Assignment 6 – Nicolas Andre Caronte Grønland

a)
To obtain reduced integrating of the shear stiffness matrix I have introduced the following K_s matrix:

```
K_s = [   1    ,   len/2 ,   -1    ,   len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
         -1    ,  -len/2 ,    1    ,  -len/2 ;
        len/2 , len^2/4 , -len/2 ,  len^2/4 ];
```

b)

Using the MatLab scripts that where found on Cimne, and adjusting the variables to fit this problem the following max displacements, moments and shear forces where found for eight different a-values. These values were extracted from the file "SimpleSupUL_Beam_64.flavia. res"(A file that is created every time the script is run).

Since there is no information about the units of the problem, everything is stated without units. My analysis is only comparing the different elements, so this won't effect the results.

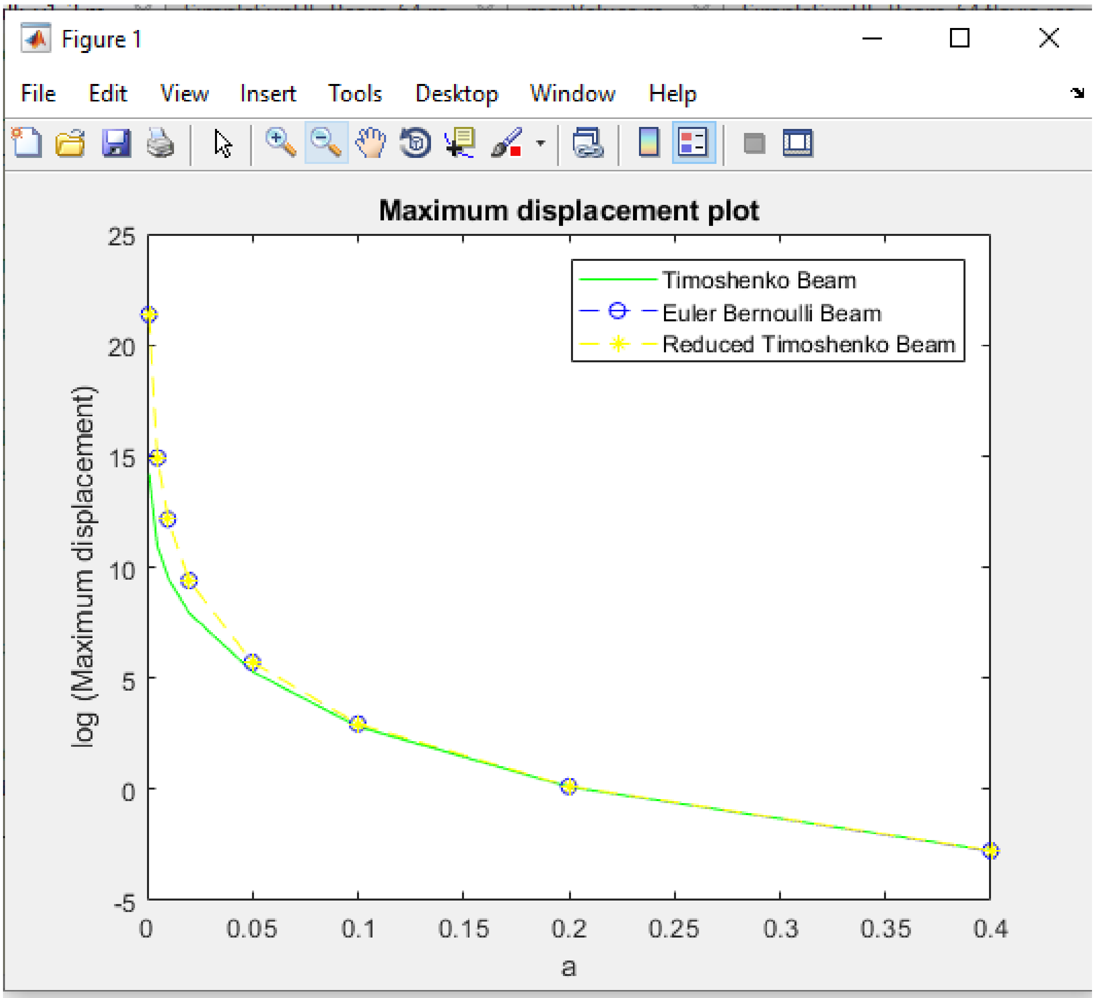
Results

Euler-Bernoulli	a dimension	Max displacement	Max moment	Max shear force
	0.001	1.9048e+09	2.0003	2.0000
	0.005	3047540	2.0003	1.9999
	0.010	190457	2.0001	1.9998
