Universitat Politecnica de Catalunya CIMNE
Master in Computational Mechanics

Computational Structural Mechanics & Dynamics

Assignment 6

Beams

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26th March 2020

Part a:

Program in MATLAB the Timoshenko 2 nodes beam element with reduced integration for the shear stiffness matrix:

$$K_s = \begin{bmatrix} 1 & \frac{l}{2} & -1 & \frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \\ -1 & -\frac{l}{2} & 1 & -\frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \end{bmatrix}$$

Solution:

The code for the stiffness matrix for Timoshenko reduced integration case is as below:

```
% Reduced Intergration case
K_s = [1, len/2, -1, len/2;
       len/2, len^2/4, -len/2, len^2/4;
       -1, -len/2, 1, -len/2;
       len/2, len^2/4, -len/2, len^2/4];
```

This was the only small change needed to be done in the Matlab code of existing Timoshenko beam element.

Part b :

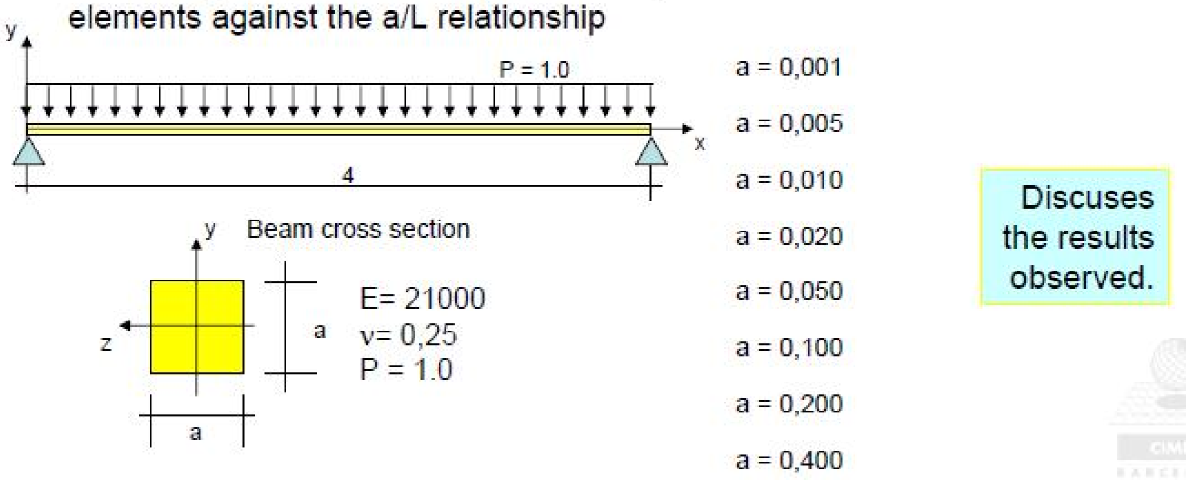
b) Solve the following problem with a 64 element mesh with the

2 nodes Euler Bernulli element

2 nodes Timoshenko Full Integrate element

2 nodes Timoshenko Reduce Integration element.

Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship



Solution:

After modifying the code for Timoshenko reduced integration, the results have been implemented by loading the problemtype available from MAT-FEM website in GiD.
Shown below are the tables consisting values for max. displacement, bending moments and shear forces for different values of 'a/L' ratio.
According to the Euler Bernoulli beam theory, when the thickness of the beam is very very small as compared to the length (thin beams), then the shear effect is not considered. But, when the thickness of the beam can be compared to the length (thick beams), then the shear effect becomes prominent which is the Timoshenko theory. But, however discretization of this theory produces an error in the finite element method used for it, which can be overcame by using the reduced integration method, which is the purpose of this assignment.

