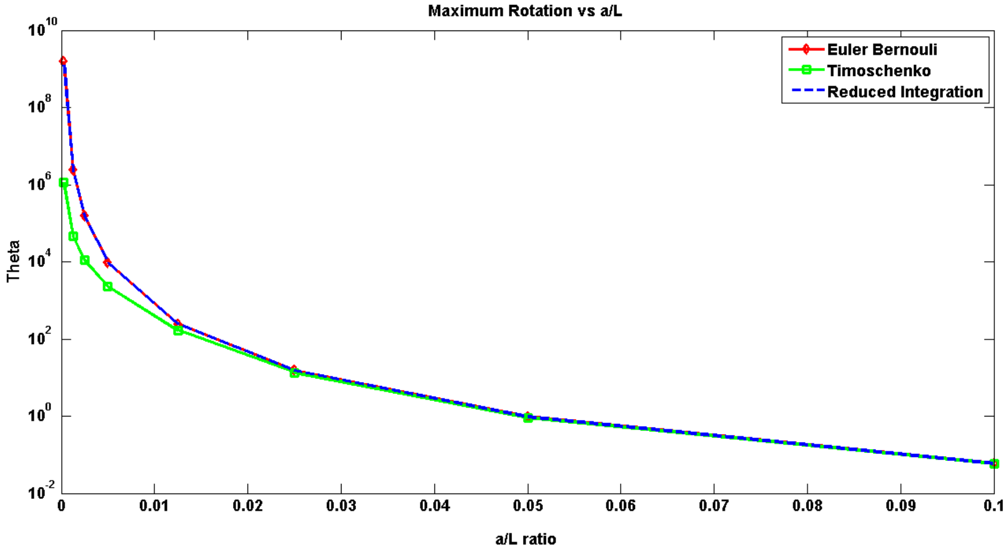

Figure 4: Maximum rotation vs. a/ L

(b) Maximum Moment Comparison:-

a/L	Euler Bernoulli	Timoshenko	Reduced Integration
0.00025	1.9999	0.0015341	1.999
0.00125	1.9999	0.037658	1.999
0.0025	1.9999	0.14257	1.999
0.005	1.9999	0.59364	1.999
0.0125	1.9999	1.3144	1.999
0.025	1.9999	1.7687	1.999
0.05	1.9999	1.9528	1.999
0.1	1.9999	1.9829	1.999
1	1.9999	1.9989	1.999

The obtained maximum moment values have been plotted versus a/L values in the **Figure 5**. It can be observed that for thick beams Timoshenko beam elements provide similar results as that of the results obtained from the reduced integration theory. Timoshenko beam element with reduced integration is valid for both thick and slender beams.

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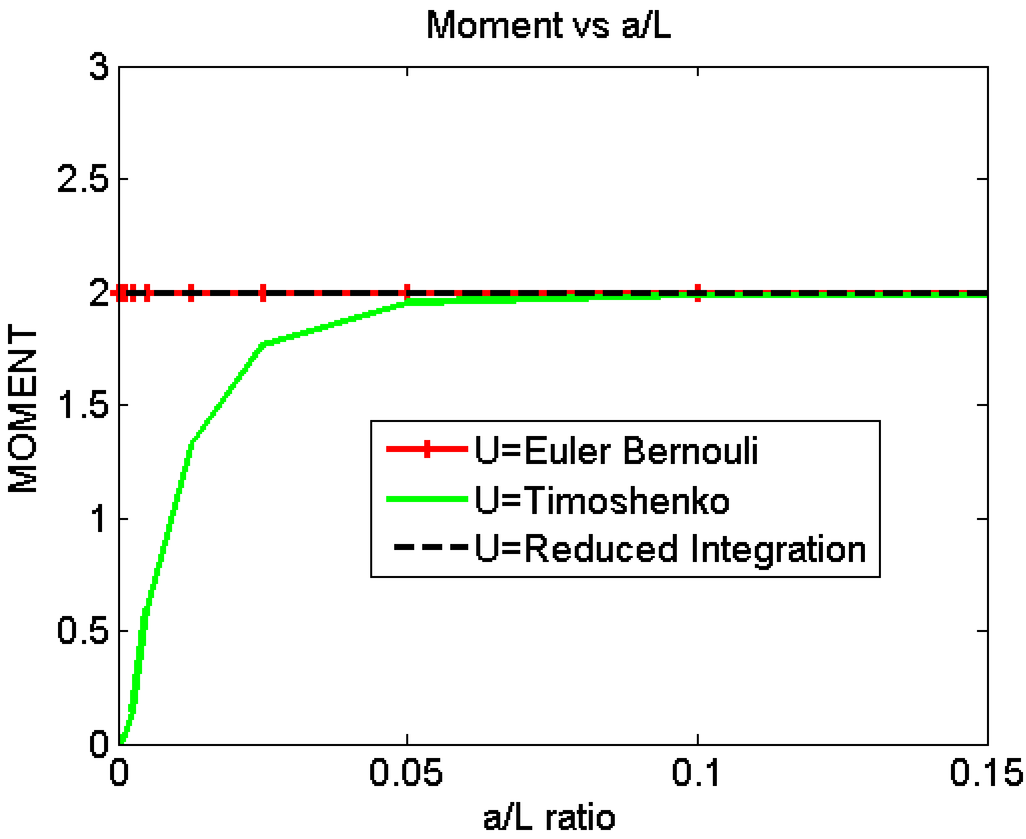

Figure 5: Moment vs. a/L ratio

(c) **Maximum Shear force Comparison:-**

The effect of transverse shear deformation is negligible for a slender beam. So, the Euler Bernoulli theory does not provide any information about the shear force. The obtained shear forces for both the Timoshenko and the reduced integration theory have been plotted below.