	0.020	11900	1.9995	1.9992
	0.050	304	1,9953	1.9950
	0.100	18.8571	1.9803	1.9800
	0.200	1.1429	1.9203	1.9200
	0.400	0.0625	1.6803	1.6800

Timoshenko Full integration	a dimension	Max displacement	Max moment	Max shear force
	0.001	14614550	0.0015	1.9688
	0.005	5.7400e+04	0.0377	1.9687
	0.010	1.3581e+04	0.1426	1.9686
	0.020	2.7962e+03	0.4696	1.9680
	0.050	199.9260	1.3111	1.9638
	0.100	16.7064	1.7510	1.9491
	0.200	1.1133	1.8586	1.8900
	0.400	0.00635	**1.6656**	1.6538

Timoshenko Reduced integration	a dimension	Max displacement	Max moment	Max shear force
	0.001	1.9040e+09	1.9990	1.9688
	0.005	3046360	1.9990	1.9687
	0.010	190386	1.9988	1.9686
	0.020	1.1896e+04	1.9982	1.9680
	0.050	303.9950	1.9940	1.9638
	0.100	18.8781	1.9790	1.9491
	0.200	1.1493	1.9191	1.8900
	0.400	0.0640	1.6792	1.6538

This gives the following plots.



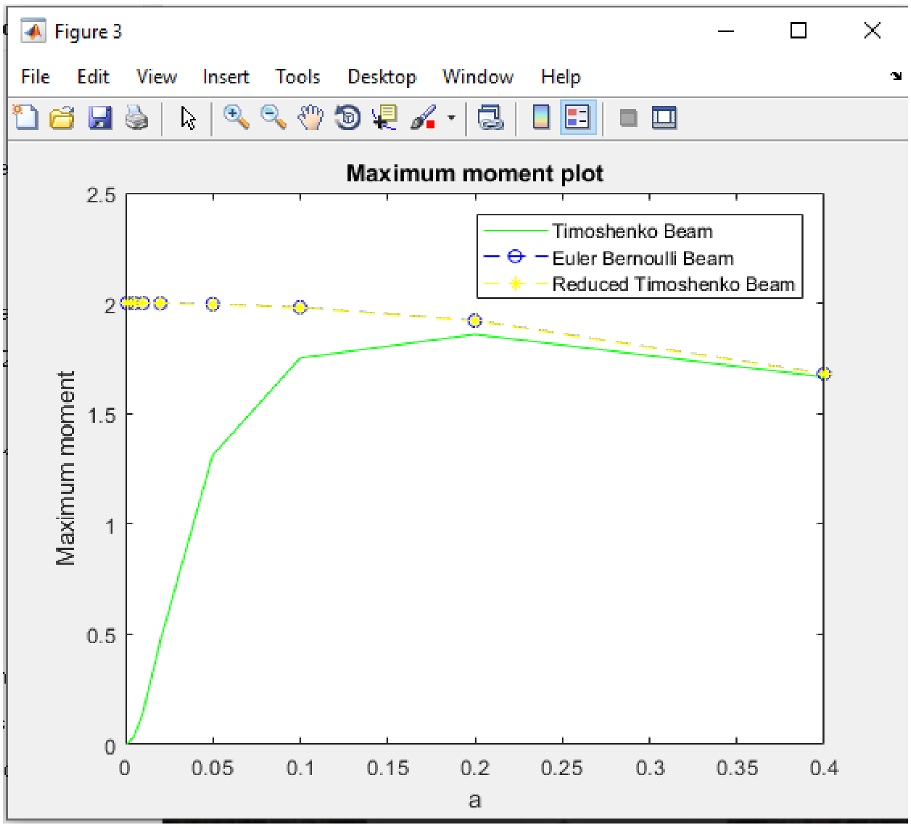


Figure 3
File Edit View Insert Tools Desktop Window Help
Maximum moment plot
Timoshenko Beam
Euler Bernoulli Beam
Reduced Timoshenko Beam
Maximum moment
2.5
2
1.5
1
0.5
0
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
a