Euler Bernoulli				L = 4
a	**a/L**	**Max. Disp.**	**Max. Moment**	**Max. Shear**
0.001	0.00025	1.90E+09	1.9999	2
0.005	0.00125	3.05E+06	1.9999	2
0.01	0.0025	1.90E+05	1.9999	2
0.02	0.005	1.19E+04	1.9999	2
0.05	0.0125	3.05E+02	1.9999	2
0.1	0.025	1.90E+01	1.9999	2
0.2	0.05	1.19E+00	1.9999	2
0.4	0.1	7.44E-02	1.9999	2

Results for Euler Bernoulli Beam

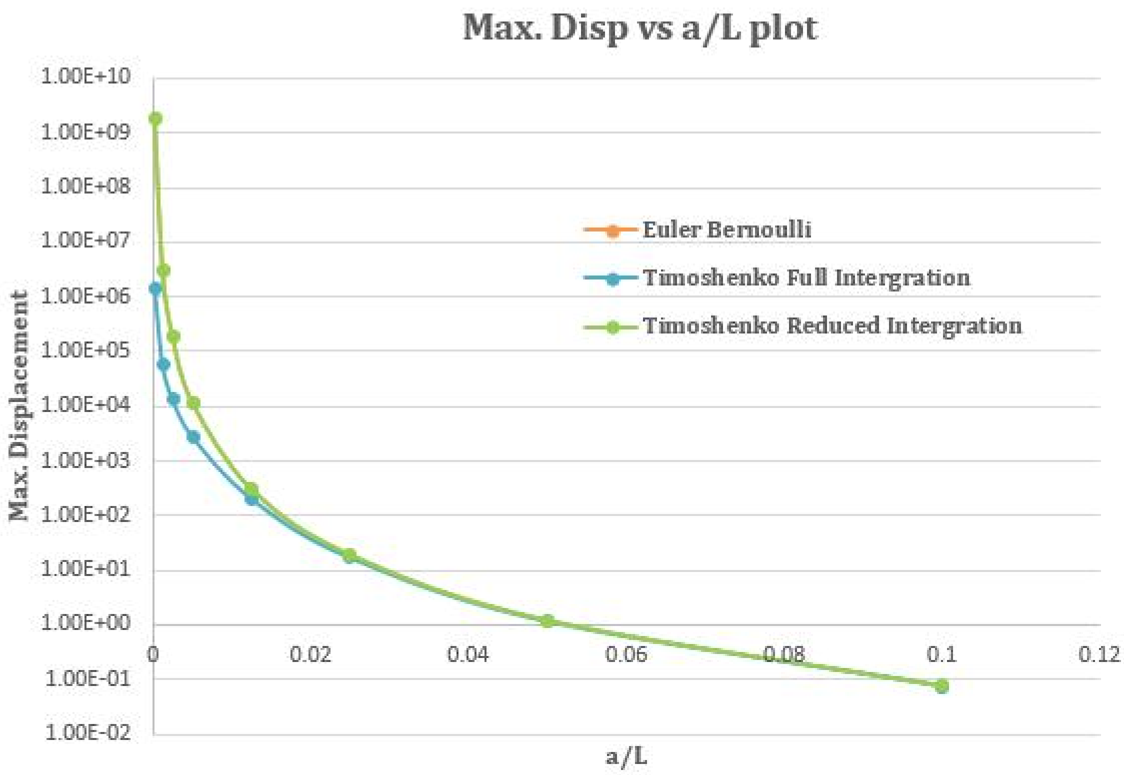
Timoshenko Full Intergration				L = 4
a	**a/L**	**Max. Disp.**	**Max. Moment**	**Max. Shear**
0.001	0.00025	1.46E+06	0.001534	1.9687
0.005	0.00125	5.74E+04	0.0376	1.9687
0.01	0.0025	1.36E+04	0.143	1.9687
0.02	0.005	2.80E+03	0.4697	1.9687
0.05	0.0125	2.00E+02	1.3144	1.9687
0.1	0.025	1.69E+01	1.7687	1.9687
0.2	0.05	1.15E+00	1.936	1.9687
0.4	0.1	7.55E-02	1.9828	1.9687

