Assignment 6: Beams

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Introduction

Beams are one of the most common structures when dealing with structures. From an engineering point of view, the most important features is bending, which characterizes the deformation of a beam when loads are applied perpendicular to the longitudinal axis. In this report two theories will be studied, Euler-Bernouilli beams, which doesn't consider shear effects and Timoshenko beams, which it does account for shear effects. Those theories applied to finite elements will be studied for the simple case of a beam under uniformly distributed force.

Problem statement

The problem to be solved is a beam fixed at the endsunder uniformly distributed load (Figure 1.). Two theories will be studied, Euler-Bernouilli beam theory and Timoshenko and will be compared.

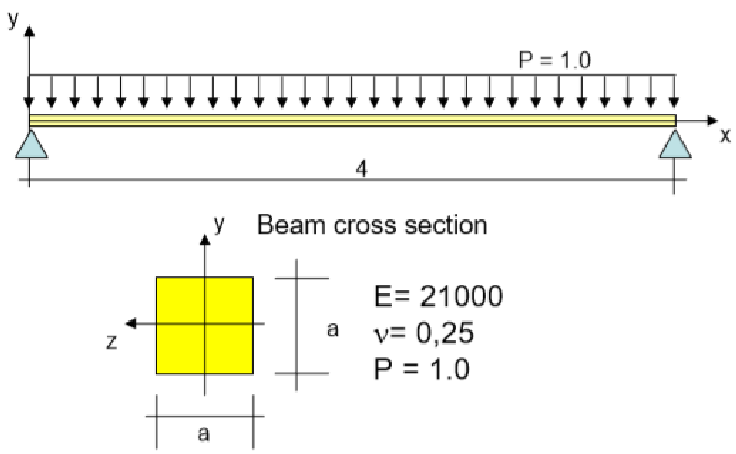
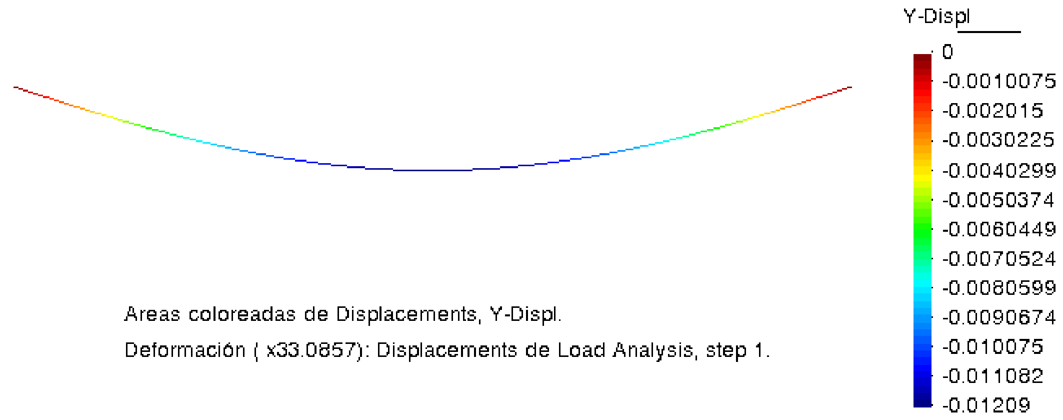


Figure 1. Geometry and material properties of the problem.

The objective of this work is to compare the performance of different type of elements (Euler,Timoshenko and reduced integration elements) and how they behave compared with the others. To do so, a beam with 64 elements will be considered.

The deformation of the beam is as seen in Figure 2.



