Computational Structural Mechanics and Dynamics

<u>Assignment</u>
on
<u>Beam</u>

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Ques 1. Program in Matlab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix

Answer 1 >> In the given fully implemented codes for the Euler-Bernoulli and Timoshenko, we are required to add the reduced integration Timoshenko Algorithm. The reduced integration technique computes the matrix $\mathbf{K_s^{(e)}}$ using a quadrature of one order less than is needed for exact integration. The matrix is

$$\boldsymbol{K}_s^{(e)} = \left(\frac{GA^*}{L}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \ddots & \frac{(l^{(e)})^2}{4} & -\frac{l^{(e)}}{2} & \frac{(l^{(e)})^2}{4} \\ & \ddots & 1 & -\frac{l^{(e)}}{2} \\ Symm & & \ddots & \frac{(l^{(e)})^2}{4} \end{bmatrix}$$

after that we edited the code provided for the Timoshenko case, as shown in the Fig 1.

```

        const = D_matsREDU/len;

        K_s = [   1    ,    len/2 ,    -1    ,    len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ;
                -1    ,  -len/2 ,     1    ,  -len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ];

        K_s = K_s * const;
```

Fig. 1: Definition of the $\mathbf{K_s^{(e)}}$ matrix for the case of reduced integration in the Timoshenko code

Ques 2. Solve the following problem with a 64 element mesh with the
2 nodes Euler Bernoulli element
2 nodes Timoshenko Full Integrate element
2 nodes Timoshenko Reduce Integration element.
Compare maximum displacements, moments and shear for the elements against the a/L relationship

Answer 2 >> The stiffer approximation is a result of Timoshenko beam element, which occurs generally at low a/L (a/L < 1/10) ratio. This is avoided by the use of Reduced Integration Method where the gauss point number used are less than that is required. This was clearly observed in the graphs obtained, presented below. The Reduced Integration Method values matches the Euler Method results.

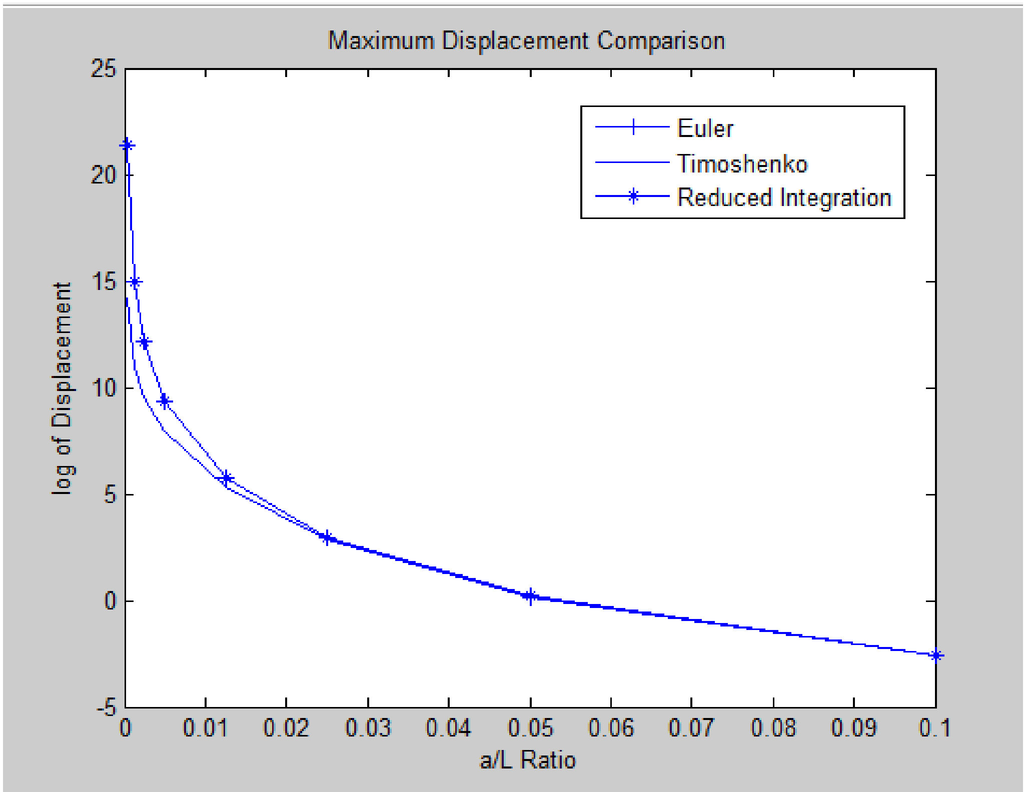

Fig.2: Representation of the natural logarithm of displacement considering different cases versus the ratio a/L

where a is the width of the square section of the beam and L is the total length of the beam.