Results for Timoshenko Full Integration Beam

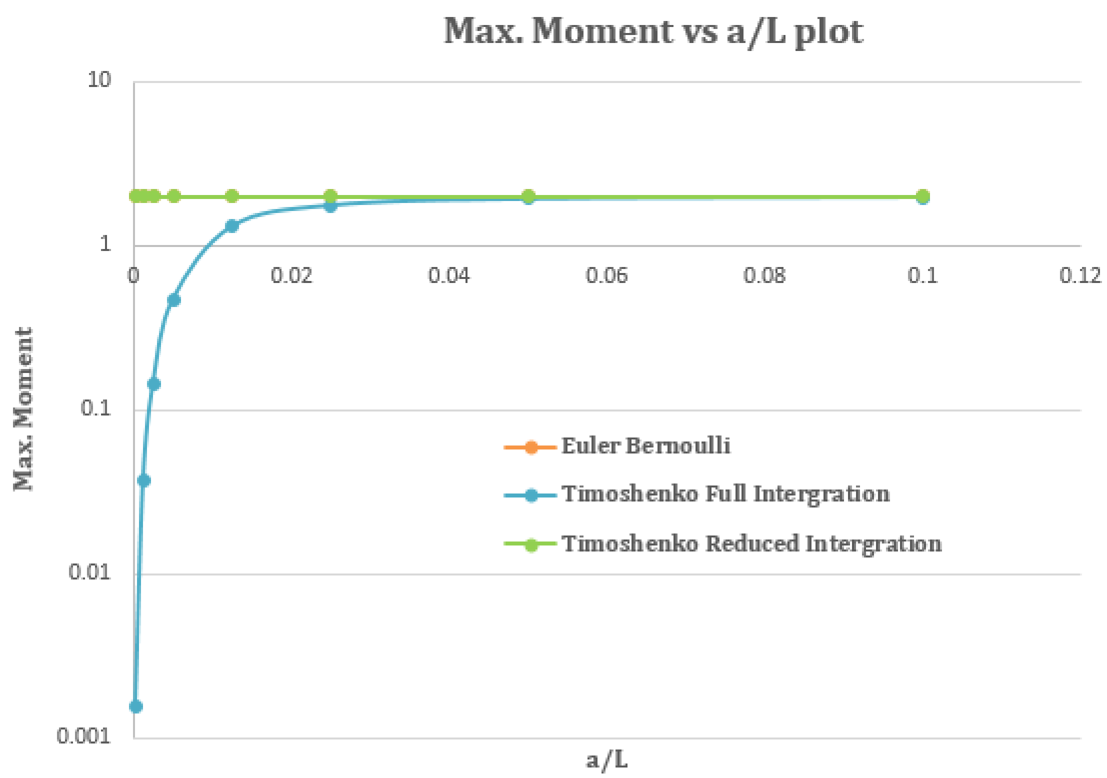
Timoshenko Reduced Intergration				L = 4
a	**a/L**	**Max. Disp.**	**Max. Moment**	**Max. Shear**
0.001	0.00025	1.90E+09	1.9999	1.9687
0.005	0.00125	3.05E+06	1.9999	1.9687
0.01	0.0025	1.90E+05	1.9999	1.9687
0.02	0.005	1.19E+04	1.9999	1.9687
0.05	0.0125	3.05E+02	1.9999	1.9687
0.1	0.025	1.91E+01	1.9999	1.9687
0.2	0.05	1.197E+00	1.9999	1.9687
0.4	0.1	7.61E-02	1.9999	1.9687

