Universitat Politècnica de Catalunya
Master of Science in Computational Mechanics
Computational Structural Mechanics and Dynamics
CSMD
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Assignment 6 - Euler-Bernoulli and Timoshenko Beam theories

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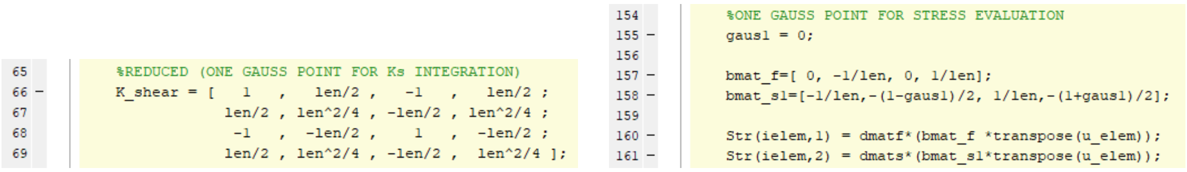
Assignment 6.1

For the first part of the assignment, two small changes in the code file named as *Beam_Timoshenko.m* were done to accomplish the following:

- Use a one gauss point integration rule for the K_s stiffness matrix of Timoshenko model, leading to the "reduced" Timoshenko model. The resultant code lines are shown in Figure (1) (a).
- Use only one gauss point to evaluate the stresses in the reduced Timoshenko model. This results in the code lines of Figure (1) (b).



```
    %REDUCED (ONE GAUSS POINT FOR Ks INTEGRATION)
    K_shear = [   1    ,   len/2 ,   -1    ,   len/2 ;
                len/2 , len^2/4 , -len/2 , len^2/4 ;
                 -1    ,  -len/2 ,    1    ,  -len/2 ;
                len/2 , len^2/4 , -len/2 ,  len^2/4 ];
```

(a)

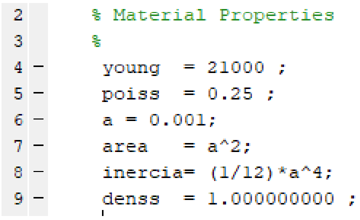
```
    %ONE GAUSS POINT FOR STRESS EVALUATION
    gaus1 = 0;

    bmat_f=[ 0, -1/len, 0, 1/len];
    bmat_s1=[-1/len,-(1-gaus1)/2, 1/len,-(1+gaus1)/2];

    Str(ielem,1) = dmatf*(bmat_f *transpose(u_elem));
    Str(ielem,2) = dmats*(bmat_s1*transpose(u_elem));
```

(b)

Figure 1: (a) One Gauss point shear stiffness matrix K_s; (b) One gauss point for evaluation of shear forces

Assignment 6.2

For the second part of the assignment the matlab code file named as *SimpleSupUL_Beam_64.m* had the changes in the *Material Properties* section as shown in Figure (2).

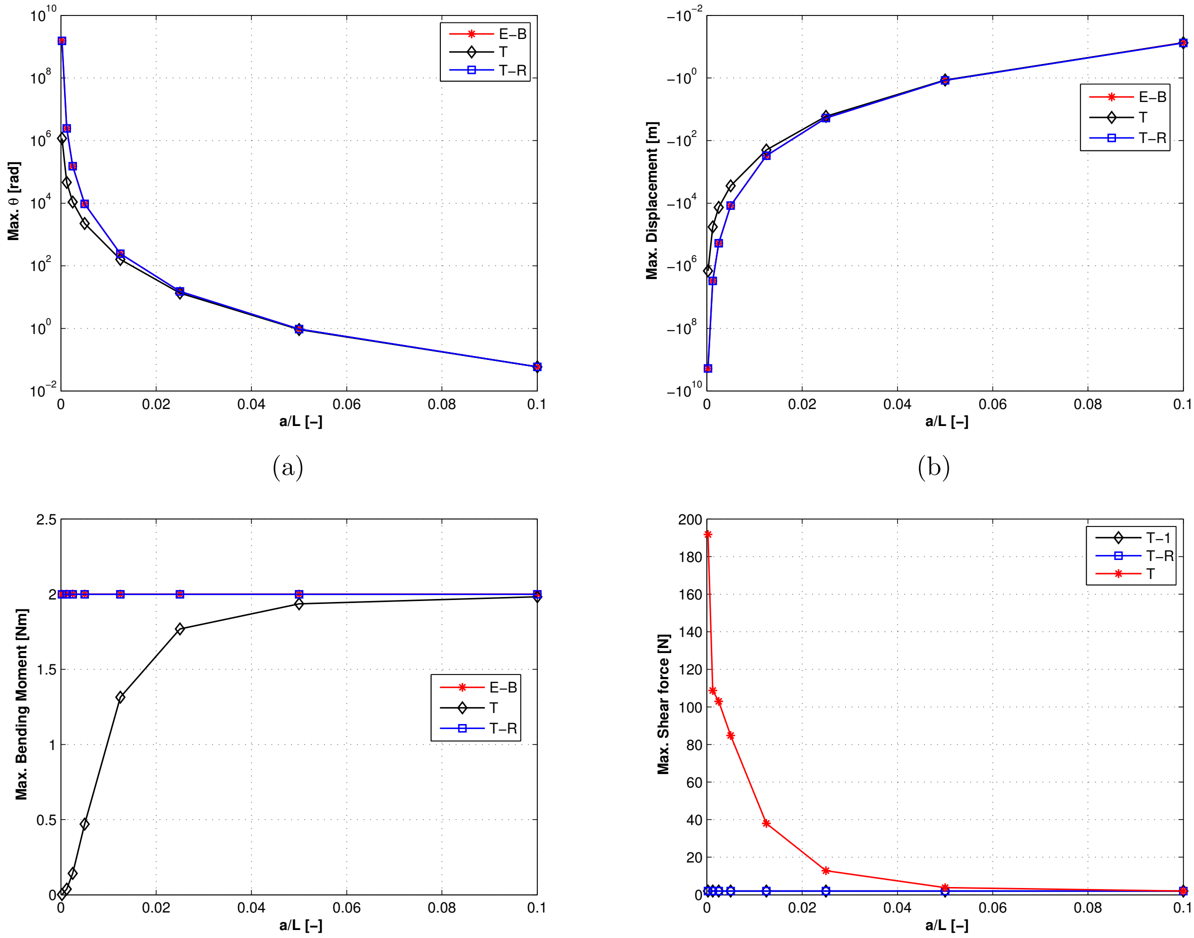


```
% Material Properties
%
 young  = 21000 ;
 poiss  = 0.25 ;
 a = 0.001;
 area   = a^2;
 inercia= (1/12)*a^4;
 denss  = 1.000000000 ;
```