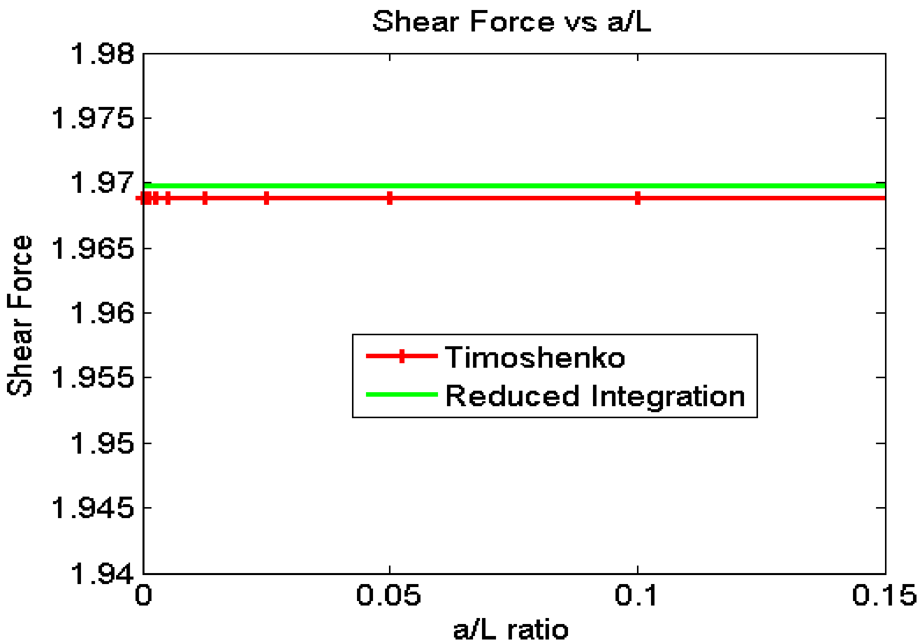

Figure: 6: Shear force vs. a/L ratio

Conclusion:

We conclude that the one point reduced quadrature for K_s yields a 2-noded Timoshenko beam element which is called reduced integration element, which is valid for both thick and slender beams. Once the nodal displacements have been obtained, the bending moment and the shear force are computed at the element mid-point which is "optimal" for the evaluation of stresses. For thick beams the Timoshenko beam element is more realistic than the Euler Bernoulli element as it includes transverse shear deformations.

Annex:-

Modifications to $K_s{}^e$:

```
Element cycle
  for ielem = 1 : nelem

    lnods(1:nnode) = elements(ielem,1:nnode);

    coor_x(1:nnode) = coordinates(lnods(1:nnode),1);    % Elem. X coordinate

    len = coor_x(2) - coor_x(1);  % x_j > x_i

    const = D_matb/len;

    K_b = [ 0 ,  0 ,  0 ,  0 ;
            0 ,  1 ,  0 , -1 ;
            0 ,  0 ,  0 ,  0 ;
            0 , -1 ,  0 ,  1 ];

    K_b = K_b * const;

    const = D_mats/len;

    K_s = [   1    ,   len/2 ,   -1    ,   len/2 ;
            len/2 , len^2/4 , -len/2 , len^2/4 ;
             -1    ,  -len/2 ,    1    ,  -len/2 ;
            len/2 , len^2/4 , -len/2 , len^2/4 ];

    K_s = K_s * const;

    K_elem = K_b + K_s;

    f       = (-denss*area + uniload(ielem))*len/2;
    ElemFor = [ f, 0, f, 0];
```

Stress Evaluation:-

```
% One gauss point for stress evaluation
    gaus1 =0;
    gaus2 = 0;    % One Gauss point for stresses evaluation

    bmat_f=[ 0, -1/len, 0, 1/len];
bmat_s1=[-1/len,-(1-gaus1)/2, 1/len,-(1+gaus1)/2];
bmat_s2=[-1/len,-(1-gaus2)/2, 1/len,-(1+gaus2)/2];

 Str1_g0 = D_matb*(bmat_f *transpose(u_elem));
Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));
Str3_g0 = D_mats*(bmat_s2*transpose(u_elem));
    Strnod(lnods(1),1)  = Strnod(lnods(1),1)+Str1_g0;
    Strnod(lnods(2),1)  = Strnod(lnods(2),1)+Str1_g0;
    Strnod(lnods(1),2)  = Strnod(lnods(1),2)+Str2_g0;
    Strnod(lnods(2),2)  = Strnod(lnods(2),2)+Str3_g0;
    Strnod(lnods(1),3)  = Strnod(lnods(1),3)+1;
    Strnod(lnods(2),3)  = Strnod(lnods(2),3)+1;

  end
for i = 1 : npnod
    Strnod(i,1)  = Strnod(i,1)/Strnod(i,3);
    Strnod(i,2)  = Strnod(i,2)/Strnod(i,3);
  end
```

End

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