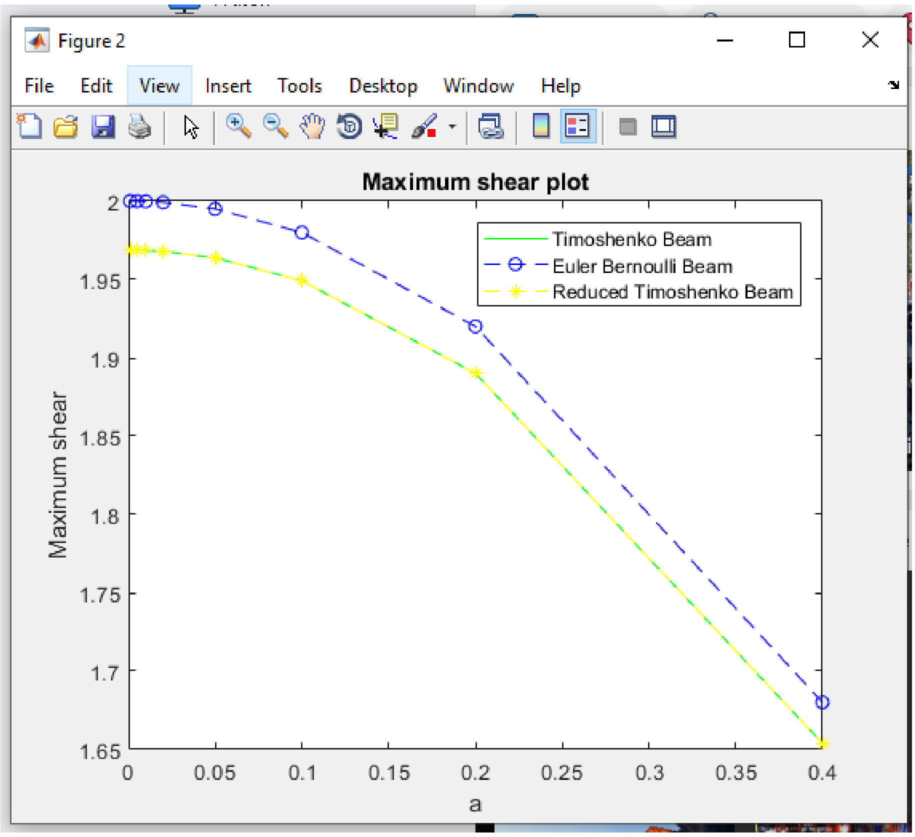


Figure 2
File Edit View Insert Tools Desktop Window Help
Maximum shear plot
Timoshenko Beam
Euler Bernoulli Beam
Reduced Timoshenko Beam
Maximum shear
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
a

Discussion

Firstly, we see that all three elements will give large displacements, larger the slenderer the beam is. Nevertheless, the Timoshenko fully integrated element gave the best results. This is because this element experiences shear locking, absorbing some of the energy that otherwise would create larger displacements. Thus, the Timoshenko fully integrated element is stiffer, and the displacements are smaller.

Secondly, we see that both Euler-Bernoulli and Timoshenko reduced integration, give very good results for max moment, considering that the exact results following beam theory would be ql^2/8=2. More accurate results are obtained for slenderer beams. When it comes to the Timoshenko fully integrated element, we see that the results are bad when it comes to maximum moments. Again, this is because of the shear locking that this element experiences. So, the fully integrated Timoshenko beam element is good at displacements but bad at moments.

Lastly, when it comes to the maximum shear force, all three elements give roughly the same results.

In general, the difference between the Timoshenko beam element, both full and reduced, and the Euler-Bernoulli element is that the Timoshenko beam elements will include the shear stiffness of the beam through the K_s matrix, while the Euler-Bernoulli elements will neglect this one. For slenderer beams this won’t matter too much, but for more massive cross sections the shear contribution is quite considerable and should probably be taken into account.