Figure 2: Physical and Geometrical properties.

After this changes the three beam models defined as follows: Euler-Bernounolli (E-B), Timoshenko (T) and Timoshenko Reduced (T-R), were tested for the given geometrical/load given case. The cases were run for different values of the relation a/L, where a is the cross section height and width, and L the beam length (4 m in this problem).

Figure 3 (a) and (b) shows the results for Maximum angular deformation θ and displacement in y direction w respectively for the three models (E-B, T and T-R) with respect to the relation a/L.



(a)

(b)

(c)

(d)

Figure 3: Maximum: (a) Angular displacement θ; (b) displacement w; (c) Bending Moment; (d) Shear Force.

It can be noticed that the T model underestimate the magnitude of the maximum angular deformation θ and maximum displacement w, with respect to E-B model results, when the relation a/L is small. This is a expected result due to the shear locking effect that provides a stiffer element model when L/a is too high, or in other words, when the beam is slender.

The reduced Timoshenko model (T-R) is employed to overcome this limitation, and in can be noticed in Figure (3) (a) and (b) that over all considered range of a/L, the results for θ and w agrees with the E-B model results. It can be also noticed that, for this given case, when $a/L > 0.025$ the three models presents almost same results for θ and w.

Results for internal Bending Moment and Shear forces are given in Figure (3) (c) and (d) respectively. For the maximum Bending Moment, Figure (3) (c), both models E-B and T-R predict the same value in the whole range of a/L, meanwhile, the T model only converges to almost the same results when $a/L \geq 0.1$.

With regard to Maximum Shear Force, Figure (3) (d), two options for the evaluation of the shear forces in the T model were tested. First one, as a natural procedure, using two gauss points ($\xi_1 = -1/\sqrt{3}$ and $\xi_2 = 1/\sqrt{3}$) for the B_s matrix evaluation, named here as T simply. Second one using only one gauss points for the B_s matrix evaluation ($\xi_1 = 0$, center of element), named here as T-1. Both Timoshenko reduced (T-R) and Timoshenko with one gauss point for stress computation (T-1) accounts for the same value of shear force ($Q \approx 2$[N]) in the whole range of a/L. However, the Timoshenko with 2 gauss points (T) shows a over prediction of shear forces for ($a/L < 0.1$).

These results shows that the Timoshenko (T) solution for shear forces, being a linear function of ξ, provides a high variation within the element for low values of a/L. Further, as the T-1 solution for Shear forces agrees with T-R solution, it can be said that the T solution for Shear forces in the middle of the element is the same as the T-R solution . When $a/L > 0.1$, for this case, both evaluation with one and two gauss points (T-1 and T) provides same result for Shear forces, as here locking effects are diminished and low varition of shear forces within the element is obtained for T model. Again this is a result of the Shear locking effect, were the Shear effects dominates the Timoshenko (T) solution for small values of a/L which is not reasonable for slender beams under the physical point of view .