Comparing each element for the displacements:

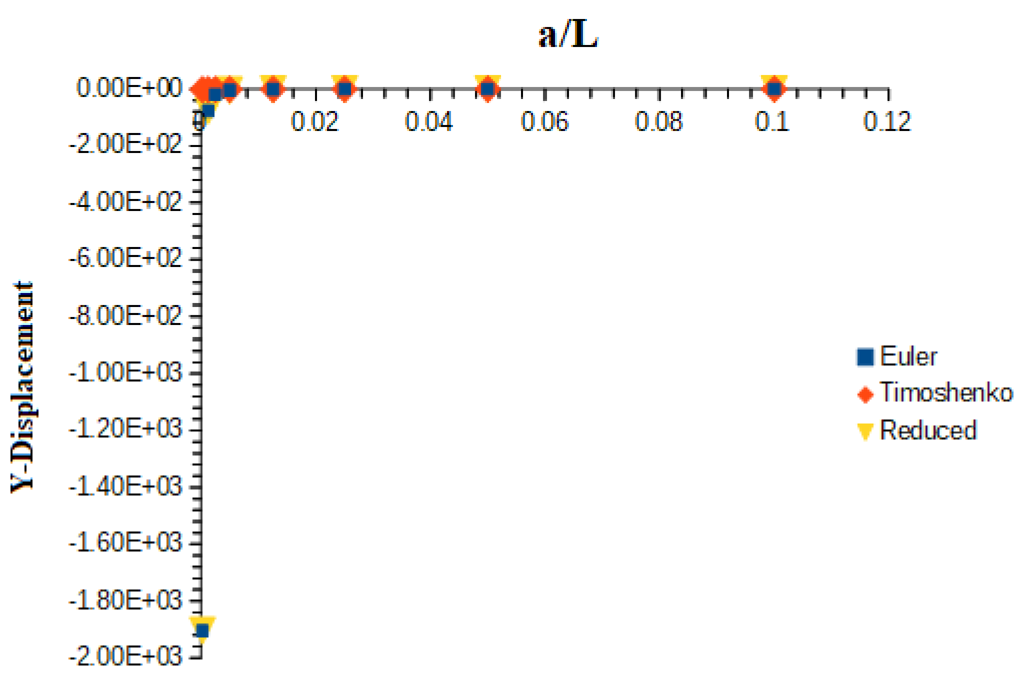


Figure 3. Maximum displacement against the relation between thickness (a) and longitude of the beam (L) for different beam type.

As it can be seen in Figure 3. when thickness is very small, for Euler and Reduced integration Timoshenko elements the displacements became huge due to the stiffness is reduced drastically, so since the loads remain the same for all problems, the beam bends drastically. However, for normal Timoshenko the displacements are not as big as for other methods. This is due to the shear locking, in that case, for this kind of element the shear stiffness matrix decays proportionally to the thickness t, however, the bending stiffness decays as t^3. So when Timoshenko elements are used, there is an stiffness that should not exist and makes the beam more resistant to the bending.

In the next figure, the shear locking effect is seen in more detail. Deleting the point for smaller thickness:

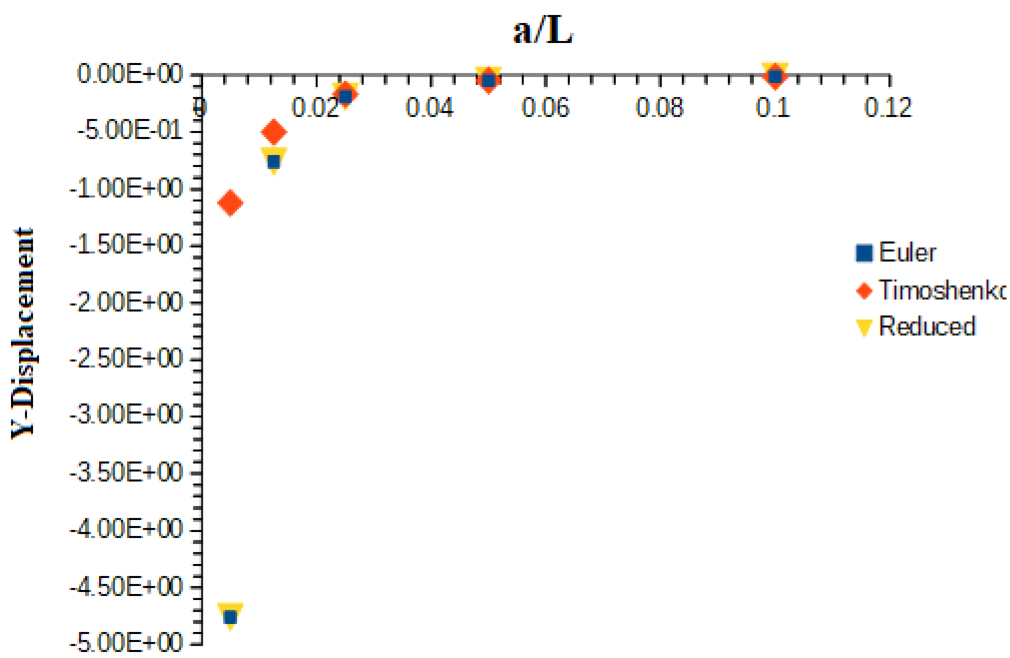


Figure 4. Maximum displacement against the relation between thickness (a) and longitude of the beam (L) for different beam type.

In this case, it can be seen how for approximately a/L greater than 0.02 all elements behave the same. However, for smaller values of a/L, there is a tendency for the Timoshenko elements to separate from other points. This is due to the shear locking. At this point, the bending stiffness vanishes but shear stiffness is still considerable due to it goes proportional to the thickness. So when the total stiffness should have vanished, for Timoshenko elements there is still some extra stiffness which should be there. That's the reason why reduced integration is used to overcome this problem.

If we compare the maximum moment on the beams:

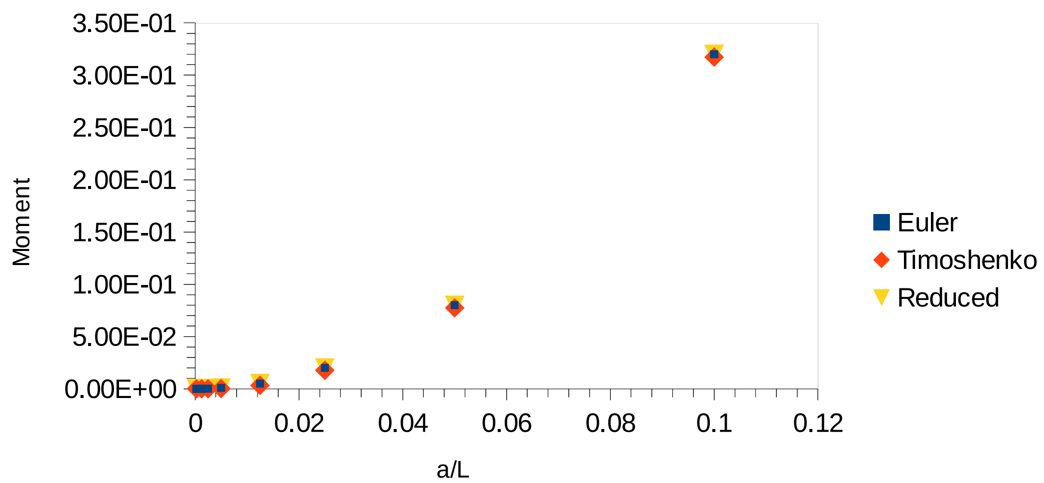


Figure 5. Maximum moment against a/L for different types of elements.

And for shear moment:

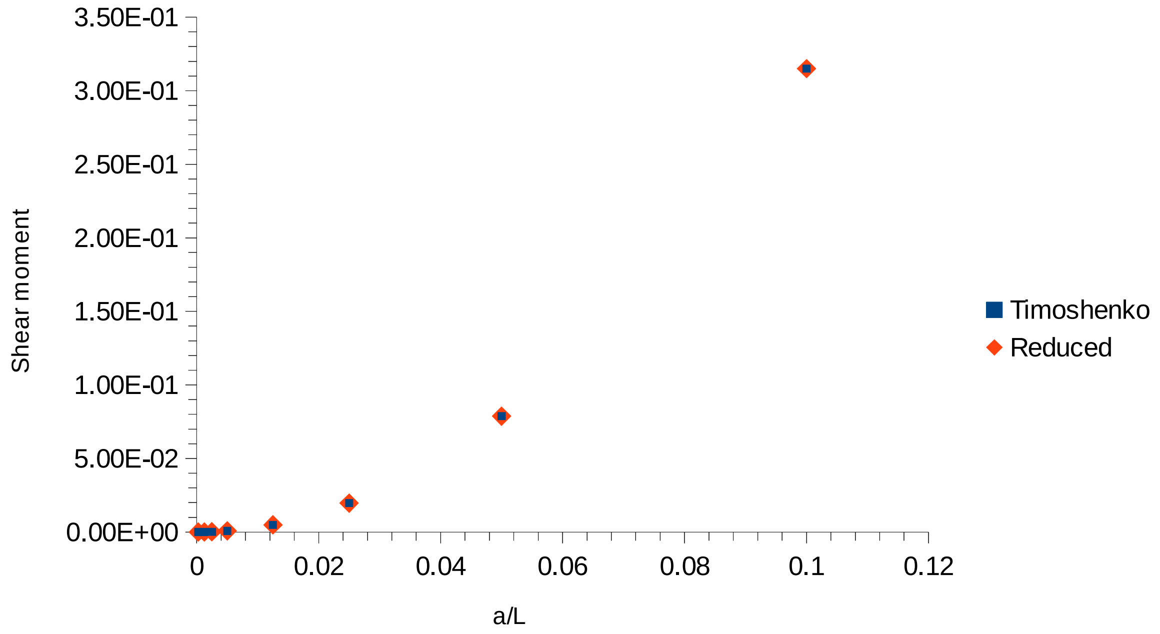


Figure 6. Maximum shear moment against a/L for different types of elements.

It must be remarked that Euler-bernouilli elements doesn't appear here because in this theory shear effects are not taken into account.

The distribution of the bending moments across the beam is shown in Figure 7.

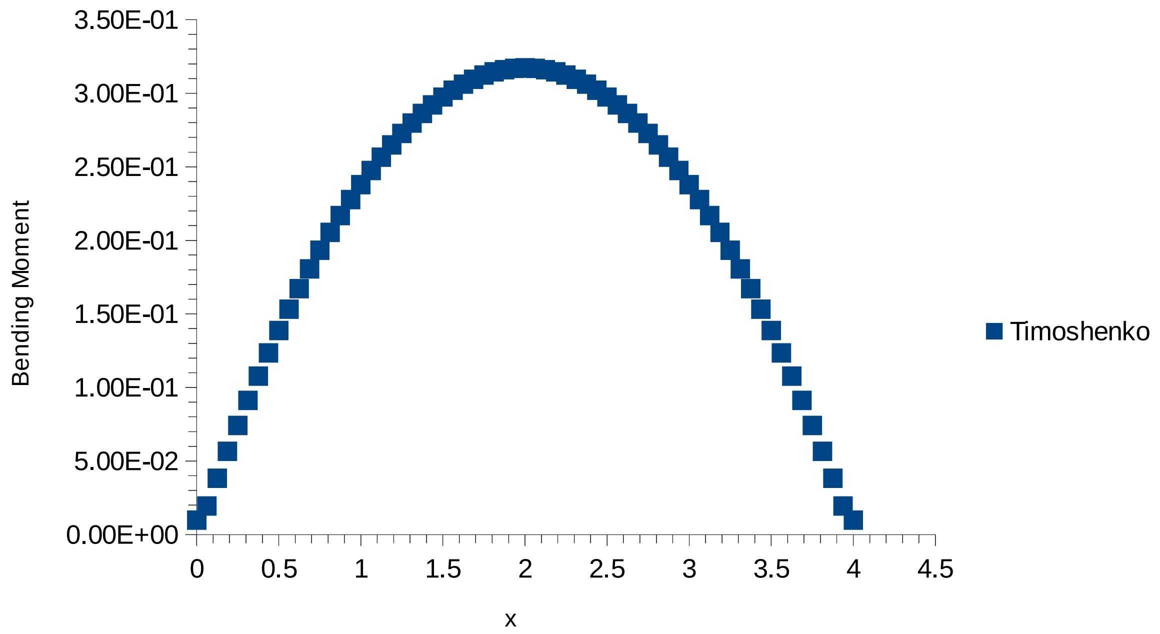


Figure 7. Bending moment across the beam for a=0.4 and Timoshenko beams.