Universitat Polytechnica de Catalunya
MSc Computational Mechanics
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Computational Structural Mechanics and Dynamics

Practice 1

Due 19/03/2018

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Section a)

We will begin this assignment by implementing the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix. The theory of this matrix along with the code implementation can be seen below in figure 1a and figure 1b respectively.

$$\begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \cdots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \cdots & 1 & -\frac{l^{(e)}}{2} \\ \text{Simetr.} & & \cdots & \frac{(l^{(e)})^2}{4} \end{bmatrix}$$

Figure 1a: Reduced Timoshenko Theory

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%IMPLEMENTATION OF TIMOSHENKO REDUCED INTEGRATION MATRIX

  K_shear= [   1    ,    len/2 ,    -1    ,    len/2 ;

             len/2 , len^2/4 , -len/2 , len^2/4 ;

             -1    ,  -len/2 ,    1    ,  -len/2 ;

             len/2 , len^2/4 , -len/2 ,  len^2/4 ];
```

Figure 1b: Implemented Reduced Timoshenko

Here we can see the properly implemented timoshenko reduced integration matrix. It is worth noting that the previous full integration timoshenko matrix was commented out so this matrix could replace it and take its place.

Section b)

Now that we have successfully implemented the Timoshenko Reduced integration matrix, we can now begin to compare the results produced from this method with the other previously implemented methods. We will begin with a comparison of the Timoshenko Reduced integration method, the Euler Bernoulli method, and the Timoshenko full integration method. The comparison graph can be seen in figure 2 below.

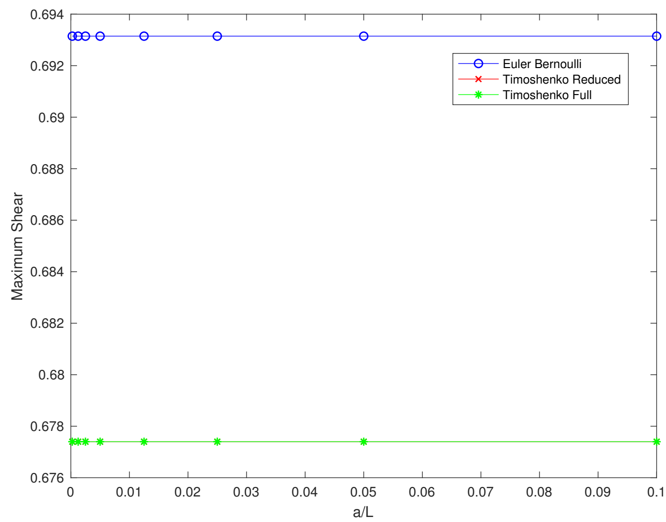

Figure 2: Log(Maximum Shear) plotted against a/L ratio

Here we can see the Maximum shear of each method with increasing a/L values. It should be noted that these values stay constant because maximum shear is independent of thickness. This is consistent with the theory. It is also worth noting that the Timoshenko reduced and full methods produce identical maximum shear values.

We will now take a look at a Maximum Moment comparison of the 3 implemented methods. This can be seen below in figure 3.

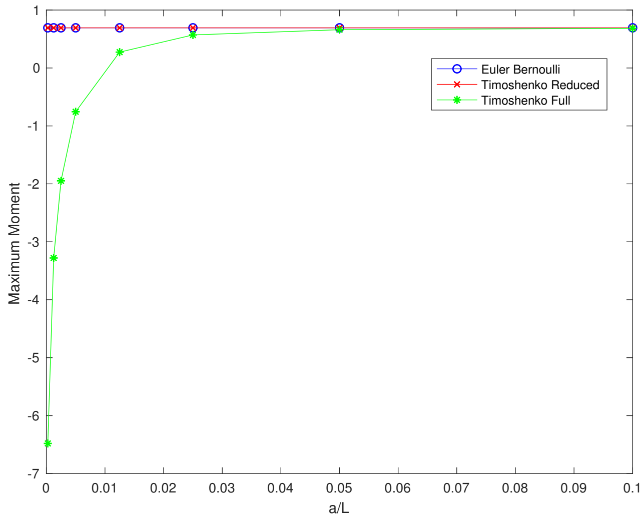

Figure 3: Log(Maximum Moment) plotted against a/L ratio

Above we can see that both Euler Bernoulli and the Timoshenko Reduced produce identical values for Maximum Shear. The Timoshenko Reduced method however, exhibits increasing maximum moment as the cross sectional area increases.

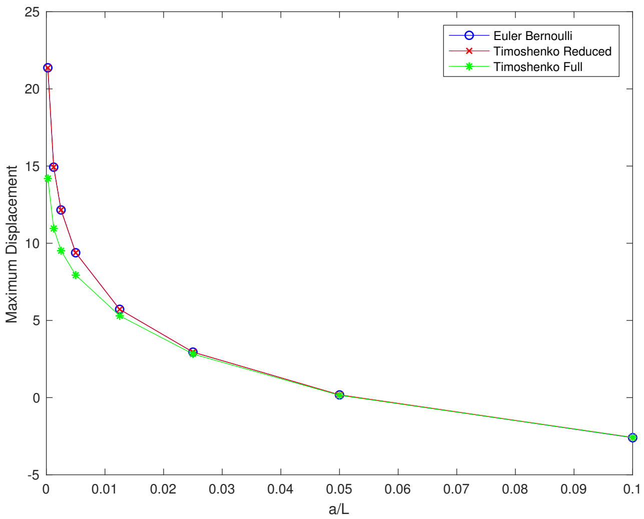

Figure 4: Log(Maximum Displacement) plotted against a/L ratio

Above we can see a plot of the Logarithm of the maximum displacements plotted against the a/L ratio. All three methods exhibit a high degree of change with increasing cross sectional area. One can notice that the full integration Timoshenko method has less initial displacements. This can be attributed to the "shear locking" effect, which increases the stiffness while decreasing the strain.

In conclusion, we can say that the Euler Bernoulli method is the best of the three methods while the Timoshenko reduced performs relatively poorly in comparison and should be avoided if possible.