Results for Timoshenko Reduced Integration Beam

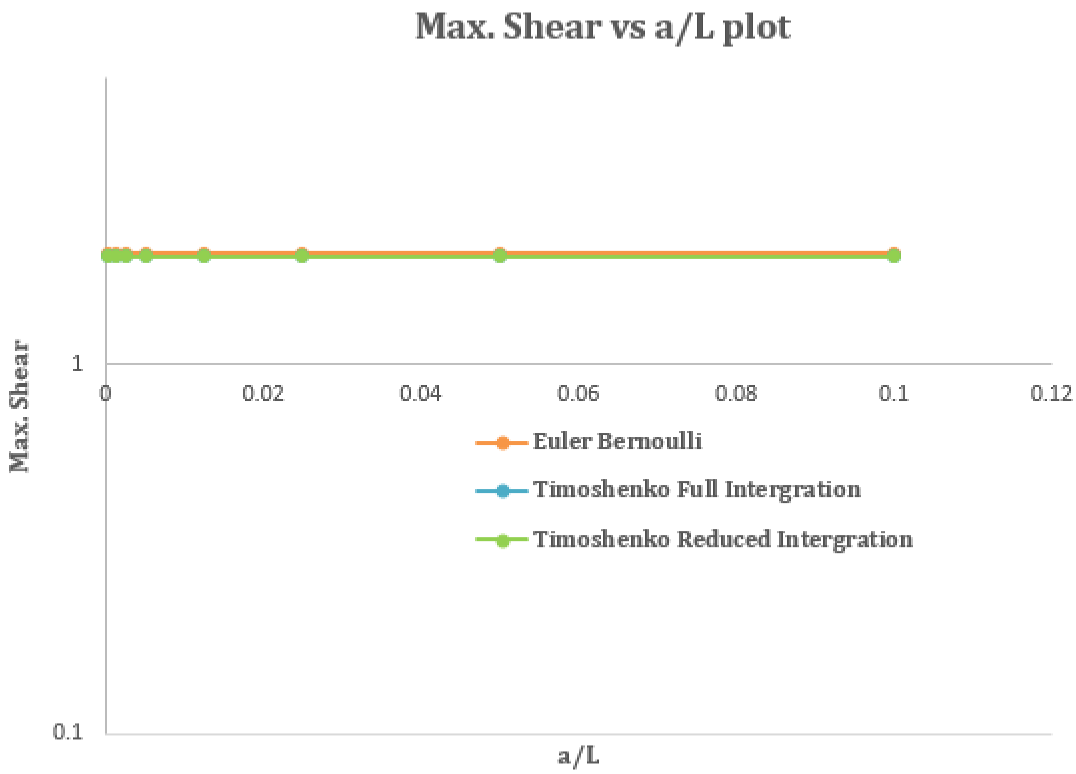
Shown below are the log plots of max. displacement, moment and shear against 'a/L' ratio. These graphs represent the comparison of Euler Bernoulli, Timoshenko Full and Reduced Integration beams.



Comparison of graphical results for all beams with respect to max. displacement



Comparison of graphical results for all beams with respect to max. bending moment



Comparison of graphical results for all beams with respect to max. shear force

Discussions:

- From the theories and results obtained, an important point can be noted that the Euler Bernoulli beam shows good results for low ‘a/L’ ratio values.
- It is observed that as the ‘a/L’ ratio value increases, the displacements exhibited by the beams is lower. The Euler Bernoulli beam and Timoshenko reduced integration beam show conveniently the same values for all ‘a/L’ ratio values. When the ‘a/L’ ratio value is lower, that means, when the beams are more slender, the Timoshenko full integration beam shows a lower displacement value. But, it can be noted that after a certain value all the beams exhibit similar results till convergence.
- For bending moment graphs, it is seen that the Timoshenko full integration beam does not exhibit good results initially, as compared to the other types of beams. It requires some threshold value beyond which it shows a converged result. Whereas, the Euler Bernoulli and Timoshenko Reduced Integration yield almost the same results for all values of ‘a/L’, which are far more better than the Timoshenko Full Integration beam.
- For the Shear graphs, Euler Bernoulli beam, the Timoshenko reduced and full integration beams show similar results which work good for all values of the ratio. There is a very minute negligible difference between the results of Euler Bernoulli ad the 2 Timoshenko beams, while both the Timoshenko reduced integration and full integration show almost exactly the same result.