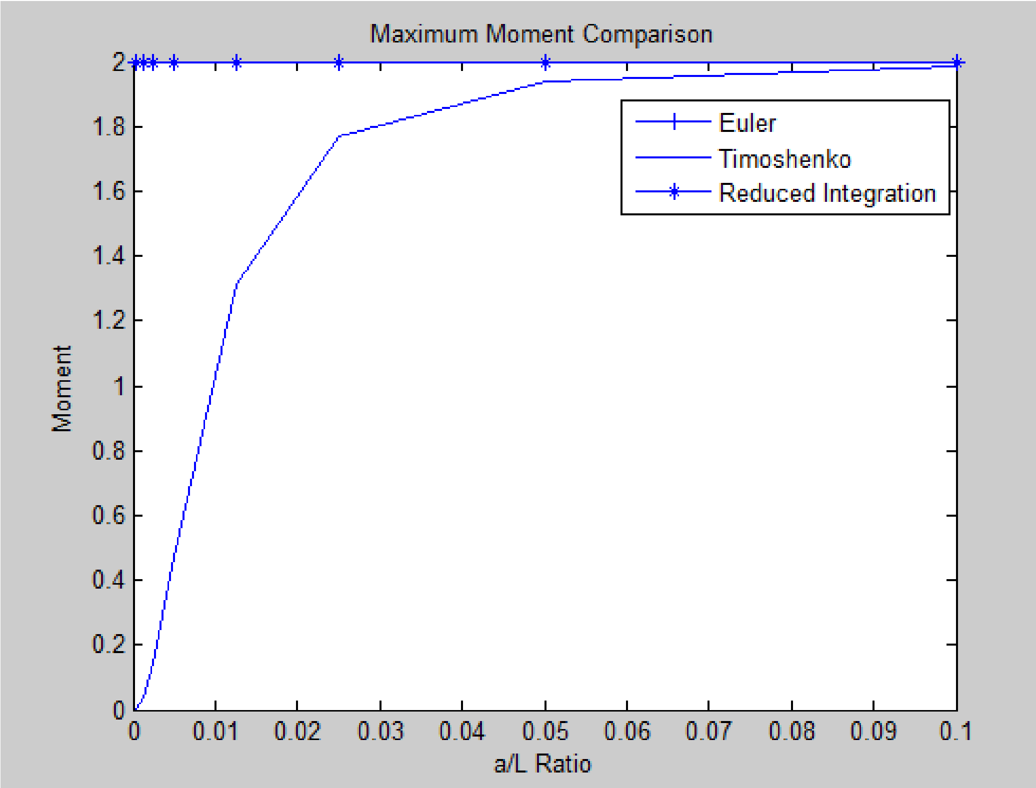

Fig.3: Representation of the moment considering different cases versus the ratio a/L.

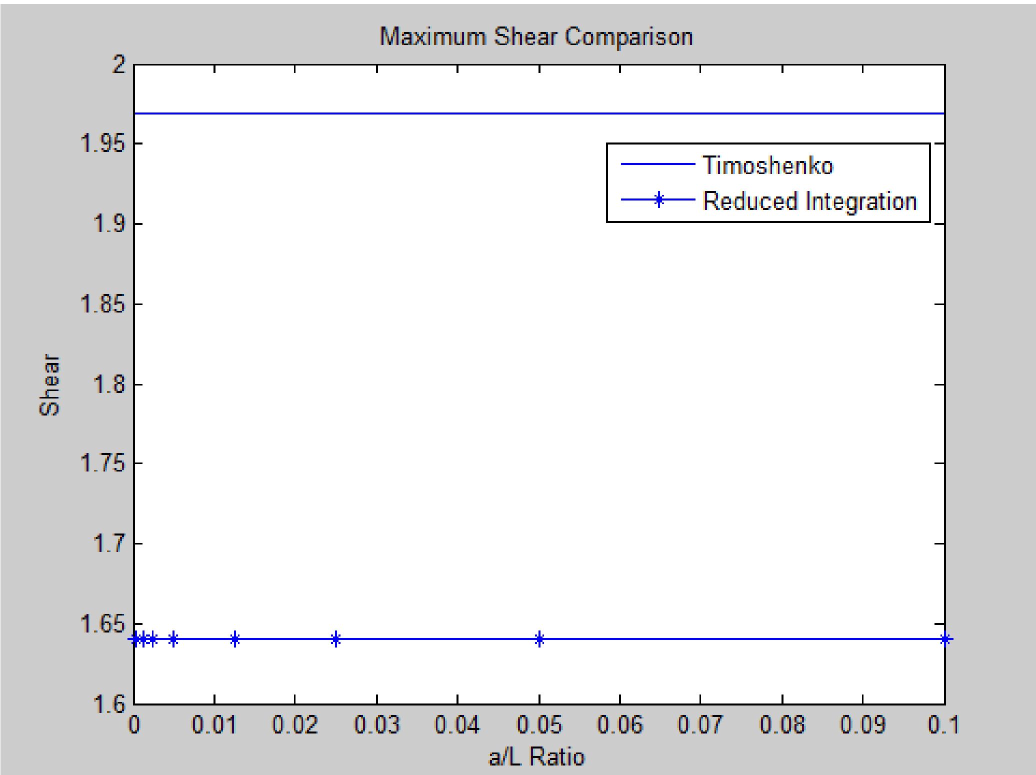

Fig.4: Representation of the Shear considering different cases versus the ratio a/L